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A JOINT INVENTORY MODEL WITH RELIABILITY, CARBON EMISSION, AND INSPECTION ERRORS IN A DEFECTIVE PRODUCTION SYSTEM

Isha SANGAL
Department of Mathematics & Statistics, Banasthali Vidyapith, Rajasthan, 304 022, India
isha.sangal@gmail.com

Bijoy Kumar SHAW
Department of Mathematics & Statistics, Banasthali Vidyapith, Rajasthan, 304 022, India
bkshaw.math@gmail.com

Biswajit SARKAR*
Department of Industrial Engineering, Yonsei University, 50 Yonsei-ro, Sinchon-dong, Seodaemun-gu, Seoul-03 722, South Korea
bsbiswajitsarkar@gmail.com

Rekha GUCHHAIT
School of Mechanical, Aerospace, and Nuclear Engineering, Ulsan National Institute of Science and Technology, Ulsan 44 919, South Korea.
rg.rekhaguchhait@gmail.com

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Abstract: Nowadays, environment is an important concern of industries parallel to the economy. In this direction, a joint vendor-buyer model is exhibited where the system reliability and inspection errors are discussed along with the carbon emission issue. The main goal of this model is to obtain the optimum investment, shipment size, reliability and lead time even the inspection errors present in the system. A reliability dependent unit production cost is utilized to raise the machinery system reliability. Transportation of products use the single-setup-multi-unequal-delivery (SSMUD) policy to reduce carbon emission. Mathematical problem is solved analytically and a quasi-closed-form solution is found. Total cost is minimized with the optimum level of decision variables. Globality of the decisions is proved by Hessian matrix. Results demonstrate that the total cost is minimized even though the optimum solutions are obtained in quasi-closed-form. Numerical example is elaborated to test the validity of the model and to clarify the comparison among SSSD, SSMD, and SSMUD policies.

Keywords: Joint Inventory Model, Imperfect Production, Unequal Shipment Size, Reliability, Carbon Emission.

MSC: 90B05, 90B50.

1. INTRODUCTION

Everyday life can not be imagined without technology, from home to industry, but its side effect is great amount of carbon emitted in nature. TThough, it can be reduced by using the advanced technology in manufacturing system, at the same time producing good quality products. Recent studies show that carbon emission reduction is one of the main aims of industries besides the profit or loss. Sarkar *et al.* [17] studied the carbon emission with variable setup cost, and again, Sarkar *et al.* [18] discussed it within the three-echelon model.

The make-to-order (MTO) policy is a production policy where items are produced while an order is received, i.e., producer starts the production process only after receiving the order of buyers. The elementary economic production quantity (EPQ) model along MTO strategy introduced by Hadley and Whitin [6], and Silver *et al.* [28]. Finite production quantity and demand rates were considered in both of the models. An unlimited production rate was considered in the model of Goyal [4]. Banerjee [1] extended that model by using a lot-for-lot production/delivery policy. To avoid shortages, Sarker and Parija [24] introduced the periodic ordering policy. The vendor-buyer model with deterioration and transportation were established by Yan *et al.* [29]. Sarkar [15] extended this model by introducing classical optimizing method for finding the optimal results.

Transportation of items is a leading issue between the players involved in the supply chain management. Three transportation policies are used: single-setup-single-delivery (SSSD), single-setup-multi-delivery (SSMD), and multi-setup-multi-delivery (MSMD). In the SSSD policy, all items are manufactured at a single setup and delivered to the customer at a single shipment.In the SSMD policy, though all items are manufactured in a single setup, they are delivered in multiple deliveries. In MSMD policy, the manufacturer uses multiple setup system for production and delivers the products in multiple shipments. Nowadays, the global supply chain uses SSMD policy instead of SSSD and MSMD for transporting the products. By introducing SSMD policy, Goyal [5] expanded the Banerjee's [1] model. The delivery cost is usually provided by the vendor and can be fixed as well as the variables. The distance and capacity of the container dependent transportation cost was discussed by Sarkar *et al.* [20]. In general, the shipment size is always equal for the whole cycle length, but Hill [7] considered the unequal shipment size, and recently, Ganguly *et al.* [3] did the same in details with consideration of environmental issues. The SSMD policy with unequal delivery size, i.e., single-setup-multi-unequal-delivery (SSMUD) policy was illustrated by them.

As a result of long-run manufacturing process or the machinery problem, the machine can go *in-control* state to *out-of-control* state. In *in-control* state, most machines manufacture good quality products, but in *out-of-control* state some machines manufacture defect products. Thus, there should be a technique (or separation process) to identify perfect and imperfect products. This technique is known as inspection process and is essential for the brand image of a company. The vendor-buyer model with imperfect products was discussed by Ouyang *et al.* [12] and Huang [8]. The perfect products go to the market and the defective ones are returned to the manufacturer for modification. And finally, the buyers receive the perfect products, and defective products may be disposed or sell out with some discount. In the paper of Lee and Fu [10], the production rate was fixed but the delivery quantity was periodic. This study used the SSMD policy with variable transportation cost but they did not use the inspection concept. By introducing two-stage inspection process, Sarkar *et al.* [19] extended thei paper. The inspection policy may not be always performed perfectly. So,two types of error in the inspection result are considered, namely, Type I and Type II. The inspection errors in an economic production system with return sales was discussed by Yoo *et al.* [31] and Khan *et al.* [9].

The *out-of-control* manufacturing system is controllable by a parameter, which is known as reliability. The reliability can act as a decision variable in the machinery system. The increasing value of reliability implies the decreasing investment for the technology cost. This study considers a reliability dependent production cost of a product, which is the sum of reliability dependent material cost and exponential development cost. The reliability was considered in an imperfect manufacturing system by Sarkar *et al.* [13]. Deterioration with the time varying demand and partial backlogging was studied by Chang and Dye [2]. Singh and Singh [27] and Shaw *et al.* [25] discussed deteriorate products behaviour in a vendor-buyer model. Also, the lead time of the buyer was considered and the lead time dependent crashing cost was explained by Sarkar and Majumder [14] with setup cost reduction (Majumder *et al.* [11]). By considering some investment amount, the setup cost reduction was introduced Sarkar *et al.* [16]. The lead time dependent crashing cost was described by Yang [30] and Shin *et al.* [26].

This study considers a joint inventory model of vendor and buyer with system reliability and container management. The investment for setup and reliability of the system are optimized with the reduced carbon emission. Total cost of the entire system is minimized. In Section 2, we give the problem analysis within three subsections. Section 3 describes the mathematical modelling and solution procedure of the model, Section 4 contains the numerical experiment and results of the mathematical model, Section 5 explains the sensitivity of the cost parameters. Finaly, Section 6 gives the conclusions and suggestions about the further study.

2. PROBLEM ANALYSIS

A brief problem definition, notation used in the model, and major assumptions are given in this part.

2.1. Problem definition

The studding model describes a joint decisions between vendor and buyer, where the vendor produces defect products but sells only authentic perfect products to the customer. An inspection process is utilized by the vendor to remove defects, but inspection errors are still in the system. The setup cost of the vendor is optimised by investment, i.e., the setup cost is a function of investment amount. The delivery cost of products is dependent on the capacity of the container and distance among the vendor and buyer. Also, the production cost is depended on the material cost and reliability dependent development cost. To reduce the earbon emission, vendor has to pay the carbon tax which is included in the delivery cost, a carbon emission cost. The demand of the buyer during the lead time is of normal distribution and an additional cost is applied to decrease the lead time. Figure 1 gives the flowchart of the resultant production rate by using inspection and the inspection errors in the manufacturing process of the model.

2.2. Notation

The similar notation of Shaw *et al.* [25] is used in this model, which are as follows:

Decision variables

A investment amount to form the setup of manufacturer (\$/cycle)

q receiving quantity per shipment for the buyer (units)

ϕ reliability of machinery system

L lead time length (time)

λ rate of increasing shipment size (≥ 1)

Parameters

T cycle time (time unit)

T_b time between two successive deliveries to the buyer (time unit)

$A_1(A)$ investment dependent setup cost for vendor (\$/setup)

A_2 per shipment buyer's setup cost (\$/unit time/shipment)

h_1 holding charge of vendor (\$/unit/unit time)

h_2 holding charge of buyer (\$/unit/unit time)

Q_0 manufacturing lot size (units)

p_0 production rate at manufacture (units/unit time)

d average demand for the buyer (units/cycle time)

n shipment number in the whole time cycle ($n \in \mathbb{N}$)

σ standard deviation

k safety factor

α probability of defective products in the production

m_1 classifying a perfect product as defect (type-I error) (%/unit)

m_2 classifying a defect product as good (type-II error) (%/unit)

ϕ_{max} maximum reliability of machinery system

ϕ_{min} minimum reliability of machinery system

p_c vendors production cost ($/unit)

I^v venders on-hand inventory (unit)

I^b buyers on-hand inventory (unit)

c_t per container vendors delivery cost ($/container/unit distance)

γ capacity of the container (unit)

l distance between vendor and buyer (unit)

c_f per shipment carbon emission cost ($/shipment)

c_v per product carbon emission cost ($/unit)

C_0 normal inspection charge ($/units)

C_3 cost for wrongly accepting a defect product (type-I error) ($/unit)

C_4 cost for wrongly rejecting a good product (type-II error) ($C_3 > C_4$) ($/unit)

C_2 disposal cost ($/unit)

I_c total inspection, inspection errors, and disposal cost ($/cycle)

2.3. Assumptions

This is a single type of product manufacturing integrated model between vendor & buyer and $\alpha\%$ of defective products that can be identified by inspection. Thus, $(1-\alpha)Q_0$ and αQ_0 are the number of perfect and defective products, respectively.

For the presence of inspection errors, $(1-\alpha)(1-m_1)Q_0$ and$(1-\alpha)m_1Q_0$ are the actual perfect and defective products for the perfect products $(1-\alpha)Q_0$, respectively. Similarly, αm_2Q_0 and $(1-m_2)\alpha Q_0$ are the actual perfect and defective products for the defective products αQ_0. Thus, the total actual perfect and defective products are as $Q=(1-\alpha)(1-m_1)+\alpha m_2Q_0=u_1Q_0$ and $Q_0-Q=\{(1-\alpha)m_1+(1-m_2)\alpha\}Q_0=(1-u_1)Q_0$, respectively, where $u_1=(1-\alpha)(1-m_1)+\alpha m_2$. Accordingly, the perfect items production rate is $p=u_1p_0$.

The vendor or manufacturer delivers only good products to the buyer or supplier at a q $(q\leq Q)$ quantity for the first delivery, but for every next delivery, the quantity increases by the ratio $\lambda(\geq)$. Thus, the last delivery quantity is $\lambda^{n-1}q$. The good products have been transported to the open market at n shipment and the time period of i^{th} shipment is $(\lambda^{i-1}q)/d$, where the demand rate of the buyer is $d(d\leq q)$.

The delivery cost is dependent on the capacity of the container (γ) and the distance (l) between vendor and buyer, which is $\frac{c_tl(q+\lambda q+\ldots+\lambda^{n-1}q)}{\gamma}$ for the cycle time T.

The material cost $(b_1-b_2\phi)$ and machinery system reliability dependent development cost $(c_D(\phi)/p_0)$ are added to the production cost p_c, where $c_D(\phi)=a_1+a_2e^{\frac{a_3(\phi_{max}-\phi)}{\phi-\phi_{min}}}$.

The buyers demand within the lead time L is distributed normally with μL and $\sigma\sqrt{L}$ mean and standard deviations, respectively.

Per shipment, the carbon emission cost is constant in the transportation process, and per unit, the emission cost is variable in the production process.

The setup cost of the vendor depend upon the investment amount.

No shortages are considered for the defective items (Q_0-Q).

3. MATHEMATICAL MODEL

The vendor receives the order quantity Q_0 from the buyer for the whole planning horizon T. After the order is received, the vendor starts the production, i.e., the vendor does not have any previous stock to deliver. Thus, the model follows a MTO policy. As a result, the vendor starts the production at a rate p_0. During

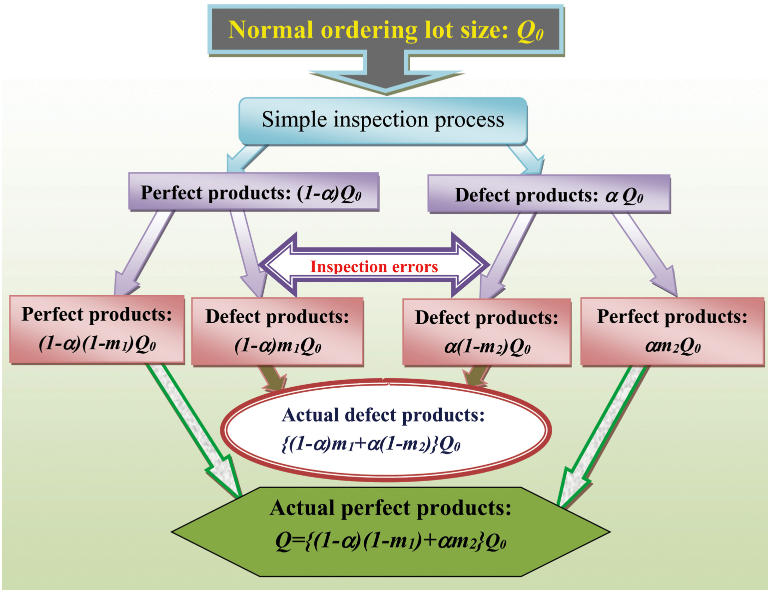

Figure 1: Flowchart of the perfect and imperfect products in the system.

production process, the defect production rate is $\alpha\%$. The rate of imperfect and perfect productions are $p_0\alpha$ and $p_0(1-\alpha)$ units, respectively. For the two-types of error in inspection process, m_1 and m_2 are the probability false acceptance (Type-I error) and false rejection (Type-II error), respectively. Therefore, the actual rate of imperfect and perfect production rates are $(1-m_2)p_0\alpha$ and $m_2p_0\alpha$, respectively. Similarly, the actual rate of imperfect and perfect product production rates are $p_0(1-\alpha)(1-m_1)$ and $p_0m_1(1-\alpha)$, respectively within the perfect production rate $p_0(1-\alpha)$ according to figure 2. Hence, the total rate of imperfect and perfect production are $\{(1-\alpha)m_1+(1-m_2)\alpha\}p_0 = (1-u_1)p_0$ and $p = \{(1-\alpha)(1-m_1)+\alpha m_2\}p_0 = u_1p_0$, respectively, where $u_1 = (1-\alpha)(1-m_1)+\alpha m_2$. Finally, the total perfect and imperfect products in the system are $Q = u_1Q_0$ and $Q_0(1-u_1)$ units, respectively, for the time horizon T. The imperfect products are disposed at per unit cost C_2 and the disposal cost is $C_2Q_0(1-u_1)$. On the other hand, the vendor sends the good items to the buyer in a small quantity size q ($\leq Q$) for the first shipment. The next shipment quantities are λ^2q, λ^3q,....,$\lambda^{n-1}q$, where $1 \leq \lambda \leq p/d$ (for instance, see Hill [7]). Therefore, the perfect quantity is $Q = q+\lambda^2q+\lambda^3q+...+\lambda^{n-1}q = \frac{q(\lambda^{n-1})}{\lambda-1}$ and the cycle time is $T = \frac{Q}{d} = \frac{q(\lambda^{n-1})}{d(\lambda-1)}$. Consequently, the time between delivery i and $(i+1)$ is $\lambda^{i-1}q/d$ for $i = 1, 2, ..., (n-1)$. Now, the production up-time is $t_1 = Q_0/p_0 = Q/p$, where the manufacturing and inspection processes are done, the production is stopped during the time period $[t_1, t_2]$, where $t_2 = T - t_1$.

3.1. Vendors model

The setup's cost $(A_1(A))$ is the decreasing function of investment amount A and is assumed to be $A_1(A) = A_0 e^{-a_0 A}$, where $A_0(\geq A)$ and $1/a_0$ are the initial investment and percentage of decrease in A, respectively. Therefore, the total setup cost and investment per unit time are $\frac{A_1(A)+A}{T} = \frac{A+A_0 e^{-a_0 A}}{T}$. From Sarkar *et al.* [13], the machinery system reliability dependent development cost is $c_D(\phi) = a_1 + a_2 e^{\frac{a_3(\phi_{max}-\phi)}{(\phi-\phi_{min})}}$, where $\phi = \frac{\text{number of failures}}{\text{total operating hours}}$, and a_1, a_2, $a_3 \geq 0$. The unit production cost (p_c) is the sum of material cost $(b_1 - b_2\phi)$ and unit development cost. Therefore, $p_c = b_1 - b_2\phi + \frac{c_D(\phi)}{p_0} = b_1 - b_2\phi + \frac{u_1}{p}\left\{a_1 + a_2 e^{\frac{a_3(\phi_{max}-\phi)}{\phi-\phi_{min}}}\right\}$, where $a_1, a_2, a_3, b_1, b_2 \geq 0$.

For the holding cost, first investigation is about the on-hand inventory for the vendor, given by the area calculations of triangle and rectangle in Figure 2. The on-hand inventory is calculated from the area calculations of the vendor's inventory figure in Figure 2 of the vendor and this is

$$\begin{aligned} I^v &= Q_0 T - \frac{Q_0 t_1}{2} - \left\{\frac{q}{d}q + \frac{\lambda q}{d}(q+\lambda q) + \frac{\lambda^2 q}{d}(q+\lambda q+\lambda^2 q) + ...\right. \\ &+ \left.\frac{\lambda^{n-1}q}{d}(q+\lambda q+...+\lambda^{n-1}q)\right\} \\ &= \frac{qT}{2pu_1(\lambda^2-1)}\{(\lambda^n-1)(\lambda+1)(2p-du_1) - 2pu_1(\lambda^{n+1}-1)\}, \end{aligned}$$

and consequently per unit time the total holding/carrying cost is

$$\frac{h_1 I^v}{T} = \frac{qh_1}{2pu_1(\lambda^2-1)}\{(\lambda^n-1)(\lambda+1)(2p-du_1) - 2pu_1(\lambda^{n+1}-1)\}.$$

Again, per unit inspection cost C_0 affects the total produced products Q_0 and the total inspection cost is $C_0 Q_0$. To dispose the imperfect products of the system, per unit disposal cost C_2 is applied to the imperfect products $(Q_0 - Q)$ and the disposal cost is $C_2 Q_0(1-u_1)$. The cost of falsely acceptance for a defective product and rejection of a perfect product is $C_3\alpha m_2 Q_0 + C_4(1-\alpha)Q_0 m_1$, respectively. Thus, the inspection and the related cost are

$$\begin{aligned} I_c &= C_0 Q_0 + C_2(1-u_1)Q_0 + C_3\alpha m_2 Q_0 + C_4(1-\alpha)m_1 Q_0 \\ &= C_0 + C_2(1-u_1) + C_3\alpha m_2 + C_4(1-\alpha)m_1 Q_0 \\ &= u_2 Q_0, \end{aligned}$$

and per unit time, this cost is $\frac{I_c}{T} = \frac{du_2}{u_1}$, where $u_2 = C_0 + C_2(1-u_1) + C_3\alpha m_2 + C_4(1-\alpha)m_1$.

Per shipment delivery cost is the product of per container delivery cost (c_t), distance between vendor to buyer (l), and total number of container required $\sum_{i=1}^{n-1}\frac{\lambda^{i-1}q}{\gamma}$, where i^{th} shipment required $(\frac{\lambda^{i-1}q}{\gamma})$ containers i.e., the total delivery cost is $\frac{lc_t(q+\lambda^2 q+....+\lambda^{n-1}q)}{\gamma} = \frac{dlc_t T}{\lambda}$, where γ is the capacity of the container.

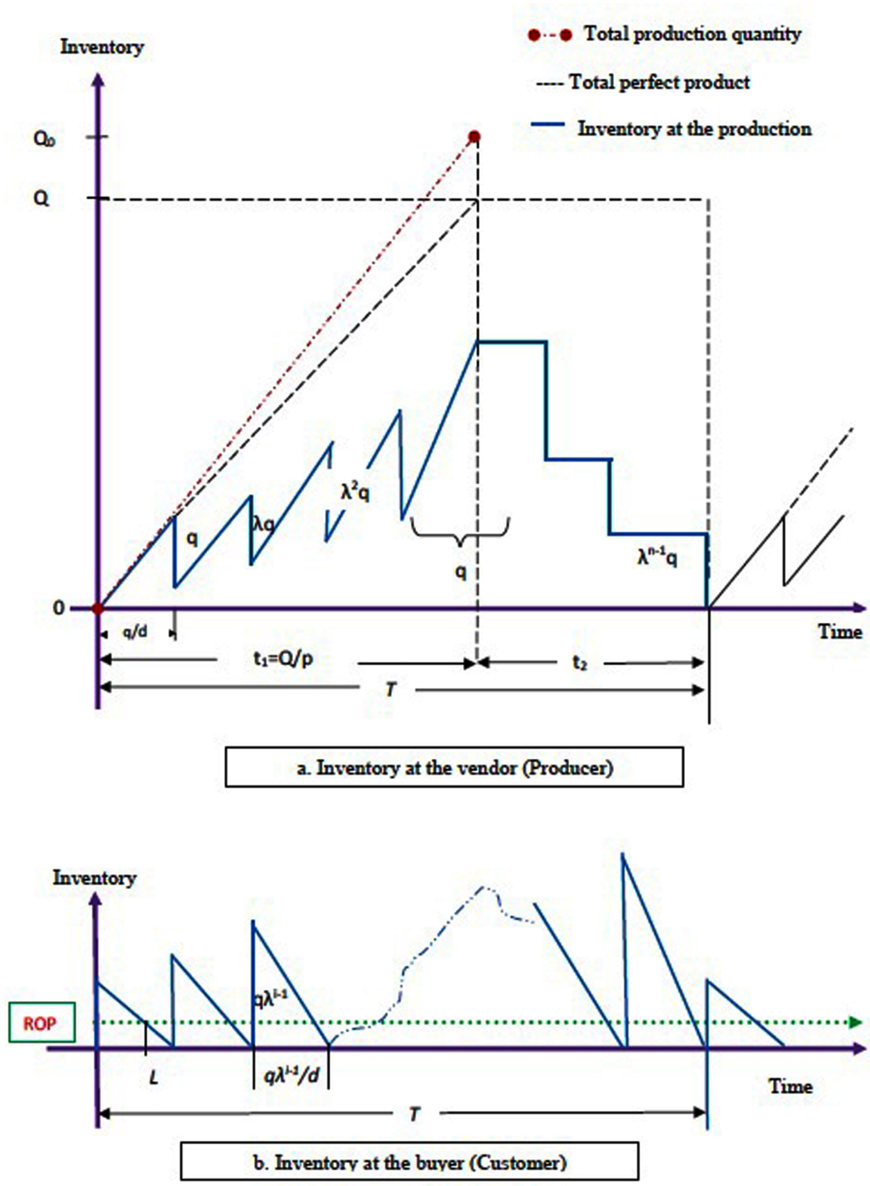

Figure 2: Time vs Inventory positions of the vendor-buyer manufacturing-inventory model.

Lastly, the constant carbon emission cost (c_f) is applied to every shipments (n) and the variable carbon emission cost (c_f) is applied to all products (Q_0). Thus, the total carbon emission cost is $nc_f + Q_0 c_v$, and per unit time, this cost is $\frac{nc_f+Q_0c_v}{T}$.

By using $T = \frac{q(\lambda^n-1)}{d(\lambda-1)}$, per unit time the joint total cost of the vendor is $TC^v = \frac{1}{T}$[setup cost+production cost+holding cost+inspection and related cost+delivery cost+carbon emission cost] and thus,

$$
\begin{aligned}
TC^v &= \frac{d(\lambda-1)\left[p(A + A_0 e^{-a_0 A} + b_1 - b_2\phi + nc_f) + u_1\left\{a_1 + a_2 e^{\frac{a_3(\phi_{max}-\phi)}{\phi-\phi_{min}}}\right\}\right]}{pq(\lambda^{n-1})} \\
&+ \frac{qh_1}{2pu_1(\lambda^2-1)}\{(\lambda^n-1)(\lambda+1)(2p-du_1) - 2pu_1(\lambda^{n+1}-1)\} \\
&+ d\left(\frac{u_2+c_v}{u_1} + \frac{lc_t}{\gamma}\right). \qquad (1)
\end{aligned}
$$

3.2. Buyers model

Normally, the area formed by the buyer in i^{th} shipment is $\frac{\lambda^{i-1}q}{2d} \times \lambda^{i-1}q$ for $i = 1, 2, ..., (n-1)$ and the total area formed by the buyer is $\frac{1}{T}\sum_{i=1}^{n-1} \frac{\lambda^{i-1}q}{2d} \times \lambda^{i-1}q = \frac{q^2(\lambda^{2n}-1)}{2Td(\lambda^2-1)} = \frac{q(\lambda^n+1)}{2(\lambda+1)}$. The mean and standard deviation of the lead time demand (follow normal distribution) of the buyer are μL and $\sigma\sqrt{L}$, respectively. This gives the reorder point (ROP) is $\mu L + k\sigma\sqrt{L}$, where safety factor is k. According to this (Sarkar *et al.* [19]), the buyers on-hand inventory per unit time is

$$
\frac{I^b}{T} = \frac{(\lambda^n+1)q}{2(\lambda+1)} + ROP - \mu L = \frac{q(\lambda^n+1)}{2(\lambda+1)} + k\sigma\sqrt{L}.
$$

By using the model of Yang [30], the lead time dependent crashing cost is $R(L) = a_4 L^{-a_5}$ per shipment for getting quick delivery, where $L_e \le L \le L_s$ and $a_4, a_5 \ge 0$. Thus, the buyer's total cost is the sum of handling cost (A_2) per shipment, holding cost ($h_2 I^b$), and crashing cost ($R(L)$) per shipment. Thus, per unit time, the buyer's total cost is

$$
\begin{aligned}
TC^b &= \frac{nA_2}{T} + \frac{h_2 I^b}{T} + \frac{nR(L)}{T} \\
&= \frac{nd(\lambda-1)(A_2 + a_4 L^{-a_5})}{q(\lambda^n-1)} + h_2\left\{\frac{q(\lambda^n+1)}{2(\lambda+1)} + k\sigma\sqrt{L}\right\}. \qquad (2)
\end{aligned}
$$

3.3. Coordination of vendor and buyer

The model is coordinated between vendor and buyer by transforming their information. Here, per unit time, the joint total cost of the integrated inventory model using SSMUD policy is given by the sum of Equations 1 and 2

$$TC(A,q,\phi,L,\lambda) = TC^v + TC^b$$
$$= \frac{d(\lambda-1)}{pq(\lambda^n-1)}\Bigg[p\left\{A + A_0e^{-a_0A} + b_1 - b_2\phi + n(A_2 + c_f + a_4L^{-a_5})\right\}$$
$$+u_1\left\{a_1 + a_2e^{\frac{a_3(\phi_{max}-\phi)}{\phi-\phi_{min}}}\right\}\Bigg] + \frac{qh_1}{2pu_1(\lambda^2-1)}\Bigg\{(\lambda^n-1)(\lambda+1)(2p-du_1)$$
$$-2pu_1(\lambda^{n+1}-1)\Bigg\} + h_2\left\{\frac{q(\lambda^n+1)}{2(\lambda+1)} + k\sigma\sqrt{L}\right\} + d\left(\frac{u_2+c_v}{u_1} + \frac{lc_t}{\gamma}\right). \quad (3)$$

Again, per unit time, the total cost by the SSMD policy is obtained from the above equations with consideration of $\lambda = 1$ i.e., $n = \frac{\lambda^n-1}{\lambda-1}$

$$TCM(A,q,\phi,L) = \frac{d}{pnq}\Bigg[p\left\{A + A_0e^{-a_0A} + b_1 - b_2\phi + n(A_2 + c_f + a_4L^{-a_5})\right\}$$
$$+u_1\left\{a_1 + a_2e^{\frac{a_3(\phi_{max}-\phi)}{\phi-\phi_{min}}}\right\}\Bigg] + \frac{qh_1}{2pu_1}\{n(2p-du_1) - pu_1(n+1)\}$$
$$+h_2\left(\frac{q}{2} + k\sigma\sqrt{L}\right) + d\left(\frac{u_2+c_v}{u_1} + \frac{lc_t}{\gamma}\right). \quad (4)$$

3.4. Solution procedure

Now, differentiating Equation 3 with respect to the variables A, q, ϕ, L, and λ then, the next results are found:

$$\frac{\partial TC(A,q,\phi,L,\lambda)}{\partial A} = \frac{d(\lambda-1)}{q(\lambda^n-1)}(1 - a_0A_0e^{-a_0A}),$$
$$\frac{\partial TC(A,q,\phi,L,\lambda)}{\partial q} = -\frac{d(\lambda-1)}{pq^2(\lambda^n-1)}\Bigg[p\{A + A_0e^{-a_0A} + b_1 - b_2\phi + n(A_2 + c_f$$
$$+a_4L^{-a_5})\} + u_1\left\{a_1 + a_2e^{\frac{a_3(\phi_{max}-\phi)}{\phi-\phi_{min}}}\right\}\Bigg] + \frac{h_1}{pu_1(\lambda^2-1)}$$
$$\left\{(\lambda^n-1)(\lambda+1)(2p-du_1) - 2pu_1(\lambda^{n+1}-1)\right\} + \frac{h_2(\lambda^n+1)}{2(\lambda+1)},$$
$$\frac{\partial TC(A,q,\phi,L,\lambda)}{\partial \phi} = \frac{d(\lambda-1)}{pq(\lambda^n-1)}\left[-pb_2 + \frac{a_2a_3(\phi_{min}-\phi_{max})}{(\phi-\phi_{min})^2}e^{\frac{a_3(\phi_{max}-\phi)}{\phi-\phi_{min}}}\right],$$
$$\frac{\partial TC(A,q,\phi,L,\lambda)}{\partial L} = -\frac{dna_4a_5(\lambda-1)}{q(\lambda^n-1)}L^{-1-a_5} + \frac{h_2k\sigma}{2\sqrt{L}},$$
$$\frac{\partial TC(A,q,\phi,L,\lambda)}{\partial \lambda} = \frac{d\{n\lambda^{n-1} - 1 - (n-1)\lambda^n\}}{pq(\lambda^n-1)^2}\Bigg[p\{A + A_0e^{-a_0A} + b_1$$
$$-b_2\phi + n(A_2 + c_f + a_4L^{-a_5})\} + u_1\left\{a_1 + a_2e^{\frac{a_3(\phi_{max}-\phi)}{\phi-\phi_{min}}}\right\}\Bigg]$$

$$+\frac{qh_1}{2pu_1(\lambda-1)^2}\left[\{(n-1)\lambda^n - n\lambda^{n-1} - 1\}(2p-du_1) - 2pnu_1(\lambda^{n+1}-1)\right]$$
$$+\frac{h_2q\{(n-1)\lambda^n - n\lambda^{n-1} - 1\}}{(\lambda+1)^2}.$$

Equating to zero, all the above partial derivatives give

$$A = \frac{ln(a_0A_0)}{a_0}, \tag{5}$$

$$\frac{2d(\lambda-1)}{pq^2(\lambda^n-1)}\left[p\{A + A_0e^{-a_0A} + b_1 - b_2\phi + n(A_2 + c_f + a_4L^{-a_5})\}\right.$$
$$\left.+u_1\left\{a_1 + a_2e^{\frac{a_3(\phi_{max}-\phi)}{\phi-\phi_{min}}}\right\}\right] = \frac{1}{2pu_1(\lambda^2-1)}[h_1(\lambda^n-1)(\lambda+1)(2p-du_1)$$
$$+2pu_1\{h_2(\lambda-1)(\lambda^n+1) - h_1(\lambda^{n+1}-1)\}], \tag{6}$$

$$e^{\frac{a_3(\phi_{max}-\phi)}{\phi-\phi_{min}}} = \frac{pb_2(\phi-\phi_{min})^2}{a_2a_3(\phi_{min}-\phi_{max})}, \tag{7}$$

$$q = \frac{2da_4a_5(\lambda-1)}{h_2k\sigma(\lambda^n-1)L^{a_5+1/2}}, \tag{8}$$

$$\frac{d\{n\lambda^{n-1} - 1 - (n-1)\lambda^n\}}{pq(\lambda^n-1)^2}\left[p\{A + A_0e^{-a_0A} + b_1 - b_2\phi + n(A_2 + c_f + a_4L^{-a_5})\}\right.$$
$$\left.+u_1\left\{a_1 + a_2e^{\frac{a_3(\phi_{max}-\phi)}{\phi-\phi_{min}}}\right\}\right] + \frac{qh_1}{2pu_1(\lambda-1)^2}\left[\{(n-1)\lambda^n - n\lambda^{n-1} - 1\}(2p-du_1)\right.$$
$$\left.-2pnu_1(\lambda^{n+1}-1)\right] + \frac{h_1q\{(n-1)\lambda^n - n\lambda^{n-1} - 1\}}{(\lambda+1)^2} = 0. \tag{9}$$

From these equations and using following sequence, the optimal decision variables A^*, q^* ,ϕ^*, L^*, and λ^* are found.

For finding the optimum values A^*, q^* ,ϕ^*, L^*, and λ^*

The optimum values of investment amount A, delivery quantity q, reliability parameter ϕ, and lead time L are calculated by using the Equations 5 to 9. First, the optimum investment amount A^* is calculated from the Equation 5 and then, the Equation 7 gives the optimum value of reliability ϕ^*. Putting these optimum values A^* and ϕ^* in Equations 6, 8 and 9, then the three simultaneous equations are given for three variables q, L, and λ. By solving them, the optimum values q^*, L^* and λ^* are found corresponding the variables. Thus, all optimum values are calculated.

Lemma

The Hessian matrix for $TC(A, q, \phi, L, \lambda)$ is always positive definite and the joint total cost is global minimum for the optimum decision variables $A = A^*$, $q = q^*$, $\phi = \phi^*$, $L = L^*$, and $\lambda = \lambda^*$.

Proof

To prove this, the Hessian matrix is shown by

$$H(TC)=\begin{pmatrix}\frac{\partial^2TC(.)}{\partial A^2} & \frac{\partial^2TC(.)}{\partial A\partial q} & \frac{\partial^2TC(.)}{\partial A\partial\phi} & \frac{\partial^2TC(.)}{\partial A\partial L} & \frac{\partial^2TC(.)}{\partial A\partial\lambda}\\ \frac{\partial^2TC(.)}{\partial q\partial A} & \frac{\partial^2TC(.)}{\partial q^2} & \frac{\partial^2TC(.)}{\partial q\partial\phi} & \frac{\partial^2TC(.)}{\partial q\partial L} & \frac{\partial^2TC(.)}{\partial q\partial\lambda}\\ \frac{\partial^2TC(.)}{\partial\phi\partial A} & \frac{\partial^2TC(.)}{\partial\phi\partial q} & \frac{\partial^2TC(.)}{\partial\phi^2} & \frac{\partial^2TC(.)}{\partial\phi\partial L} & \frac{\partial^2TC(.)}{\partial\phi\partial\lambda}\\ \frac{\partial^2TC(.)}{\partial L\partial A} & \frac{\partial^2TC(.)}{\partial L\partial q} & \frac{\partial^2TC(.)}{\partial L\partial\phi} & \frac{\partial^2TC(.)}{\partial L^2} & \frac{\partial^2TC(.)}{\partial L\partial\lambda}\\ \frac{\partial^2TC(.)}{\partial\lambda\partial A} & \frac{\partial^2TC(.)}{\partial\lambda\partial q} & \frac{\partial^2TC(.)}{\partial\lambda\partial\phi} & \frac{\partial^2TC(.)}{\partial\lambda\partial L} & \frac{\partial^2TC(.)}{\partial\lambda^2}\end{pmatrix}$$

$$=\begin{pmatrix}\frac{\partial^2TC(.)}{\partial A^2} & 0 & 0 & 0 & 0\\ 0 & \frac{\partial^2TC(.)}{\partial q^2} & 0 & \frac{\partial^2TC(.)}{\partial q\partial L} & \frac{\partial^2TC(.)}{\partial q\partial\lambda}\\ 0 & 0 & \frac{\partial^2TC(.)}{\partial\phi^2} & 0 & 0\\ 0 & \frac{\partial^2TC(.)}{\partial L\partial q} & 0 & \frac{\partial^2TC(.)}{\partial L^2} & \frac{\partial^2TC(.)}{\partial L\partial\lambda}\\ 0 & \frac{\partial^2TC(.)}{\partial\lambda\partial q} & 0 & \frac{\partial^2TC(.)}{\partial\lambda\partial L} & \frac{\partial^2TC(.)}{\partial^2},\end{pmatrix}$$

where $TC(.)=TC(A,q,\phi,L,\lambda)$. At the optimum point $(A^*,q^*,\phi^*,L^*,\lambda^*)$ and from simple calculations, the second order partial derivatives are found in the following way

$$\frac{\partial^2TC(.)}{\partial A^2}=\frac{da_0^2A_0(\lambda-1)}{q(\lambda^n-1)}e^{-a_0A}=\frac{da_0(\lambda-1)}{q(\lambda^n-1)}>0,$$

$$\frac{\partial^2TC(.)}{\partial q^2}=\frac{2d(\lambda-1)}{pq^3(\lambda^n-1)}\Bigg[p\{A+A_0e^{-a_0A}+b_1-b_2\phi+n(A_2+c_f+a_4L^{-a_5})\}$$
$$+u_1\left\{a_1+a_2e^{\frac{a_3(\phi_{max}-\phi)}{\phi-\phi_{min}}}\right\}\Bigg]$$
$$=\frac{1}{2pu_1}[h_1(\lambda^n-1)(\lambda+1)(2p-du_1)+2pu_1\{h_2(\lambda-1)(\lambda^n+1)-h_1(\lambda^{n+1}-1)\}]>0,$$

$$\frac{\partial^2TC(.)}{\partial\phi^2}=\frac{da_2a_3(\lambda-1)(\phi_{max}-\phi_{min})\{a_3(\phi_{max}-\phi_{min})+2(\phi-\phi_{min})\}}{pq(\lambda^n-1)(\phi-\phi_{min})^4}e^{\frac{a_3(\phi_{max}-\phi)}{\phi-\phi_{min}}}>0,$$

$$\frac{\partial^2TC(.)}{\partial L^2}=\frac{da_4a_5(a_5+1)(\lambda-1)}{q(\lambda^n-1)}L^{-2-a_5}-\frac{h_2k\sigma}{4\sqrt{L}^3}$$
$$=\frac{da_4a_5(\lambda-1)}{2qL(\lambda^n-1)}\left\{2(2a_5+1)L^{-1-a_5}-1\right\}>0,$$

$$\frac{\partial^2TC(.)}{\partial A\partial q}=-\frac{d(\lambda-1)}{q^2(\lambda^n-1)}(1-a_0A_0e^{-a_0A})=0=\frac{\partial^2TC(.)}{\partial q\partial A},$$

$$\frac{\partial^2TC(.)}{\partial q\partial\phi}=-\frac{d(\lambda-1)}{pq^2(\lambda^n-1)}\left[-pb_2+\frac{a_2a_3(\phi_{min}-\phi_{max})}{(\phi-\phi_{min})^2}e^{\frac{a_3(\phi_{max}-\phi)}{\phi-\phi_{min}}}\right]=0=\frac{\partial^2TC(.)}{\partial\phi\partial q},$$

$$\frac{\partial^2TC(.)}{\partial q\partial L}=\frac{dna_4a_5(\lambda-1)}{q^2(\lambda^n-1)}L^{-1-a_5}=\frac{h_2qk\sigma\sigma}{2\sqrt{L}}>0,$$

$$\frac{\partial^2TC(.)}{\partial\lambda^2}>0,$$

and, $0 = \frac{\partial^2 TC(.)}{\partial A \partial \phi} = \frac{\partial^2 TC(.)}{\partial \phi \partial A} = \frac{\partial^2 TC(.)}{\partial A \partial L} = \frac{\partial^2 TC(.)}{\partial L \partial A} = \frac{\partial^2 TC(.)}{\partial \phi \partial L} = \frac{\partial^2 TC(.)}{\partial L \partial \phi} = \frac{\partial^2 TC(.)}{\partial \lambda \partial A} = \frac{\partial^2 TC(.)}{\partial A \partial \lambda} = \frac{\partial^2 TC(.)}{\partial \lambda \partial \phi} = \frac{\partial^2 TC(.)}{\partial \phi \partial \lambda}$ for the non-negative parametric values. Clearly, all first order principle minors are positive. By using those results, the next principle minors are

$$
\begin{aligned}
det(H_{22}) &= \frac{\partial^2 TC(.)}{\partial A^2} \times \frac{\partial^2 TC(.)}{\partial q^2} > 0,\\
det(H_{33}) &= \frac{\partial^2 TC(.)}{\partial A^2} \times \frac{\partial^2 TC(.)}{\partial q^2} \times \frac{\partial^2 TC(.)}{\partial \phi^2} > 0,\\
det(H_{44}) &= \frac{\partial^2 TC(.)}{\partial A^2} \times \frac{\partial^2 TC(.)}{\partial \phi^2} \left[\frac{\partial^2 TC(.)}{\partial q^2} \frac{\partial^2 TC(.)}{\partial L^2} - \left\{ \frac{\partial^2 TC(.)}{\partial q \partial L} \right\}^2 \right] > 0,\\
\text{and } det(H_{55}) &= \frac{\partial^2 TC(.)}{\partial A^2} \times \frac{\partial^2 TC(.)}{\partial \phi^2} \left[\frac{\partial^2 TC(.)}{\partial \lambda^2} \left\{ \frac{\partial^2 TC(.)}{\partial q^2} \frac{\partial^2 TC(.)}{\partial L^2} \right.\right.\\
&\quad \left. + \left(\frac{\partial^2 TC(.)}{\partial q \partial L} \right)^2 + \frac{\partial^2 TC(.)}{\partial q \partial L} \frac{\partial^2 TC(.)}{\partial \lambda \partial q} \right\} - \frac{\partial^2 TC(.)}{\partial q \partial \lambda}\\
&\quad \left. \left\{ \frac{\partial^2 TC(.)}{\partial q \partial L} \frac{\partial^2 TC(.)}{\partial \lambda \partial L} + \frac{\partial^2 TC(.)}{\partial q \partial \lambda} \frac{\partial^2 TC(.)}{\partial L^2} \right\} \right] > 0.
\end{aligned}
$$

Hence, the total cost $TC(A, q, \phi, L, \lambda)$ is found as a global minimum value at $(A^*, q^*, \phi^*, L^*, \lambda^*)$.

4. NUMERICAL EXAMPLE

This example is to elaborate the above model. Generally, the cost for accepting a defective item is more than the cost for rejecting a good item, i.e., the Type-I error cost is higher than the cost of Type-II error ($C_3 > C_4$). The values of parameters are: $n = 8$ shipments, $d = 60$units/month, $p_0 = 100$ units/month, $h_1 =$ \$0.08/unit/month, $A_0 =$ \$500 /order, $a_0 = 0.06$, $b_1 = 20$, $b_2 = 0.2$, $\phi_{max} = 0.9$, $\phi_{min} = 0.1$, $a_1 = 150$, $a_2 = 70$, $a_3 = 0.51$, $a_4 = 85$, $a_5 = 12$, $c_f =$ \$1.5/shipment, $c_v =$ \$0.63/unit, $l = 250$miles, $c_t =$ \$20/container/mile, $\gamma = 5$ units/container, $C_0 =$ \$0.1/unit, $C_2 =$ \$1/unit, $C_3 =$ \$0.023/unit, $C_4 =$ \$0.01/unit, $\alpha = 5\%$, $m_1 = 1\%$, $m_2 = 4\%$, $h_2 =$ \$0.1/unit/month, $A_2 =$ \$5/month, $k = 2.33$, and, $\sigma = 4$ units/week.

Then, the minimum joint total cost for the studdied model is \$60,134.06 per cycle, and the corresponding optimum values of decision variables are $q^* = 14.18$ units, $A^* =$ \$18.31, $\phi^* = 0.61$, $L^* = 1.33$ months, and $\lambda^* = 1.43$.

At the optimum point, the principal minors are $det(H_{11}) = 0.14$, $det(H_{22}) = 0.0009$, $det(H_{33}) = 0008$, $det(H_{44}) = 0.145$, $det(H_{55}) = 4.422$. This indicates that the joint total cost function $TC(q, A, \phi, L, \lambda)$ has the global minimum value for the optimum point $(q^*, A^*, \phi^*, L^*, \lambda^*)$.

4.1. Comparison table for SSSD, SSMD, and SSMUD policy

The optimum total costs got by appyling SSMUD, SSMD, and SSSD policies are depicted in Table 1, so getting the insight in which is the best policy. By considering $\lambda = 1$, the SSMUD policy is transformed to SSMD for any shipment number and the cost related to the SSMD policy is focussed on Function 4. Again, by considering one shipment, the SSMD is transferred to SSSD. The following table gives the total costs and decision variables from Equation 4 in the given example for SSMD and SSSD policies by considering $n = 1$ and $n = 8$. Lastly, the optimum total cost for SSMUD policy from Equation 3 is described.

Table 1: Comparison of SSSD, SSMD, and SSMUD policies

Policy	Total cost (\$)	q^*(units)	A^*(\$)	ϕ^*	L^* (months)	λ^*
SSSD ($n = 1 \& \lambda = 1$)	75,054.89	139.34	18.42	0.21	0.61	
SSMD ($n = 8 \& \lambda = 1$)	71,294.61	77.77	19.89	0.2	0.65	
SSMUD ($n = 8$)	60,134.06	14.18	18.31	0.61	1.33	1.43

Table 1 shows that the optimum total cost in SSMUD policy is for 16% smaller than in the SSMD policy and similarly, the optimum total cost in SSMD policy is for 5% smaller than in the SSSD. Therefore, the SSMUD policy is better than SSMD, and SSMD policy is better than SSSD.

5. SENSITIVITY ANALYSIS

Table 2 shows sensitivity analysis for some cost parameters, where parametric values are changed by −50%, −25%, +25%, and +50% of the original value. It also shows the percentage of changes in total joint cost $TC(A, q, \phi, L, \lambda)$, accordingly.

Table 2: Table of sensitivity analysis for the cost parameters

Parameters	Changes	TC(.) (in %)	Parameters	Changes	TC(.) (in %)
h_1	−50%	-0.0068	c_f	-50%	-0.0011
	−25%	-0.0031		-25%	-0.0005
	+25%	0.0027		+25%	0.0005
	+50%	0.0052		+50%	0.0011
h_2	−50%	-0.0081	c_v	−50%	-0.0333
	−25%	-0.0033		−25%	-0.0167
	+25%	0.0030		+25%	0.0167
	+50%	0.0026		+50%	0.0333
A_2	−50%	-0.0035	c_t	−50%	-46.888
	−25%	-0.0017		−25%	-24.944
	+25%	0.0016		+25%	24.944
	+50%	0.0033		+50%	46.888

Here $TC(.)$ referred $TC(A, q, \phi, L, \lambda)$.

The joint total cost increase or decrease accordingly to the increase or decrease of all cost parameters shown in Table 2 and all the changes are significant. The vendor's holding cost (h_1) is more sensitive than the buyer's holding cost (h_2). Except the changing of transportation cost parameter (c_t), the joint total cost changes briefly with all other parameters changes. With the increasing value of buyers handling cost or vendors transportation cost for a container, the joint total cost is also increasing. Transportation cost for a container is more sensitive and fixed carbon emission cost (c_f) is less sensitive than the other cost parameters (see Figure 3). The total cost is increasing or decreasing proportionally according to the increasing or decreasing cost of the parameters fixed (c_f) or variable (c_v) carbon emission cost or transportation cost (c_t).

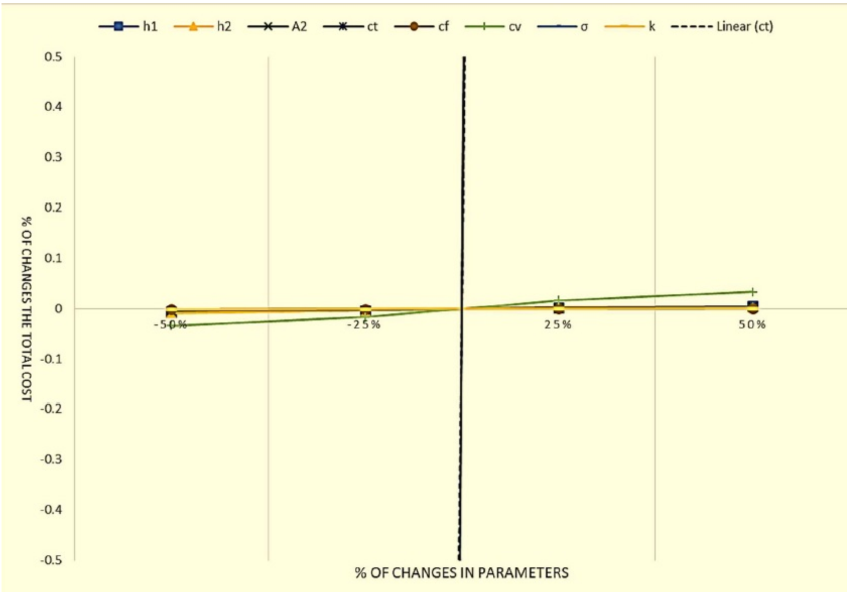

Figure 3: Graphical representations for the percentage changes of joint total cost with changes the percentage of parameters.

6. CONCLUSIONS and SUGGESTIONS

This study investigated the effect of investment in setup cost, carbon emission, and system reliability. Here, the setup cost of vendor is the decreasing function of investment amount. Results show that the system's total cost is minimized for the SSMUD policy. For this case, the system reliability is also higher than in the other transportation policies. Even though the lead time is high in SSMUD policy as the shipment size is unequal, the investment for the setup is relatively lower than in other policies. Based on the lead time of the system, the SSSD policy is good but the joint total cost of the system is relatively high, which may effect the whole integrated system. The SSMD policy is better according to the joint total cost from the comparison between SSSD & SSMD policy in Table 1. Manager of the industry can choose either option based on the total cost, i.e., transportation policy or lead time reduction policy. The model can be extended by adding the concept of multi-product and multi-echelon. The uncertainty concept for the system reliability is more practical (see the model of Sarkar and Mahapatra [21]). This model contains constant defective rate but can be extended by introducing random defects production rate. Energies can be calculated for the system and their reduction can be a new topic, as in (Sarkar *et al.* [22]; Sarkar and Sarkar, [23]).

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