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EFFECT OF CARBON-TAX AND CAP-AND-TRADE MECHANISM ON AN INVENTORY SYSTEM WITH PRICE-SENSITIVE DEMAND AND PRESERVATION TECHNOLOGY INVESTMENT

Aditi KHANNA
Department of Operational Research, Faculty of Mathematical Sciences, New Academic Block, University of Delhi, Delhi 110007, India
dr.aditikhanna.or@gmail.com

Shikha YADAV*
Department of Operational Research, Faculty of Mathematical Sciences, New Academic Block, University of Delhi, Delhi 110007, India
shikhayadav2492@gmail.com

PRIYAMVADA
Department of Operational Research, Faculty of Mathematical Sciences, New Academic Block, University of Delhi, Delhi 110007, India
priyamvada@yahoo.com

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Abstract: In today's global decision-making context, government and organizations are highly concerned with environmental degradation caused by carbon emissions.Being environmental conscious, this paper investigates two different carbon policies viz., "Carbon tax and Cap-and-trade mechanism".It is observed that the main sources of carbon emissions are transhipments, inventory holding, inventory deterioration, and its preservation.Demand for the item is considered to be selling price dependent. Further, a comparison between a "carbon tax" and "cap-and-trade" policies has been illustrated.Some

*Corresponding Author. Tel-Fax: 91-11-27666672, email: shikhayadav2492@gmail.com

important managerial insights are obtained from numerical and sensitivity analyses. The present paper contributes to the existing literature of carbon control policies by developing optimal inventory models dealing with deteriorating items with preservation technology. Results suggest that firms should implement "Cap-and-trade" policy to increase their total profit, which at the same time, will help in reducing the carbon emissions.

Keywords: Carbon-Tax, Cap-and-Trade, Preservation Technology, Selling-price dependent demand, Deterioration.

MSC: 90B05, 13P25.

1. INTRODUCTION

The gradual contamination of the environment due to the discharge of greenhouse gases (GHGs) is a risk to all ecological systems and to human civilization (Ghosh et al. [7]). Thus, for all the firms, a stringent policy involving measures for the lessening carbon emissions is the need of the hour. With the foreseen crisis, the government has implemented several mechanisms to put control over the firms emissions to the environment, viz., "carbon emission tax, cap-and-trade, carbon offsets" etc. With such a setting in execution, a firm needs to optimize its operational decision making relevant to all the supply chain activities to cope with the government initiatives (Hua et al. [12]). Transhipments, inventory holding, and deterioration are major causes of carbon emission. Deterioration of goods is a well-established phenomenon. In normal storage scenario, an ample amount of goods deteriorate. Increasing deterioration rate decreases the profit, therefore, the deterioration effect cannot be neglected (Sarkar et al. [18]). Initially, the investigation of inventory models with deterioration started by Ghare and Schrader [16], and by Wee [20], Chaudhari and Chakrabarti [5]. In most of the practical scenarios, many practical changes and specific equipment are used to reduce and control the deterioration rate. Preservation technology is specially adopted for those products which have high deterioration rates to lessen their deterioration. Decomposition of items due to various factors viz. environment, temperature, pollution, etc. causes carbon emissions. Thus, an appropriate procedure is used to reduce deterioration by investing in different preservation technology, for eg., refrigerators, air-conditioner and drying machines, used for different products. This concept is studied by Blackburn and Scudder [4], Dye and Hsieh [6], Hsu et al. [11], Giri et al. [9], Huang et al. [13].

In a more realistic and practical system, the demand is reliant to price. Price reliant demand rate is given by Abad [1], [2], Abad and Jaggi [3], and Polatoglu and Sahin [17]. With the law of economics, a lowprice can fetch a high demand for firms. Therefore, setting an optimal price for a product in order to obtain maximum profit is one of the challenges for the firm (You [21]). Here, the demand is selling price dependent. Mo et al. [15] analyzed the best conclusions for the price changes.

Developing an inventory system to relieve the environment from carbon emissions is the major challenge in the present era. Government and non-governmental organizations have been introducing policies, rules, and regulations to mitigate the impact of carbon emissions such as the development of renewable sources, and promoting the use of natural fuels and eco-friendly objects. Two popular policies are "carbon taxes and carbon cap-and-trade" mechanism among the other carbon policies. "Under the cap-and-trade mechanism, firms initially obtain a pre-determined amount of carbon allowances (carbon quotas) from the government agencies, and the total carbon emissions generated at a certain period should be lower than the carbon quotas (He et al. [10]). Firms could buy/sell carbon allowances in the carbon trading market when they have lack/surplus allowances, where allowance price is determined by the trading market (Singh and Weninger [19])".

Initially, USA has executed a tax policy on carbon, which is imputable (Metcalf [14]). A "carbon tax" is a toll that imposes on carbon discharges; it is a form of carbon pricing. The returns made by the tax would then be applied to a payroll tax rebate of revenues to taxpayers. A carbon tax can reduce emissions very efficiently and effectively. Thus, the idea of managing supply chains with various carbon policies is an attraction to researchers worldwide. It is noted that efficiently designed carbon policy can reduce carbon emission in a supply chain to a great extent (Ghosh et al. [8]).

The present research is organized in the given manner, i.e., Section 2, 3, 4, 5, 6, and 7 present the problem definition, notations, assumptions, mathematical formulation, solution procedure, numerical and sensitivity analysis, and lastly, the conclusion and future directions.

	Sources of carbon emissions						
Author(s)	**Shipping**	**Inventory holding**	**Deterioration**	**Preservation**	**Selling price dependent demand**	**Carbon Cap-and-trade**	**Carbon tax**
Ghare and Schrader [16]			Y				
Chakrabati and Chaudhuri [5]			Y				
Polatoglu and Sahin [17]					Y		
You [21]					Y		
Blackburn and Scudder [4]			Y				
Metcalf [14]						Y	Y
Hsu et al. [11]			Y	Y			
Dye and Hsieh [6]			Y	Y			
He et al. [10]						Y	Y
Hua et al. [12]	Y	Y	Y			Y	Y
Singh and Weninger [19]						Y	
Sarkar et al. [18]			Y	Y			
Giri et al. [9]			Y	Y			
Ghosh et al. [7]	Y	Y					
Ghosh et al. [8]	Y	Y					Y
Huang et al. [13]			Y				
This paper	Y	Y	Y	Y	Y	Y	Y

Table 1: Author's contribution table

2. PROBLEM DESCRIPTION, NOTATIONS, AND ASSUMPTIONS

2.1. Problem Description

The motive of our study is to lessen the carbon emanations by implementing a "Carbon tax and Cap-and-trade mechanism". Vital elements of carbon emanations are shipments, inventory holding, deterioration, and preservation. The demand is price-sensitive in nature and unfilled demand is fully backlogged. Thus, the aim of our models is to maximize the profit by improving the selling price, inventory time, and an investment in preservation technology.

2.2. Notations

Notations are as follows:

Decision Variables

s	Selling price($/unit)
t_0	Time period during which on hand inventory is available (weeks)
α	Preservation technology investment cost ($/unit/unit time)

Constant parameters

K	Cost of Ordering per order
T	Cycle length (weeks)
C	Unit cost ($/unit)
h	Inventory carrying cost($/unit/unit time)
$y_1(\alpha)$	Deterioration rate with preservation technology(units/unit time),
y_0	Constant deterioration rate without investing in preservation technology(unit/unit time)
u	Sensitive parameter of preservation technology investment, $0 < u < 1$
$I_1(t)$	Inventory level on hand at time $t \in [0, t_0)$
$I_2(t)$	Inventory level on hand at time $t \in (t_0, T]$
Q	Order quantity in a cycle (units)
B	Number of backorders
$D(s)$	Selling price-dependent demand
C_1	Shortage cost ($/unit/unit time)
z	Emissions quota of carbon per unit time (tonnes)
C_p	Quota price of carbon ($/ per unit(tonnes))
w	Tax charged on carbon ($/per unit(ton))
TP_1	Profit per unit time with a "carbon emissions tax"
TP_2	Profit per unit time with "cap-and-trade"

2.3. Assumptions

1. The selling price dependent demand $D(s)$, is:

$$D(s) = a - bs$$

Here $a > 0$ and $0 < b < 1$ are scale and shape parameters, and both are positive known constants.

2. Infinite time horizon with zero lead-time.
3. Fully backlogged shortages are allowed.
4. Transhipment cost is $SC = V_1 + V_2Q$, where Q units is the shipment size, and V_1, V_2 are positive known constants.
5. Emissions of carbon are caused by transhipment, inventory holding, deteriorating items and their preservation.
6. For shipping an order of Q units, the carbon emission is $e_0 + e_1Q$.
7. For holding Q units, the carbon emission is $g_0 + g_1AI$.
8. Carbon dioxide emitted due to the deterioration of an item, and γ is the emissions per deteriorated item.
9. For the preservation of Q units, the carbon emission is $p_0 + p_1AI$.
10. Model assumes reduced deterioration rate $y_1(\alpha) = y_0e^{-u\alpha}$, where $0 < u < 1$ is a sensitive parameter. The following conditions $\frac{\partial y_1(\alpha)}{\partial \alpha} < 0$, $\frac{\partial^2 y_1(\alpha)}{\partial \alpha^2} > 0$ are satisfied in the relationship between $y_1(\alpha)$ and α, (where e_1, g_1, and p_1 are the variable emission factors).

3. MATHEMATICAL MODELLING

Using the given assumptions, Figure 1 shows the inventory scenario. The effect of demand and deterioration is shown in $[0, t_0)$, and the deterioration is controlled by the preservation technology. Finally, inventory reaches the zero level at $t = t_0$, and in the $(t_0, T]$, demand is fully backlogged.

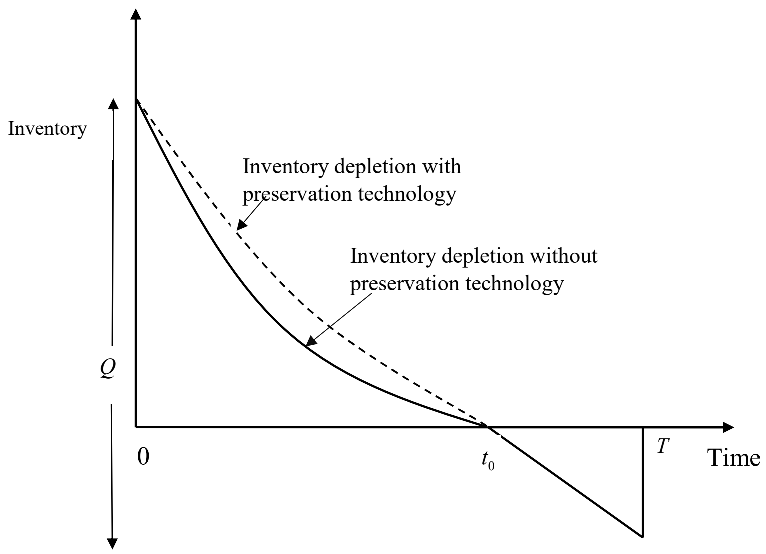

Figure 1: Graph of the inventory depletion

$$\frac{dI_1(t)}{dt} + y_1(\alpha)I_1(t) = -D(s); \quad 0 \le t \le t_0 \tag{1}$$

$$\frac{dI_2(t)}{dt} = D(s); \quad t_0 \le t \le T \tag{2}$$

Using $I_1(0) = Q$ in equations (1), and $I_2(t_0) = 0$ in equation (2), the solution is given by:

$$I_1(t) = \frac{D(s)}{y_1(\alpha)}(e^{y_1(\alpha)(t_0-t)} - 1); \quad 0 \le t \le t_0 \tag{3}$$

$$I_2(t) = D(s)(t - t_0); \quad t_0 \le t \le T \tag{4}$$

- The backorders are given by

$$B = \int_{t_0}^{T} D(s)dt = D(s)(T - t_0) \tag{5}$$

- The order quantity is given by

$$Q = I_1(0) + B = \frac{D(s)}{y_1(\alpha)}(e^{y_1(\alpha)t_0} - 1) + D(s)(T - t_0) \tag{6}$$

- The total average inventory is given by

$$AI = \int_0^{t_0} I_1(t)dt = \frac{D(s)}{(y_1(\alpha))^2}[e^{y_1(\alpha)t_0} - y_1(\alpha)t_0 - 1] \tag{7}$$

- The number of deteriorated items is given by

$$DI = \int_0^{t_0} y_0 I_1(t)dt = \frac{y_0 D(s)}{(y_1(\alpha))^2}[e^{y_1(\alpha)t_0} - y_1(\alpha)t_0 - 1] \tag{8}$$

- The carbon emission in shipping, inventory holding, deterioration and preservation of deteriorating items is as follows:
    - Carbon emission in shipping Q units is

$$e_0 + e_1\left(\frac{D(s)}{y_1(\alpha)}(e^{y_1(\alpha)t_0} - 1) + D(s)(T - t_0)\right)$$

- Carbon emission in holding Q units is

$$g_0 + g_1\left(\frac{D(s)}{(y_1(\alpha))^2}[e^{y_1(\alpha)t_0} - y_1(\alpha)t_0 - 1]\right)$$

- Carbon emission due to deteriorating items is

$$\gamma\left(\frac{y_0 D(s)}{(y_1(\alpha))^2}[e^{y_1(\alpha)t_0} - y_1(\alpha)t_0 - 1]\right)$$

- Carbon emission due to the preservation is

$$p_0 + p_1\left(\frac{D(s)}{(y_1(\alpha))^2}[e^{y_1(\alpha)t_0} - y_1(\alpha)t_0 - 1]\right)$$

Therefore, the total carbon emissions is

$$\begin{aligned} CE &= e_0 + e_1 Q(s,t_0,\alpha) + g_0 + g_1 AI(s,t_0,\alpha) + p_0 \\ &\quad + p_1 AI(s,t_0,\alpha) + \gamma DI(s,t_0,\alpha) \\ &= e_0 + e_1 D(s)T + \frac{e_1 D(s) y_1(\alpha) t_0^2}{2} \\ &\quad + g_0 + \frac{g_1 D(s) t_0^2}{2} + p_0 + \frac{p_1 D(s) t_0^2}{2} + \frac{\gamma y_0 D(s) t_0^2}{2} \end{aligned} \tag{9}$$

Components of the profit function are:

- Ordering cost

$$OC = K \tag{10}$$

- Holding cost

$$\begin{aligned} HC &= h \times C \times (AI) \\ &= h \times C \frac{D(s)}{(y_1(\alpha))^2}[e^{y_1(\alpha)t_0} - y_1(\alpha)t_0 - 1] \end{aligned} \tag{11}$$

- Shortage cost

$$\begin{aligned} BC &= C_1 \int_{t_0}^{T} I_2(t)dt \\ &= \frac{C_1}{2} D(s)(T - t_0)^2 \end{aligned} \tag{12}$$

- Purchase cost

$$\begin{aligned} PC &= C \times Q \\ &= \frac{CD(s)}{(y_1(\alpha))}(e^{y_1(\alpha)t_0} - 1) + CD(s)(T - t_0) \end{aligned} \tag{13}$$

- Shipping cost

$$\begin{aligned} SC &= V_1 + V_2 Q \\ &= V_1 + V_2\left(\frac{D(s)}{(y_1(\alpha))}(e^{y_1(\alpha)t_0} - 1) + D(s)(T - t_0)\right) \end{aligned} \tag{14}$$

- Preservation investment cost

$$\Pr C = \alpha \times T \tag{15}$$

All the costs in equations (10, 11, 12, 13, 14, 15)are the cost per cycle

- Carbon emission cost in shipping, inventory holding, deteriorating items, and preservation of deteriorating items under carbon emission tax is

$$Tax^c = w(CE)$$

Using (CE) from equation (9)

$$\begin{aligned} &= w\left(e_0 + e_1 D(s)T + \frac{e_1 D(s) y_1(\alpha) t_0^2}{2} + g_0 + \frac{g_1 D(s) t_0^2}{2}\right. \\ &\qquad \left. + p_0 + \frac{p_1 D(s) t_0^2}{2} + \frac{\gamma y_0 D(s) t_0^2}{2}\right) \end{aligned} \tag{16}$$

- Carbon emission cost in transshipment, inventory holding, deteriorating items, and preservation under Cap-and-trade mechanism is

$$Cap^c = C_p(CE - z)$$

Using (CE) from equation (9)

$$\begin{aligned} &= C_p\left\{\left(e_0 + e_1 D(s)T + \frac{e_1 D(s) y_1(\alpha) t_0^2}{2} + g_0 + \frac{g_1 D(s) t_0^2}{2}\right.\right. \\ &\qquad \left.\left. + p_0 + \frac{p_1 D(s) t_0^2}{2} + \frac{\gamma y_0 D(s) t_0^2}{2}\right) - z\right\} \end{aligned} \tag{17}$$

- Sales Revenue per cycle is

$$R = s \times D(s) \times T \tag{18}$$

Under the carbon emissions, the present paper investigates two scenarios, i.e., Carbon “cap-and-trade and Carbon tax”.

Case1:“Carbon tax case”:
Profit function of “carbon tax” is given as “(sales revenue − ordering cost −

holding cost − backorder cost − purchase cost − shortage cost − preservation cost − tax cost)”

$$
\begin{aligned}
TP_1(s,t_0,\alpha) &= \frac{1}{T}(R - OC - HC - BC - PC - SC - \Pr C - Tax^c)\\
&= \frac{1}{T}\Bigg\{sD(s)T - K - \frac{hCD(s)t_0^2}{2} - \frac{C_1D(s)(T-t_0)^2}{2}\\
&\quad - CD(s)T - \frac{CD(s)(y_1(\alpha))t_0^2}{2} - V_1 - V_2D(s)T\\
&\quad - \frac{V_2D(s)y_1(\alpha)t_0^2}{2} - \alpha T - we_0 - we_1D(s)T\\
&\quad - \frac{we_1D(s)y_1(\alpha)t_0^2}{2} - wg_0 - wp_0 - \frac{wg_1D(s)t_0^2}{2}\\
&\quad - \frac{wp_1D(s)t_0^2}{2} - \frac{w\gamma y_0D(s)t_0^2}{2}\Bigg\}
\end{aligned}
\tag{19}
$$

Case 2:“Cap-and-trade case”:
Profit function of “cap-and-trade” is given as “(sales revenue − ordering cost − holding cost − backorder cost − purchase cost − shortage cost − preservation cost − cap-and-trade cost)”

$$
\begin{aligned}
TP_2(s,t_0,\alpha) &= \frac{1}{T}(R - OC - HC - BC - PC - SC - \Pr C - Cap^c)\\
&= \frac{1}{T}\Bigg\{sD(s)T - K - \frac{hCD(s)t_0^2}{2} - \frac{C_1D(s)(T-t_0)^2}{2}\\
&\quad - CD(s)T - \frac{CD(s)(y_1(\alpha))t_0^2}{2} - V_1 - V_2D(s)T\\
&\quad - \frac{V_2D(s)y_1(\alpha)t_0^2}{2} - \alpha T - c_pe_0 - c_pe_1D(s)T\\
&\quad - \frac{c_pe_1D(s)y_1(\alpha)t_0^2}{2} - c_pg_0 - c_pp_0 - \frac{c_pg_1D(s)t_0^2}{2}\\
&\quad - \frac{c_pp_1D(s)t_0^2}{2} - \frac{c_p\gamma y_0D(s)t_0^2}{2} + c_pz\Bigg\}
\end{aligned}
\tag{20}
$$

4. SOLUTION PROCEDURE

The optimality of an inventory cycle lenght (t_0), investment in preservation technology (α), and selling price (p) are given below:

Case 1: “Carbon tax”
Now, the necessary condition, which should be satisfied for the optimality of the profit function, is $\frac{\partial TP_1(s,t_0,\alpha)}{\partial s} = 0$, $\frac{\partial TP_1(s,t_0,\alpha)}{\partial t_0} = 0$ and $\frac{\partial TP_1(s,t_0,\alpha)}{\partial \alpha} = 0$

$$
\frac{\partial TP_1(s,t_0,\alpha)}{\partial s} = \frac{1}{T}\Bigg\{aT - 2bsT + \frac{bhCt_0^2}{2} + \frac{bC_1(T-t_0)^2}{2} + bCT
$$

$$+\frac{bV_2y_1(\alpha)t_0^2}{2}+we_1bT+\frac{bCy_1(\alpha)t_0^2}{2}+bV_2T$$
$$+\frac{we_1by_1(\alpha)t_0^2}{2}+\frac{wg_1bt_0^2}{2}+\frac{wp_1bt_0^2}{2}+\frac{w\gamma y_0bt_0^2}{2}\Bigg\} \tag{21}$$

The optimal value of s is given as s^*:

$$s^*=\frac{1}{2bT}\Bigg\{aT+\frac{bhCt_0^2}{2}+\frac{bC_1(T-t_0)^2}{2}+bCT+\frac{bCy_1(\alpha)t_0^2}{2}+bV_2T$$
$$+\frac{bV_2y_1(\alpha)t_0^2}{2}+we_1bT+\frac{we_1by_1(\alpha)t_0^2}{2}+\frac{wg_1bt_0^2}{2}+\frac{wp_1bt_0^2}{2}$$
$$+\frac{w\gamma y_0bt_0^2}{2}\Bigg\} \tag{22}$$

$$\frac{\partial TP_1(s,t_0,\alpha)}{\partial t_0}=\frac{1}{T}\{-hCD(s)t_0+C_1D(s)(T-t_0)-CD(s)(y_1(\alpha))t_0$$
$$-V_2D(s)(y_1(\alpha))t_0-we_1D(s)(y_1(\alpha))t_0-wg_1D(s)t_0$$
$$-wp_1D(s)t_0-w\gamma y_0D(s)t_0\} \tag{23}$$

The optimal value of t_0 is given as t_0^*:

$$t_0^*=\frac{C_1T}{\{hC+C_1+C(y_1(\alpha))+V_2(y_1(\alpha))+we_1(y_1(\alpha))+wg_1+wp_1+w\gamma y_0\}} \tag{24}$$

$$\frac{\partial TP_1(s,t_0,\alpha)}{\partial\alpha}=\left(\frac{CD(s)t_0^2+V_2D(s)t_0^2+we_1D(s)t_0^2}{2T}\right)uy_0e^{-u\alpha}-T \tag{25}$$

The optimal value of α is given as α^*:

$$\alpha^*=\frac{-1}{u}\log\left\{\frac{2T^2}{uy_0(CD(s)t_0^2+V_2D(s)t_0^2+we_1D(s)t_0^2)}\right\} \tag{26}$$

For the sufficient condition of optimality, refer to Appendix A1.

Case 2: "Carbon cap-and-trade"

Now, the necessary condition, which should be satisfied for the optimality of the profit function, is $\frac{\partial TP_2(s,t_0,\alpha)}{\partial s}=0$, $\frac{\partial TP_2(s,t_0,\alpha)}{\partial t_0}=0$ and $\frac{\partial TP_2(s,t_0,\alpha)}{\partial\alpha}=0$

$$\frac{\partial TP_2(s,t_0,\alpha)}{\partial s}=\frac{1}{T}\Bigg\{aT-2bsT+\frac{hCbt_0^2}{2}+\frac{C_1b(T-t_0)^2}{2}+bCT$$
$$+\frac{bC(y_1(\alpha))t_0^2}{2}+bV_2T+\frac{bV_2y_1(\alpha)t_0^2}{2}+bc_pe_1T$$
$$+\frac{bc_pe_1y_1(\alpha)t_0^2}{2}+\frac{bc_pg_1t_0^2}{2}+\frac{bc_pp_1t_0^2}{2}+\frac{bc_p\gamma y_0t_0^2}{2}\Bigg\} \tag{27}$$

The optimal value of s is given as s^*:

$$s^* = \frac{1}{2bT}\left\{aT + \frac{hCbt_0^2}{2} + \frac{C_1 b(T-t_0)^2}{2} + bCT + \frac{bC(y_1(\alpha))t_0^2}{2} + bV_2T + \frac{bV_2y_1(\alpha)t_0^2}{2} + bc_pe_1T + \frac{bc_pe_1y_1(\alpha)t_0^2}{2} + \frac{bc_pg_1t_0^2}{2} + \frac{bc_pp_1t_0^2}{2} + \frac{bc_p\gamma y_0t_0^2}{2}\right\} \tag{28}$$

$$\frac{\partial TP_2(s,t_0,\alpha)}{\partial t_0} = \frac{1}{T}\{-hCD(s)t_0 + C_1D(s)(T-t_0) - CD(s)(y_1(\alpha))t_0 - V_2D(s)(y_1(\alpha))t_0 - c_pe_1D(s)(y_1(\alpha))t_0 - c_pg_1D(s)t_0 - c_pp_1D(s)t_0 - c_p\gamma y_0D(s)t_0\} \tag{29}$$

The optimal value of t_0 is given as t_0^*:

$$t_0^* = \frac{C_1T}{\{hC + C_1 + C(y_1(\alpha)) + V_2(y_1(\alpha)) + c_pe_1(y_1(\alpha)) + c_pg_1 + c_pp_1 + c_p\gamma y_0\}} \tag{30}$$

$$\frac{\partial TP_2(s,t_0,\alpha)}{\partial \alpha} = \left(\frac{CD(s)t_0^2 + V_2D(s)t_0^2 + c_pe_1D(s)t_0^2}{2T}\right)uy_0e^{-u\alpha} - T \tag{31}$$

The optimal value of α is given as α^*:

$$\alpha^* = \frac{-1}{u}\log\left\{\frac{2T^2}{uy_0(CD(s)t_0^2 + V_2D(s)t_0^2 + c_pe_1D(s)t_0^2)}\right\} \tag{32}$$

For the sufficient condition of optimality, refer to Appendix A2.

Further, Figures 2-4 display optimal values of case 1.

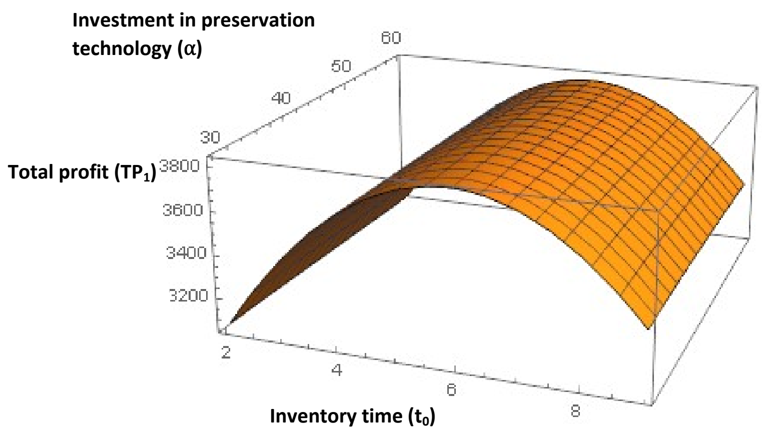

Figure 2: Concavity of (TP_1) w.r.t. (t_0) and (α).

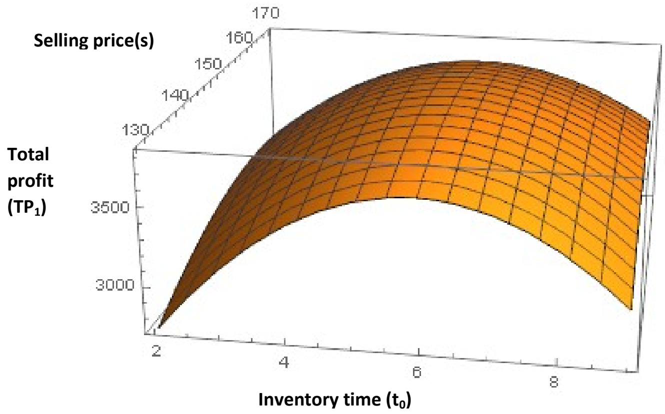

Figure 3: Concavity of (TP_1) w.r.t. (t_0) and (s).

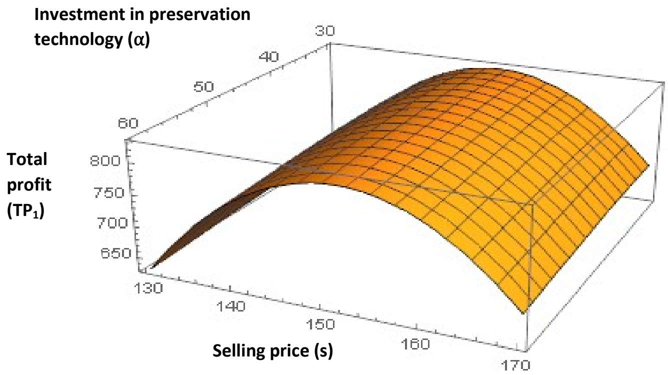

Figure 4: Concavity of (TP_1) w.r.t. (α) and (s).

5. NUMERICAL ANALYSIS

Consider an inventory system dealing with perishable products (for e.g., frozen food in departmental store), where special handling is needed in storage, by considering the shelf life and storage time to prevent damage and decay. Frozen food require freezers as the storage media and airtight packaging to maintain the product's durability. Freezing and air-conditioning are the essential procedures for the frozen food business, which can be considered as the preservation technology investment for a store. Freezing slowdowns microbial activity and nutritional loss, which makes the freight smooth and flexible so to easily access the remote areas. Moreover, demand of such products is also sensitive to price. Hence, setting the optimum price is a crucial aspect of an inventory system.

Let us consider the following data for such a system.

Example 1 (Carbon tax case). The following parameter values taken ,in appropriate units, for the numerical illustration

$K = \$80$/order	$a = 100$	$C_1 = \$15$/unit/unit time	$e_1 = 1$ tonn/unit	$p_1 = 1$ tonn/unit
$C = \$20$/unit	$b = 0.4$	$V_1 = \$10$/unit/unit time	$g_0 = 5$ tonnes/unit	$\gamma = 0.2$ tonns/unit
$h = \$0.3$/unit/unit time	$y_0 = 0.2$ units/unit time	$V_2 = \$0.1$/unit/unit time	$g_1 = 1$ ton/unit	$w = \$2$/ton
$T = 10$ weeks	$u = 0.1$	$e_0 = 10$ tonnes/unit	$p_0 = 8$ tonnes/unit	

The optimal results are:

Total profit $TP_1 = \$3846.303$	Selling price $s = \$151.2486$/unit
Investment in preservation technology $\alpha = \$40.8238$/unit/unit time	Inventory time $t_0 = 5.938$ weeks

Example 2 (Carbon cap-and-trade case). The following parameter values taken, in appropriate units, for the numerical illustration

$K = \$80$/order	$a = 100$	$C_1 = \$15$/unit	$e_1 = 1$ tonn/unit	$p_1 = 1$ tonn/unit
$C = \$20$/unit	$b = 0.4$	$V_1 = \$10$/unit/unit time	$g_0 = 5$ tonnes/unit	$\gamma = 0.2$ tonns/unit
$h = \$0.3$/unit/unit time	$y_0 = 0.2$ units/unit time	$V_2 = \$0.1$/unit/unit time	$g_1 = 1$ ton/unit	$z = 60$ tonnes
$T = 10$ weeks	$u = 0.1$	$e_0 = 10$ tonnes/unit	$p_0 = 8$ tonnes/unit	$C_p = \$1$/tonn

The optimal results are:

Total profit $TP_2 = \$4052.66$	Selling price $s = \$148.7595$/unit
Investment in preservation technology $\alpha = \$41.884$/unit/unit time	Inventory time $t_0 = 6.469$ weeks

From the above results, one can observe that the total profit is higher in "carbon cap-and-trade" case than that in "carbon tax" case. Hence, the decision maker should implement "carbon cap-and-trade" policy for more profit.

6. SENSITIVITY ANALYSIS

Sensitivity analysis on the carbon "cap-and-trade"case with several parameters, such as (a, b, c_p, HC, u, y_0).

Parameters	%change	%change α	%change t_0	%change s	%change Q	%change TP_2
a	−50	−23. 210	−1. 022	−41. 878	−61. 543	−86. 265
	−25	−8. 872	−0. 280	−20. 969	−30. 726	−52. 799
	+25	6. 449	0. 147	20. 987	30. 700	72. 101
	+50	11. 519	0. 239	41. 982	61. 389	163. 494
b	−50	2. 659	0. 065	84. 019	11. 666	151. 435
	−25	1. 367	0. 034	28. 004	5. 834	50. 014
	+25	−1. 450	−0. 039	−16. 800	−5. 839	−29. 450
	+50	−2. 997	−0. 083	−27. 998	−11. 684	−48. 618
C_p	−50	1. 234	4. 673	−0. 929	1. 391	2. 723
	−25	0. 622	2. 283	−0. 456	0. 682	1. 331
	+25	−0. 627	−2. 185	0. 439	−0. 657	−1. 274
	+50	−1. 260	−4. 277	0. 864	−1. 290	−2. 496
HC	−50	4. 355	15. 105	−2. 459	3. 682	7. 400
	−25	2. 200	7. 026	−1. 144	1. 710	3. 407
	+25	−2. 197	−6. 167	1. 004	−1. 497	−2. 941
	+50	−4. 365	−11. 624	1. 893	−2. 820	−5. 505
u	−50	66. 660	−0. 629	0. 083	–	−0. 937
	−25	24. 122	−0. 209	0. 027	–	−0. 332
	+25	−15. 718	0. 125	−0. 016	–	0. 212
	+50	−26. 853	0. 208	−0. 027	–	0. 360
y_0	−50	−16. 519	0. 087	−0. 014	0. 021	0. 213
	−25	−6. 853	0. 043	−0. 007	0. 0107	0. 092
	+25	5. 313	−0. 043	0. 007	−0. 010	−0. 076
	+50	9. 651	−0. 087	0. 014	−0. 021	−0. 142

Table 2: Sensitivity analysis of the key parameters

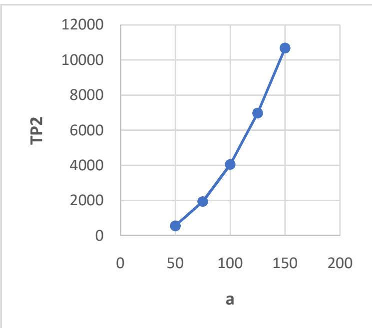

Figure 5: Effect of demand parameter (a) w.r.t. Total profit (TP_2)

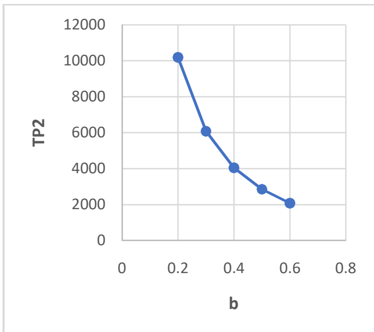

Figure 6: Effect of demand parameter (b) w.r.t. Total profit (TP_2)

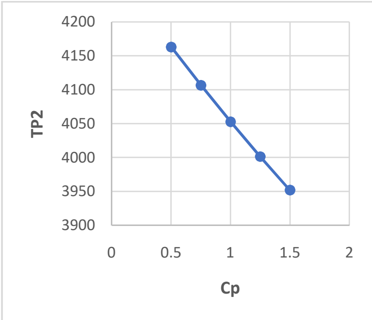

Figure 7: Effect of the carbon price (c_p) w.r.t. Total profit (TP_2) preservation technology (u)

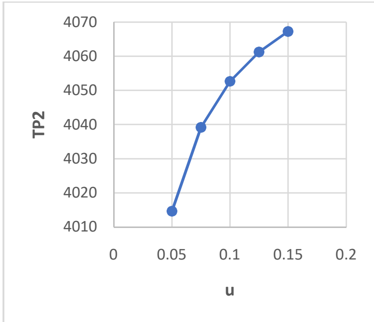

Figure 8: Effect of the effectiveness of the w.r.t. Total profit (TP_2)

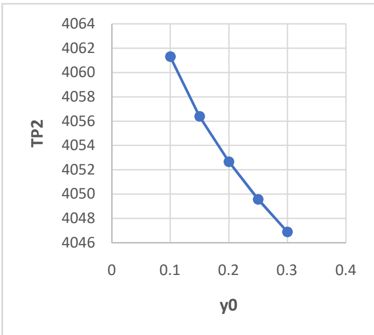

Figure 9: Effect of initial deterioration rate (y_0) w.r.t. Total profit (TP_2)

Observations and Managerial insights

From Table 2, following observations and managerial insights are made:

- With an increase in demand parameter, (a), one can boost the demand, and hence the order size increases. Larger order size also requires extra investment in preservation technology, thus, there is a slight increase in investment. However, with increasing demand, the sales also increase, which leads to high profit.
- When the price-sensitive parameter (b) of demand rises, the total profit and order quantity decrease as the parameter has an adverse effect on demand. Thus, the demand tends to decrease, and hence, the order quantity decreases. Here, it is suggested to decrease the price of the item so as to boost the declining demand, and hence to manage profit.

- An increase in carbon price (c_p) contributes to the increas in the total cost component, which, obviously, effects the total profit. Here, it is suggested to slightly increase the selling price to manage a declining profit. Moreover, the decision-maker should decrease the order size as the carbon price per unit increases.
- A higher holding cost (HC) indicates improved storage condition, and hence, one can decrease the investment in preservation technology. However, the cost increases, which decreases the total profit. Here, it is recommended to order small lots in order to manage the inventory effectively.
- With an increase in the effectiveness parameter (u), it is suggested to invest less in the preservation technology as the increase in effectiveness parameter implies better preservation condition and thus, investment decreases and at the same the profit increases. Under such a condition it is suggested to decrease the selling price so as to fetch more demand and increase the sales value, which will eventually lead to higher profit.
- A higher deterioration rate (y_0) recommended means that the investment in preservation technology should be increased to diminish the deterioration rate. It is advisable to reduce the order size so as to deal with the deterioration effectively. In this case, the selling price may be increased so as to manage declining profit.

7. CONCLUSION

A profit-maximization model with price-sensitive demand under carbon emissions has been developed. Two different scenarios (models) are proposed considering carbon policies viz., “Carbon tax” and “Cap-and-trade” mechanism for deteriorating items, where deterioration rate can be controlled by adopting preservation technology. Some important managerial insights are obtained from numerical and sensitivity analysis. Results show that “cap-and-trade” policy would be better for the decision-maker as it results in higher profit. Moreover, the deterioration rate is minimized by preservation technology, which in turn, helps lowering total carbon emissions, which is being exhibited by numerical results. Further, sensitivity analysis provides some useful insights: it is noted that the profit function rises with the increase in the demand parameter (a) and the effectiveness parameter of the preservation technology (u). With increasing carbon price (c_p) per unit, the total profit decreases so as the order size. For higher holding cost (HC) and deterioration rate (y_0), it is preferable to order small lots in order to manage the inventory effectively.

For future study, concepts like a vendor-buyer model, multiple-shipments, etc could be included, and extended by taking freshness-dependent, time-dependent, storage cost dependent demand, etc.

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Appendix A1.

$$\begin{aligned}
\frac{\partial^2 TP_1(s,t_0,\alpha)}{\partial s^2} &= \frac{1}{T}(-2bT)\\
&= -2b < 0\\
&\qquad \frac{\partial^2 TP_1(s,t_0,\alpha)}{\partial s^2} < 0 \quad \text{i.e. maxima}\\
\frac{\partial^2 TP_1(s,t_0,\alpha)}{\partial t_0^2} &= -\frac{1}{T}\{hCD(s) + C_1D(s) + CD(s)(y_1(\alpha)) + V_2D(s)(y_1(\alpha))\\
&\quad + we_1D(s)(y_1(\alpha)) + wg_1D(s) + wp_1D(s) + w\gamma y_0D(s)\}\\
&\qquad \frac{\partial^2 TP_1(s,t_0,\alpha)}{\partial t_0^2} < 0 \quad \text{i.e. maxima}\\
\frac{\partial^2 TP_1(s,t_0,\alpha)}{\partial \alpha^2} &= \left(\frac{CD(s)t_0^2 + V_2D(s)t_0^2 + we_1D(s)t_0^2}{2T}\right)(-u^2y_0e^{-u\alpha})\\
&\qquad \frac{\partial^2 TP_1(s,t_0,\alpha)}{\partial \alpha^2} < 0 \quad \text{i.e. maxima}
\end{aligned}$$

Appendix A2.

$$\begin{aligned}
\frac{\partial^2 TP_2(s,t_0,\alpha)}{\partial s^2} &= \frac{1}{T}(-2bT)\\
&= -2b < 0\\
&\qquad \frac{\partial^2 TP_2(s,t_0,\alpha)}{\partial s^2} < 0 \quad \text{i.e. maxima}\\
\frac{\partial^2 TP_2(s,t_0,\alpha)}{\partial t_0^2} &= -\frac{1}{T}\{hCD(s) + C_1D(s) + CD(s)(y_1(\alpha)) + V_2D(s)(y_1(\alpha))\\
&\quad + c_pe_1D(s)(y_1(\alpha)) + c_pg_1D(s) + c_pp_1D(s) + c_p\gamma y_0D(s)\}\\
&\qquad \frac{\partial^2 TP_2(s,t_0,\alpha)}{\partial t_0^2} < 0 \quad \text{i.e. maxima}\\
\frac{\partial^2 TP_2(s,t_0,\alpha)}{\partial \alpha^2} &= \left(\frac{CD(s)t_0^2 + V_2D(s)t_0^2 + c_pe_1D(s)t_0^2}{2T}\right)(-u^2y_0e^{-u\alpha})\\
&\qquad \frac{\partial^2 TP_2(s,t_0,\alpha)}{\partial \alpha^2} < 0 \quad \text{i.e. maxima}
\end{aligned}$$

Appendix A3.

For optimality of three variables, hessian matrix is given as

$$H = \begin{bmatrix} \frac{\partial^2 TP_1(s,t_0,\alpha)}{\partial \alpha^2} & \frac{\partial^2 TP_1(s,t_0,\alpha)}{\partial \alpha \partial t_0} & \frac{\partial^2 TP_1(s,t_0,\alpha)}{\partial \alpha \partial s} \\ \frac{\partial^2 TP_1(s,t_0,\alpha)}{\partial t_0 \partial \alpha} & \frac{\partial^2 TP_1(s,t_0,\alpha)}{\partial t_0^2} & \frac{\partial^2 TP_1(s,t_0,\alpha)}{\partial t_0 \partial s} \\ \frac{\partial^2 TP_1(s,t_0,\alpha)}{\partial s \partial \alpha} & \frac{\partial^2 TP_1(s,t_0,\alpha)}{\partial s \partial t_0} & \frac{\partial^2 TP_1(s,t_0,\alpha)}{\partial s^2} \end{bmatrix}$$

$$\frac{\partial^2 TP_1(s,t_0,\alpha)}{\partial \alpha^2} = \left(\frac{CD(s)t_0^2 + V_2 D(s)t_0^2 + we_1 D(s)t_0^2}{2T}\right)(-u^2 y_0 e^{-u\alpha})$$

$$\begin{aligned}\frac{\partial^2 TP_1(s,t_0,\alpha)}{\partial t_0^2} &= -\frac{1}{T}\{hCD(s) + C_1 D(s) + CD(s)(y_1(\alpha)) \\ &\quad + V_2 D(s)(y_1(\alpha)) + we_1 D(s)(y_1(\alpha)) + wg_1 D(s) \\ &\quad + wp_1 D(s) + w\gamma y_0 D(s)\}\end{aligned}$$

$$\frac{\partial^2 TP_1(s,t_0,\alpha)}{\partial s^2} = -2b$$

$$\begin{aligned}\frac{\partial^2 TP_1(s,t_0,\alpha)}{\partial \alpha \partial t_0} &= \frac{\partial^2 TP_1(s,t_0,\alpha)}{\partial t_0 \partial \alpha} \\ &= \left(\frac{CD(s)t_0 + V_2 D(s)t_0 + we_1 D(s)t_0}{T}\right) u y_0 e^{-u\alpha}\end{aligned}$$

$$\begin{aligned}\frac{\partial^2 TP_1(s,t_0,\alpha)}{\partial t_0 \partial s} &= \frac{\partial^2 TP_1(s,t_0,\alpha)}{\partial s \partial t_0} \\ &= \frac{1}{T}\{hCbt_0 - C_1 b(T - t_0) + Cb(y_1(\alpha))t_0 + bV_2(y_1(\alpha))t_0 \\ &\quad + we_1 b(y_1(\alpha))t_0 + wg_1 bt_0 + wp_1 bt_0 + w\gamma y_0 t_0\}\end{aligned}$$

$$\frac{\partial^2 TP_1(s,t_0,\alpha)}{\partial \alpha \partial s} = \frac{\partial^2 TP_1(s,t_0,\alpha)}{\partial s \partial \alpha} = -\left(\frac{Cbt_0^2 + V_2 bt_0^2 + we_1 bt_0^2}{2T}\right)(u y_0 e^{-u\alpha})$$

Sufficient conditions for optimality Hessian matrix H, where H_1, H_2, and H_3 are as follows:

$$H_1 = \left[\frac{\partial^2 TP_1(s,t_0,\alpha)}{\partial \alpha^2}\right]$$

$$Det(H_1) = -\left(\frac{CD(s)t_0^2 + V_2 D(s)t_0^2 + we_1 D(s)t_0^2}{2T}\right)(u^2 y_0 e^{-u\alpha}) < 0$$

$$H_2 = \begin{bmatrix} \frac{\partial^2 TP_1(s,t_0,\alpha)}{\partial \alpha^2} & \frac{\partial^2 TP_1(s,t_0,\alpha)}{\partial \alpha \partial t_0} \\ \frac{\partial^2 TP_1(s,t_0,\alpha)}{\partial t_0 \partial \alpha} & \frac{\partial^2 TP_1(s,t_0,\alpha)}{\partial t_0^2} \end{bmatrix}$$

$$X1 = \frac{\partial^2 TP_1(s,t_0,\alpha)}{\partial \alpha^2} = -\left(\frac{CD(s)t_0^2 + V_2D(s)t_0^2 + we_1D(s)t_0^2}{2T}\right)(u^2 y_0 e^{-u\alpha}) < 0$$

$$X2 = \frac{\partial^2 TP_1(s,t_0,\alpha)}{\partial t_0^2}) = -\frac{1}{T}\{hCD(s) + C_1D(s) + CD(s)(y_1(\alpha)) + V_2D(s)(y_1(\alpha)) + we_1D(s)(y_1(\alpha)) + wg_1D(s) + wp_1D(s) + w\gamma y_0 D(s)\} < 0$$

$$X3 = \frac{\partial^2 TP_1(s,t_0,\alpha)}{\partial \alpha \partial t_0} = \frac{\partial^2 TP_1(s,t_0,\alpha)}{\partial t_0 \partial \alpha} = \left(\frac{CD(s)t_0 + V_2D(s)t_0 + we_1D(s)t_0}{T}\right) u y_0 e^{-u\alpha} > 0$$

$$Det(H_2) = \{(X1 \times X2) - (X3)^2\} > 0$$

$$H_3 = \begin{bmatrix} \frac{\partial^2 TP_1(s,t_0,\alpha)}{\partial \alpha^2} & \frac{\partial^2 TP_1(s,t_0,\alpha)}{\partial \alpha \partial t_0} & \frac{\partial^2 TP_1(s,t_0,\alpha)}{\partial \alpha \partial s} \\ \frac{\partial^2 TP_1(s,t_0,\alpha)}{\partial t_0 \partial \alpha} & \frac{\partial^2 TP_1(s,t_0,\alpha)}{\partial t_0^2} & \frac{\partial^2 TP_1(s,t_0,\alpha)}{\partial t_0 \partial s} \\ \frac{\partial^2 TP_1(s,t_0,\alpha)}{\partial s \partial \alpha} & \frac{\partial^2 TP_1(s,t_0,\alpha)}{\partial s \partial t_0} & \frac{\partial^2 TP_1(s,t_0,\alpha)}{\partial s^2} \end{bmatrix}$$

$$Y1 = \frac{\partial^2 TP_1(s,t_0,\alpha)}{\partial s^2} = -2b < 0$$

$$Y2 = \frac{\partial^2 TP_1(s,t_0,\alpha)}{\partial t_0 \partial s} = \frac{\partial^2 TP_1(s,t_0,\alpha)}{\partial s \partial t_0} = \frac{1}{T}\{hCbt_0 - C_1b(T - t_0) + Cb(y_1(\alpha))t_0 + bV_2(y_1(\alpha))t_0 + we_1b(y_1(\alpha))t_0 + wg_1bt_0 + wp_1bt_0 + w\gamma y_0 t_0\} > 0$$

$$Y3 = \frac{\partial^2 TP_1(s,t_0,\alpha)}{\partial s \partial \alpha} = \frac{\partial^2 TP_1(s,t_0,\alpha)}{\partial \alpha \partial s} = -\left(\frac{Cbt_0^2 + V_2bt_0^2 + we_1bt_0^2}{2T}\right)(u y_0 e^{-u\alpha}) < 0$$

$$Det(H_3) = \{X1((X2 \times Y1) - (Y2)^2) - X3((X3 \times Y1) - (Y2 \times Y3)) + Y3((X3 \times Y2) - (Y3 \times X2))\} < 0$$

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