Yugoslav Journal of Operations Research
30 (2020), Number 4, 413–427
DOI: https://doi.org/10.2298/YJOR191115006K

On $(\lambda, \mu)-$ ZWEIER IDEAL CONVERGENCE IN INTUITIONISTIC FUZZY NORMED SPACE

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Received: October 2019 / Accepted: March 2020

Abstract: In this paper, we study and introduce a new type of convergence, namely $(\lambda, \mu)-$ Zweier convergence and $(\lambda, \mu)-$ Zweier ideal convergence of double sequences $x = (x_{ij})$ in intuitionistic fuzzy normed space (IFNS), where $\lambda = (\lambda_n)$ and $\mu = (\mu_m)$ are two non-decreasing sequences of positive real numbers such that each tending to infinity. Furthermore, we studied $(\lambda, \mu)-$ Zweier Cauchy and $(\lambda, \mu)-$ Zweier ideal Cauchy sequences on the said space and established a relation between them.

Keywords: Ideal Convergence, Zweier Operator, (λ, μ)- convergence, Intuitionistic Fuzzy Normed Spaces.

MSC: 40C05, 40J05, 46A45.

1. INTRODUCTION

The theory of statistical convergence for sequences of real numbers was initiated by Fast [5] and Steinhaus [29] independently. There has been an immense interest of researchers to find statistical convergence analogues and applications of the classical theories (see, [7], [25], [28], [30]). In 2000, Mursaleen[23] introduced the notion of λ-statistical convergence as an extension of the $[V, \lambda]$ summability, given by Leindler[21]. Subsequently, Mursaleen et al.[22] extended this concept to double sequences. One of the most important generalization of the statistical convergence was introduced by Kostyrko et al.[20] by using the ideal $\mathcal{I}$ as a subset of the set of natural number $\mathbb{N}$, which they called $\mathcal{I}$-convergence. Afterwards, it was investigated from the sequence space point of view by Salát et al.[31], Tripathy et al.[34], and Khan et al.[16]. Şengönül [32] initiated Zweier sequence space and various researchers extended this concept in different area (see [4, 9, 10, 3]).

In the crisp situation, we often come across double sequences, i.e., sequences of matrices, and certainly, there are situations where either the idea of ordinary convergence does not work or the underlying space does not serve our purpose. Therefore, to deal with such situations, we have to introduce some new type of measures in the fuzzy set theory, which can provide a better tool and a suitable framework. Fuzzy set theory is a powerful handset for modelling uncertainty and vagueness in various problems arising in the field of science and engineering. It has a wide range of applications in various fields:population dynamics [2], chaos control [6], computer programming [8], nonlinear dynamical systems [14], etc.

Zadeh [35] introduced the concept of fuzzy sets. It has extensive applications in various fields within and beyond mathematics which also includes various typos of real word problems. In 1986, Atanassov[1] generalized the fuzzy sets and introduced the notion of intuitionistic fuzzy sets. Later on, Park [26] studied the notion of intuitionistic fuzzy metric space and Saadati et al.[27] extended this concepts to topological spaces. The convergence of sequence in an IFNS is vital to fuzzy functional analysis, and we feel that $\mathcal{I}$- convergence in an IFNS would yield a more comprehensive foundation of this field. Statistical and ideal convergences of double sequences in an IFNS were studied by Mursaleen et al.[24], and Khan et al. [15] respectively. Kumar et al.[19] studied (λ, μ)-statistical convergence of double sequence on IFNS. Şengönül [33] defined Zweier Sequence Spaces of fuzzy numbers, and Hazarika et al. [11] proceed with Şengönül work in statistical convergence. Khan et al. [17] further generalized it to intuitionistic fuzzy Zweier $\mathcal{I}$-convergent in double sequence spaces.

The rest of the paper is organized in the following sections: In section 2, we state some basic definitions and notions, i.e., ideal, filter, Zweier operator, (λ, μ)-convergence and intuitionistic fuzzy normed spaces. In section 3, we introduce and define $(\lambda, \mu)-$ Zweier convergence, and $(\lambda, \mu)-$ Zweier ideal convergence of double sequences $x = (x_{ij})$ in IFNS. We also define $(\lambda, \mu)-$ Zweier Cauchy and $(\lambda, \mu)-$ Zweier ideal Cauchy sequences and give some interesting results.

2. PRELIMINERIES

In this section, we begin with some notions and definitions which are needed in our subsequent discussions.

Definition 1. *[20] A family of subsets of the power set X, i.e., $\mathcal{I} \subseteq P(X)$ is said to be an ideal in X if it satisfies the following conditions:*

(a) $\emptyset \in \mathcal{I}$,
(b) for each $A, B \in \mathcal{I} \Rightarrow A \cup B \in \mathcal{I}$,
(c) for each $A \in \mathcal{I}$ *and* $B \subseteq A \Rightarrow B \in \mathcal{I}$.

Remark 2. *[20] If $\mathcal{I} \neq P(X)$, then ideal $\mathcal{I}$ is said to be nontrivial. A nontrivial ideal $\mathcal{I}$ is called admissible if $\{\{x\} : x \in X\} \subseteq \mathcal{I}$.*

Definition 3. *[20] Suppose $\mathcal{I} \subseteq P(X)$ be a nontrivial ideal, then the class*

$$\mathcal{F}(\mathcal{I}) = \{A \subset X : A^c \in \mathcal{I}\}$$

is a filter on X, called the filter associated with the ideal $\mathcal{I}$.

Definition 4. *[22] Let $\lambda = (\lambda_n)$ and $\mu = (\mu_m)$ be two non-decreasing sequences of positive real numbers in such a way that $\lambda_n, \quad \mu_m \to \infty$ as $n, \quad m \to \infty$.*

$$\lambda_{n+1} \leqslant \lambda_n + 1, \qquad \lambda_1 = 0$$

$$\mu_{m+1} \leqslant \mu_m + 1, \qquad \mu_1 = 0$$

Let $J_n = [n - \lambda_n + 1, n]$ and $J_m = [m - \mu_m + 1, m]$. Then the number

$$\delta_{\lambda,\mu}(K) = \lim_{n,m \to \infty} \frac{1}{\lambda_n \mu_m} |\{(i,j) \in I_n \times I_m : (i,j) \in K\}|;$$

is said to be (λ, μ)-density of the set $K \subseteq \mathbb{N} \times \mathbb{N}$, provided that the limit exists. In case $\lambda_n = n, \mu_m = m$, the (λ, μ)-density reduces to the natural double density.

Now, the generalized double Valée-Pousin mean is

$$t_{n,m}(x) = \frac{1}{\lambda_n \mu_m} \sum_{i \in J_n} \sum_{j \in J_m} x_{ij},$$

where $J_n = [n - \lambda_n + 1, n]$ and $J_m = [m - \mu_m + 1, m]$. If $\lambda_n = n$ for all n and $\mu_m = m$ for all m, then (V, λ, μ) -summablility is reduced to $[C, 1, 1]$-summability.

Definition 5. *[22] A double sequence $x = (x_{ij})$ of numbers is said to be (λ, μ) statistical convergent to L if $\delta_{(\lambda,\mu)}(E) = 0$ where, $E = \{i \in I_m, j \in I_n :| x_{ij} - L |\geq \epsilon\}$, i. e., if for every $\epsilon > 0$,*

$$\lim_{n,m} \frac{1}{\lambda_n \mu_m} \mid \{i \in J_n, j \in J_m :| x_{ij} - L |\geq \epsilon\} \mid \quad = \quad 0.$$

If $\lambda_n = n$ for all n and $\mu_m = m$ for all m, then (λ, μ)-double statistical convergence is reduced to double statistical convergence.

Definition 6. *[18] A binary operation $* : [0,1] \times [0,1] \to [0,1]$ is said to be $t-$ norm if it satisfies the following conditions :*

1. *$*$ is commutative and associative,*
2. *$x * y \leq x * z$ whenever $y \leq z$ for all $x, y, z \in [0,1]$,*
3. *$x * 1 = x$.*

Example 7. *[18] The following are important examples for $t-$norms :*

1. *Minimum* $*_m(x, y) = \min\{x, y\}$,
2. *Product* $*_p(x, y) = x.y$,
3. *Lukasiewicz t-norm* $*_L = \max(x + y - 1, 0)$,
4. *Drastic product*

$$*_d = \begin{cases} 0, & \textit{if } (x, y) \in [0, 1)^2 \\ \min(x, y), & \textit{otherwise} \end{cases}$$

Definition 8. *[18] A binary operation $\diamond : [0, 1] \times [0, 1] \to [0, 1]$ is said to be continuous $t-$ conorm if it satisfies the following conditions :*

1. *$\diamond$ is commutative and associative,*
2. *$x \diamond y \leq x \diamond z$ whenever $y \leq z$ for all $x, y, z \in [0, 1]$,*
3. *$x \diamond 0 = x$.*

Example 9. *[18] The following are important examples for $t-$conorms :*

1. *Maximum* $\diamond_m(x, y) = \max\{x, y\}$,
2. *Probabilistic sum* $\diamond_p(x, y) = x + y - x.y$,
3. *Lukasiewicz $t-$conorm* $\diamond_L = \min(x + y, 1)$,
4. *Drastic sum*

$$\diamond_d = \begin{cases} 1, & \textit{if } (x, y) \in (0, 1]^2 \\ \max(x, y), & \textit{otherwise} \end{cases}$$

A binary operation $\diamond : [0, 1] \times [0, 1] \to [0, 1]$ is a $t-$conorm if and only if there exists a $t-$norm $*$ such that for all $(x.y) \in [0, 1]^2$ either one of the two equivalent equalities holds:

$$x \diamond y = 1 - (1 - x) * (1 - y), \tag{1}$$

$$x * y = 1 - (1 - x) \diamond (1 - y), \tag{2}$$

The $t-$conorm given by (1) is called the dual $t-$conorm of $*$ and, analogously, the $t-$norm given by (2) is said to be the dual $t-$norm of $\diamond$. Obviously, $(*_m, \diamond_m)$, $(*_p, \diamond_p), (*_L, \diamond_L)$ and $(*_d, \diamond_d)$ are pairs of $t-$norms and $t-$conorms which are mutually dual to each other.
The duality expressed in (1) allows us to translate many properties of $t-$norms into the corresponding properties of $t-$conorms, including the n ary and infinitary extensions of a $t-$conorm. The duality changes the order: if for some $t-$norms t_1 and t_2 we have $t_1 \leq t_2$, and if s_1 and s_2 are the dual $t-$conorms of t_1 and t_2, respectively, then we get $s_1 \geq s_2$.

A $t-$norm $* : [0,1] \times [0,1] \to [0,1]$ is continuous if for all convergent sequences $(x_n), (y_n) \in [0,1]^{\mathbb{N}}$, we have

$$\lim_{n\to\infty} x_n * \lim_{n\to\infty} y_n = \lim_{n\to\infty} x_n * y_n.$$

Obviously, the continuity of a $t-$conorm $\diamond$ is equivalent to the continuity of the dual $t-$norm $*$.

Definition 10. *[27] The five-tuple $(X, \phi, \psi, *, \diamond)$ is said to be an intuitionistic fuzzy normed space (briefly, IFNS) if X is a vector space, ϕ and ψ are fuzzy sets on $X \times (0, \infty)$, $*$ is a continuous t-norm, and $\diamond$ is a continuous t-conorm satisfying the following conditions, for every $s, t > 0$; $x, y \in X$*

1. $\phi(x,t) > 0$,
2. $\phi(x,t) + \psi(x,t) \leq 1$,
3. $\phi(ax,t) = \phi(x, \frac{t}{|a|})$ *for each* $a \neq 0,$, $a \in \mathbb{R}$
4. $\phi(x,t) = 1$ *if and only if* $x = 0$,
5. $\phi(x,t) * \phi(y,s) \leq \phi(x+y, t+s)$,
6. $\phi(x,.) : (0,\infty) \to [0,1]$ *is continuous,*
7. $\lim_{t\to\infty} \phi(x,t) = 1$ *and* $\lim_{t\to 0} \phi(x,t) = 0$,
8. $\psi(x,t) < 1$,
9. $\psi(x,t) \diamond \psi(y,s) \geq \psi(x+y, t+s)$,
10. $\psi(x,t) = 0$ *if and only if* $x = 0$,
11. $\psi(ax,t) = \psi(x, \frac{t}{|a|})$ *for each* $a \neq 0,$, $a \in \mathbb{R}$
12. $\psi(x,.) : (0,\infty) \to [0,1]$ *is continuous,*
13. $\lim_{t\to\infty} \psi(x,t) = 0$ *and* $\lim_{t\to 0} \psi(x,t) = 1$.

In this case, (ϕ, ψ) is said to be an intuitionistic fuzzy norm.

Hazarika et al. [13] introduced $\mathcal{I}_\lambda$- convergence in intuitionistic fuzzy normed linear spaces and defined it as follows :

Definition 11. *Let $(X, \mu, \nu, *, \diamond)$ be IFNS and $\mathcal{I} \subset 2^{\mathbb{N}}$ admissible ideal. A sequence $x = (x_i)$ is said to be $\mathcal{I}_\lambda$-convergent to a number $L \in X$ with respect to the intuitionistic fuzzy norm (ϕ, ψ) if, for every $\epsilon > 0$ and for all $s > 0$, the set*

$$\{n \in \mathbb{N} : \phi(T_n(x_i) - L, s) \leq 1 - \epsilon \text{ or } \psi(T_n(x_i) - L, s) \geq \epsilon\} \in \mathcal{I},$$

where $T_n(x_i) = \dfrac{1}{\lambda_n} \sum_{i \in J_n} x_i$.

Şengönül [32] defined the sequence $y = (y_j)$,

$$y_j = qx_j + (1-q)x_{j-1}$$

where $x_{-1} = 0$, $q \neq 1$, $1 < q < \infty$, which is frequently defined as the Z^q -transformation of the sequence $x = (x_j)$, where the matrix $Z^q = (z_{jk})$ is defined by

$$z_{jk} = \begin{cases} q, & (j = k) \\ 1-q, & (j-1 = k) \qquad (j,k \in \mathbb{N}) \\ 0, & \text{otherwise.} \end{cases} \tag{3}$$

Şengönül[32] introduced the Zweier sequence spaces $\mathcal{Z}$ and $\mathcal{Z}_0$ as follows

$$\mathcal{Z} = \{x = (x_k) \in \omega : Z^q x \in c\}$$
$$\mathcal{Z}_0 = \{x = (x_k) \in \omega : Z^q x \in c_0\}.$$

Moreover, Khan et al.[17] extended Şengönül work and introduced the following sequences :

$${}_2\mathcal{Z}^I = \{x = (x_{jk}) \in \omega_2 : \{(j,k) \in \mathbb{N}\times\mathbb{N} : \mathcal{I}-\lim Z^q x = L \ \text{ for some } \ L \in C\}\} \in \mathcal{I};$$

$${}_2\mathcal{Z}_0^I = \{x = (x_{jk}) \in \omega_2 : \{(j,k) \in \mathbb{N}\times\mathbb{N} : \mathcal{I} - \lim Z^q x = 0\}\} \in \mathcal{I},$$

where ω_2 denote the space of all double sequences.

3. MAIN RESULTS

Throughout the article, for the sake of convenience, we denote $Z^q x = Z^q x_{ij}$ for the sequence $x = (x_{ij}) \in X$. We also take $\mathcal{I}^2$ as a nontrivial admissible ideal of $\mathbb{N}\times\mathbb{N}$.

Definition 12. *Let $(X, \phi, \psi, *, \diamond)$ be an IFNS and $\mathcal{I}^2$ is an admissible ideal. A double sequence $x = (x_{ij})$ in X is said to be $(\lambda,\mu)-$ Zweier ideal convergent to L with respect to the intuitionistic fuzzy norm (ϕ,ψ), if for every $\epsilon > 0$ and for all $s > 0$, the set*

$$\Big\{(n,m) \in \mathbb{N}\times\mathbb{N} : \frac{1}{\lambda_n\mu_m}\sum_{i\in J_n}\sum_{j\in J_m}\phi(Z^q x_{ij} - L, s) \leq 1-\epsilon \ \mathit{or}$$

$$\frac{1}{\lambda_n\mu_m}\sum_{i\in J_n}\sum_{j\in J_m}\psi(Z^q x_{ij} - L, s) \geq \epsilon\Big\} \in \mathcal{I}^2.$$

.

We write $(\phi,\psi) - I^2_{(\lambda,\mu)} - \lim_{i,j\to\infty} Z^q x_{ij} = L$

The proof of the Lemma given below are straightforward, therefore omitted.

Lemma 13. *Let $\mathcal{I}^2$ be an admissible ideal, and $(X, \phi, \psi, *, \diamond)$ be IFNS, and $x = (x_{ij})$ be a double sequence in X. Then for each $s > 0$ and $\epsilon > 0$, the following statements are equivalent.*

1. $(\phi,\psi)-I^2_{(\lambda,\mu)}-\lim_{i,j\to\infty} Z^q x_{ij}=L$
2. $\Big\{(n,m)\in\mathbb{N}\times\mathbb{N}:\dfrac{1}{\lambda_n\mu_m}\sum\limits_{i\in J_n}\sum\limits_{j\in J_m}\phi(Z^q x_{ij}-L,s)\le 1-\epsilon\}\in\mathcal{I}^2$ *and* $\Big\{(n,m)\in\mathbb{N}\times\mathbb{N}:\dfrac{1}{\lambda_n\mu_m}\sum\limits_{i\in J_n}\sum\limits_{j\in J_m}\psi(Z^q x_{ij}-L,s)\ge\epsilon\Big\}\in\mathcal{I}^2$.
3. $\Big\{(n,m)\in\mathbb{N}\times\mathbb{N}:\dfrac{1}{\lambda_n\mu_m}\sum\limits_{i\in J_n}\sum\limits_{j\in J_m}\phi(Z^q x_{ij}-L,s)>1-\epsilon\}\in\mathcal{F}(\mathcal{I}^2)$ *and* $\Big\{(n,m)\in\mathbb{N}\times\mathbb{N}:\dfrac{1}{\lambda_n\mu_m}\sum\limits_{i\in J_n}\sum\limits_{j\in J_m}\psi(Z^q x_{ij}-L,s)<\epsilon\Big\}\in\mathcal{F}(\mathcal{I}^2)$.
4. $\Big\{(n,m)\in\mathbb{N}\times\mathbb{N}:\dfrac{1}{\lambda_n\mu_m}\sum\limits_{i\in J_n}\sum\limits_{j\in J_m}\phi(Z^q x_{ij}-L,s)\le 1-\epsilon$ *or* $\dfrac{1}{\lambda_n\mu_m}\sum\limits_{i\in J_n}\sum\limits_{j\in J_m}\psi(Z^q x_{ij}-L,s)\ge\epsilon\Big\}\in\mathcal{I}^2$.
5. $(\phi,\psi)-I^2_{(\lambda,\mu)}-\lim\limits_{i,j\to\infty}\phi(Z^q x_{ij}-L,s)=1$ *and*
$$(\phi,\psi)-I^2_{(\lambda,\mu)}-\lim_{i,j\to\infty}\psi(Z^q x_{ij}-L,s)=0.$$

Theorem 14. *Let $(X,\phi,\psi,*,\diamond)$ be IFNS and $\mathcal{I}^2\subset 2^{\mathbb{N}\times\mathbb{N}}$ be an ideal. A double sequence $x=(x_{ij})$ in X is $(\lambda,\mu)-$ Zweier ideal convergent with respect to the intuitionistic fuzzy norm (ϕ,ψ), then its limit is unique.*

Proof. Let $(\phi,\psi)-I^2_{(\lambda,\mu)}-\lim\limits_{i,j\to\infty} Z^q x_{ij}=L_1$ and $(\phi,\psi)-I^2_{(\lambda,\mu)}-\lim\limits_{i,j\to\infty} Z^q x_{ij}=L_2$. For a given $\epsilon>0$, select $\alpha>0$ in such a way that $(1-\alpha)*(1-\alpha)>1-\epsilon$ and $\alpha\diamond\alpha<\epsilon$. Then, for any $t>0$, define

$$\begin{aligned}
P_{\phi,1}(\alpha,s) &= \Big\{(n,m)\in\mathbb{N}\times\mathbb{N}:\frac{1}{\lambda_n\mu_m}\sum_{i\in J_n}\sum_{j\in J_m}\phi\Big(Z^q x_{ij}-L_1,\frac{s}{2}\Big)\le 1-\alpha\Big\},\\
P_{\phi,2}(\alpha,s) &= \Big\{(n,m)\in\mathbb{N}\times\mathbb{N}:\frac{1}{\lambda_n\mu_m}\sum_{i\in J_n}\sum_{j\in J_m}\phi\Big(Z^q x_{ij}-L_2,\frac{s}{2}\Big)\le 1-\alpha\Big\},\\
P_{\psi,1}(\alpha,s) &= \Big\{(n,m)\in\mathbb{N}\times\mathbb{N}:\frac{1}{\lambda_n\mu_m}\sum_{i\in J_n}\sum_{j\in J_m}\psi\Big(Z^q x_{ij}-L_1,\frac{s}{2}\Big)\ge \alpha\Big\},\\
P_{\psi,2}(\alpha,s) &= \Big\{(n,m)\in\mathbb{N}\times\mathbb{N}:\frac{1}{\lambda_n\mu_m}\sum_{i\in J_n}\sum_{j\in J_m}\psi\Big(Z^q x_{ij}-L_2,\frac{s}{2}\Big)\ge \alpha\Big\}.
\end{aligned}$$

Since $(\phi,\psi)-I^2_{(\lambda,\mu)}-\lim\limits_{i,j\to\infty} Z^q x_{ij}=L_1$, we obtain $P_{\phi,1}(\alpha,s)$ and $P_{\psi,1}(\alpha,s)\in\mathcal{I}^2$. Moreover, using $(\phi,\psi)-I^2_{(\lambda,\mu)}-\lim\limits_{i,j\to\infty} Z^q x_{ij}=L_2$, we have $P_{\phi,2}(\alpha,s)$ and $P_{\psi,2}(\alpha,s)\in\mathcal{I}^2$. Now, suppose that

$$P_{\phi,\psi}(\alpha,s)=[P_{\phi,1}(\alpha,s)\cup P_{\phi,2}(\alpha,s)]\cap[P_{\psi,1}(\alpha,s)\cup P_{\psi,2}(\alpha,s)].$$

Thus, $P_{\phi,\psi}(\alpha,s)\in\mathcal{I}^2$, implies $P^c_{\phi,\psi}(\alpha,s)$ is a nonempty set in $\mathcal{F}(\mathcal{I}^2)$.
If $(n,m)\in P^c_{\phi,\psi}(\alpha,s)$, then two possibilities arises :

Either $(n,m) \in P^c_{\phi,1}(\alpha,s) \cap P_{\phi,2}(\alpha,s)$ or $(n,m) \in P^c_{\psi,1}(\alpha,s) \cap P_{\psi,2}(\alpha,s)$. Firstly, we consider that $(n,m) \in P^c_{\phi,1}(\alpha,s) \cap P^c_{\phi,2}(\alpha,s)$. Then, we get

$$\frac{1}{\lambda_n \mu_m} \sum_{i\in J_n} \sum_{j \in J_m} \phi\Big(Z^q x_{ij} - L_1, \frac{s}{2}\Big) > 1-\alpha$$

and

$$\frac{1}{\lambda_n \mu_m} \sum_{i\in J_n} \sum_{j \in J_m} \phi\Big(Z^q x_{ij} - L_2, \frac{s}{2}\Big) > 1-\alpha.$$

Now, take $(k,l) \in \mathbb{N} \times \mathbb{N}$ such that

$$\phi\Big(Z^q x_{kl} - L_1, \frac{s}{2}\Big) > \frac{1}{\lambda_n \mu_m} \sum_{i\in J_n} \sum_{j \in J_m} \phi\Big(Z^q x_{ij} - L_1, \frac{s}{2}\Big) > 1-\alpha$$

and

$$\phi\Big(Z^q x_{kl} - L_2, \frac{s}{2}\Big) > \frac{1}{\lambda_n \mu_m} \sum_{i\in J_n} \sum_{j \in J_m} \phi\Big(Z^q x_{ij} - L_2, \frac{s}{2}\Big) > 1-\alpha$$

e.g., Take into account, max $\Big\{\phi(Z^q x_{ij} - L_1, \frac{s}{2}), \psi(Z^q x_{ij} - L_2, \frac{s}{2}) : i \in J_n; j \in J_m\Big\}$ and
select (i,j) as (k,l) for which maximum occurs. Then we get

$$\phi(L_1 - L_2, s) \geq \phi(Z^q x_{kl} - L_1, \frac{s}{2}) * \phi(Z^q x_{kl} - L_2, \frac{s}{2}) > (1-\alpha) * (1-\alpha) > 1-\epsilon.$$

Since $\epsilon > 0$ was arbitrary, for every $t > 0$, we obtain $\phi(L_1 - L_2, t) = 1$, which provides $L_1 = L_2$. Under other conditions, if $(n,m) \in P^c_{\psi,1}(\alpha,s) \cap P^c_{\psi,2}(\alpha,s)$. then, on similar manner we can prove that $\psi(L_1 - L_2, s) < \epsilon$ for all $t > 0$. Therefore, we have $\psi(L_1 - L_2, s) = 0$, for all $t > 0$, which yields $L_1 = L_2$. Hence, in all cases, we achieve that $I^2_{(\lambda,\mu)}-$limit is unique.

□

Now, we define the notion of (λ,μ)-Zweier convergence of double sequences in an IFNS :

Definition 15. *Let $(X, \phi, \psi, *, \diamond)$ be an IFNS. A double sequence $x = (x_{ij})$ in X is said to be (λ,μ)- Zweier convergent to L with respect to the intuitionistic fuzzy norm (ϕ,ψ), if for every ϵ, $s > 0$, there exists $n_0 \in \mathbb{N}$ such that*

$$\frac{1}{\lambda_n \mu_m} \sum_{i\in J_n} \sum_{j \in J_m} \phi(Z^q x_{ij} - L, s) > 1-\epsilon \text{ and}$$

$$\frac{1}{\lambda_n \mu_m} \sum_{i\in J_n} \sum_{j \in J_m} \psi(Z^q x_{ij} - L, s) < \epsilon, \text{ for all } m,n \geq n_0.$$

We write $(\lambda,\mu) - \lim_{i,j\to\infty} Z^q x_{ij} = L$

Theorem 16. *Let $(X,\phi,\psi,*,\diamond)$ be IFNS. A double sequence $x=(x_{ij})$ in X is $(\lambda,\mu)-$ Zweier convergent with respect to the intuitionistic fuzzy norm (ϕ,ψ), then its limit is unique.*

Proof. Let $(\lambda,\mu)-\lim_{i,j\to\infty} Z^q x_{ij}=L_1$ and $(\lambda,\mu)-\lim_{i,j\to\infty} Z^q x_{ij}=L_2$. For a given $\epsilon>0$, select $\alpha>0$ in such a way that $(1-\alpha)*(1-\alpha)>1-\epsilon$ and $\alpha\diamond\alpha<\epsilon$. Then, for any $s>0$, there exists $n_1\in\mathbb{N}$ such that

$$\frac{1}{\lambda_n\mu_m}\sum_{i\in J_n}\sum_{j\in J_m}\phi(Z^q x_{ij}-L_1,s)>1-\epsilon \text{ and}$$

$$\frac{1}{\lambda_n\mu_m}\sum_{i\in J_n}\sum_{j\in J_m}\psi(Z^q x_{ij}-L_1,s)<\epsilon, \text{ for all } m,n\geq n_1.$$

Also, there exists $n_2\in\mathbb{N}$ such that

$$\frac{1}{\lambda_n\mu_m}\sum_{i\in J_n}\sum_{j\in J_m}\phi(Z^q x_{ij}-L_2,s)>1-\epsilon \text{ and}$$

$$\frac{1}{\lambda_n\mu_m}\sum_{i\in J_n}\sum_{j\in J_m}\psi(Z^q x_{ij}-L_2,s)<\epsilon, \text{ for all } m,n\geq n_2.$$

Now, consider $n_0=\max\{n_1,n_2\}$. Then for all $n\geq n_0$, we will get a $(k,l)\in\mathbb{N}\times\mathbb{N}$ such that

$$\phi(Z^q x_{kl}-L_1,s)>\frac{1}{\lambda_n\mu_m}\sum_{i\in J_n}\sum_{j\in J_m}\phi(Z^q x_{ij}-L_1,s)>1-\alpha$$

and

$$\phi(Z^q x_{kl}-L_2,s)>\frac{1}{\lambda_n\mu_m}\sum_{i\in J_n}\sum_{j\in J_m}\phi(Z^q x_{ij}-L_2,s)>1-\alpha.$$

Hence, we get

$$\begin{aligned}\phi(L_1-L_2,s)&\geq\phi(Z^q x_{kl}-L_1,s)*\phi(Z^q x_{kl}-L_2,s)\\&>(1-\alpha)*(1-\alpha)>1-\epsilon.\end{aligned}$$

Since $\epsilon>0$ is arbitrary, we have $\phi(L_1-L_2,s)=1$ for all $s>0$.
Similarly, we can show that $\psi(L_1-L_2,s)=0$ for all $s>0$., implies $L_1=L_2$. □

Theorem 17. *Let $(X,\phi,\psi,*,\diamond)$ be IFNS and $x=(x_{ij})$ be a double sequence in X such that $(\lambda,\mu)-\lim_{i,j\to\infty} Z^q x_{ij}=L$, then $(\phi,\psi)-I^2_{(\lambda,\mu)}-\lim_{i,j\to\infty} Z^q x_{ij}=L$*

Proof. Let $(\lambda,\mu)-\lim_{i,j\to\infty} Z^q x_{ij}=L$. For each $s>0$ and $\epsilon>0$ there exists a positive integer $N_0\in\mathbb{N}$ such that

$\frac{1}{\lambda_n\mu_m}\sum_{i\in J_n}\sum_{j\in J_m}\phi(Z^q x_{ij}-L,s)>1-\epsilon$ and $\frac{1}{\lambda_n\mu_m}\sum_{i\in J_n}\sum_{j\in J_m}\psi(Z^q x_{ij}-L,s)<\epsilon$ for each $n,m\geq N_0$. Then the set

$$Q(\epsilon,s)=\Big\{(n,m)\in\mathbb{N}\times\mathbb{N}:\frac{1}{\lambda_n\mu_m}\sum_{i\in J_n}\sum_{j\in J_m}\phi(Z^q x_{ij}-L,s)\leq 1-\epsilon \text{ or}$$

$$\frac{1}{\lambda_n\mu_m}\sum_{i\in J_n}\sum_{j\in J_m}\psi(Z^q x_{ij}-L,s)\geq\epsilon\Big\}$$

$Q(\epsilon,s)\subseteq\{(1,1),(1,2),(2,1),(2,2),...,(N_0-1,N_0-1)\}$. This implies set $Q(\epsilon,s)$ has at most finitely many terms. Since, ideal $\mathcal{I}^2$ being admissible, we have $Q(\epsilon,s)\in\mathcal{I}^2$

This show that $(\phi,\psi)-I^2_{(\lambda,\mu)}-\lim_{i,j\to\infty}Z^q x_{ij}=L$

□

Theorem 18. *Let $(X,\phi,\psi,*,\diamond)$ be IFNS and $x=(x_{ij})$ be a double sequence in X such that $(\lambda,\mu)-\lim_{i,j\to\infty}Z^q x_{ij}=L$, then there exists a subsequence $(x_{i_kj_l})$ of $x=(x_{ij})$ such that $(\lambda,\mu)-\lim Z^q x_{i_kj_l}=L$.*

Proof. Let $(\lambda,\mu)-\lim_{i,j\to\infty}Z^q x_{ij}=L$. Then for every $s>0,\epsilon>0$ there exists a positive integer $n_0\in\mathbb{N}$ such that
$\frac{1}{\lambda_n\mu_m}\sum_{i\in J_n}\sum_{j\in J_m}\phi(Z^q x_{ij}-L,s)>1-\epsilon$ and $\frac{1}{\lambda_n\mu_m}\sum_{i\in J_n}\sum_{j\in J_m}\psi(Z^q x_{ij}-L,s)<\epsilon$ for each $n,m\geq n_0$.
It is clear that, for each $n,m\geq n_0$, one can choose $i_k\in J_n$ and $j_l\in J_m$ in such a way that

$$\phi(Z^q x_{i_kj_l}-L,s)>\frac{1}{\lambda_n\mu_m}\sum_{i\in J_n}\sum_{j\in J_m}\phi(Z^q x_{ij}-L,s)>1-\epsilon$$

and

$$\psi(Z^q x_{i_kj_l}-L,s)<\frac{1}{\lambda_n\mu_m}\sum_{i\in J_n}\sum_{j\in J_m}\psi(Z^q x_{ij}-L,s)<\epsilon.$$

Hence, $(\lambda,\mu)-\lim Z^q x_{i_kj_l}=L$. □

Definition 19. *Let $(X,\phi,\psi,*,\diamond)$ be an IFNS. A double sequence $x=(x_{ij})$ in X is said to be (λ,μ)- Zweier Cauchy sequence with respect to the intuitionistic fuzzy norm (ϕ,ψ), if for every ϵ, $s>0$, there exists $n_0\in\mathbb{N}$ such that*

$$\frac{1}{\lambda_n\mu_m}\sum_{i,p\in J_n}\sum_{j,q\in J_m}\phi(Z^q x_{ij}-Z^q x_{kl},s)>1-\epsilon \textit{ and}$$

$$\frac{1}{\lambda_n\mu_m}\sum_{i,p\in J_n}\sum_{j,q\in J_m}\psi(Z^q x_{ij}-Z^q x_{kl},s)<\epsilon,\ \textit{for all } m,n\geq n_0.$$

Definition 20. *Let $(X,\phi,\psi,*,\diamond)$ be an IFNS and $\mathcal{I}^2 \subset 2^{\mathbb{N}\times\mathbb{N}}$ be an ideal. A double sequence $x=(x_{ij})$ in X is said to be $(\lambda,\mu)-$ Zweier ideal Cauchy sequence with respect to the intuitionistic fuzzy norm (ϕ,ψ), if for every $\epsilon,\ \ t>0$, there exists $n_0\in\mathbb{N}$ such that*

$$\left\{(n,m)\in\mathbb{N}\times\mathbb{N}:\frac{1}{\lambda_n\mu_m}\sum_{i,k\in J_n}\sum_{j,l\in J_m}\phi(Z^q x_{ij}-Z^q x_{kl},s)>1-\epsilon\right\}\in\mathcal{F}(\mathcal{I}^2)$$

and $$\left\{(n,m)\in\mathbb{N}\times\mathbb{N}:\frac{1}{\lambda_n\mu_m}\sum_{i,k\in J_n}\sum_{j,l\in J_m}\psi(Z^q x_{ij}-Z^q x_{kl},s)<\epsilon,\right\}\in\mathcal{F}(\mathcal{I}^2)$$

In the next theorem, we establish relation between the $(\lambda,\mu)-$ Zweier ideal convergent sequences and $(\lambda,\mu)-$ Zweier ideal Cauchy sequences.

Theorem 21. *Let $(X,\phi,\psi,*,\diamond)$ be an IFNS. A double sequence $x=(x_{ij})$ in X is $(\lambda,\mu)-$ Zweier ideal convergent with respect to the intuitionistic fuzzy norm (ϕ,ψ) if and only if it is $(\lambda,\mu)-$ Zweier ideal Cauchy sequence with respect to the same norm.*

Proof. Suppose that sequence $x=(x_{ij})$ is (λ,μ)- Zweier ideal Cauchy but not (λ,μ)- Zweier ideal convergent with respect to the intuitionistic fuzzy norm (ϕ,ψ). Then there exist positive integers k,l in such a way that if we choose

$$R(\epsilon,s)=\Big\{(n,m)\in\mathbb{N}\times\mathbb{N}:\frac{1}{\lambda_n\mu_m}\sum_{i,k\in J_n}\sum_{j,l\in J_m}\phi(Z^q x_{ij}-Z^q x_{kl},s)\le 1-\epsilon\ \text{ or }$$

$$\frac{1}{\lambda_n\mu_m}\sum_{i,k\in J_n}\sum_{j,l\in J_m}\psi(Z^q x_{ij}-Z^q x_{kl},s)\ge\epsilon\Big\}$$

and

$$S(\epsilon,s)=\Big\{(n,m)\in\mathbb{N}\times\mathbb{N}:\frac{1}{\lambda_n\mu_m}\sum_{i\in J_n}\sum_{j\in J_m}\phi(Z^q x_{ij}-L,\frac{s}{2})\le 1-\epsilon\ \text{ and }$$

$$\frac{1}{\lambda_n\mu_m}\sum_{i\in J_n}\sum_{j\in J_m}\psi(Z^q x_{ij}-L,s)\ge\epsilon\Big\}.$$

Then $R(\epsilon,s)\in\mathcal{I}^2$ implies $R^C(\epsilon,s)\in\mathcal{F}(\mathcal{I}^2)$. Since

$$\frac{1}{\lambda_n\mu_m}\sum_{i,k\in J_n}\sum_{j,l\in J_m}\phi(Z^q x_{ij}-Z^q x_{kl},s)\ge\frac{2}{\lambda_n\mu_m}\sum_{i\in J_n}\sum_{j\in J_m}\phi(Z^q x_{ij}-L,\frac{s}{2})>1-\epsilon$$

and

$$\frac{1}{\lambda_n\mu_m}\sum_{i,k\in J_n}\sum_{j,l\in J_m}\psi(Z^q x_{ij}-Z^q x_{kl},s)\le\frac{2}{\lambda_n\mu_m}\sum_{i\in J_n}\sum_{j\in J_m}\psi(Z^q x_{ij}-L,\frac{s}{2})<\epsilon.$$

If $\frac{1}{\lambda_n\mu_m}\sum_{i\in J_n}\sum_{j\in J_m}\phi(Z^q x_{ij}-L,\frac{s}{2}) > \frac{1-\epsilon}{2}$ and $\frac{1}{\lambda_n\mu_m}\sum_{i\in J_n}\sum_{j\in J_m}\psi(Z^q x_{ij}-L,\frac{s}{2}) < \frac{\epsilon}{2}$, respectively. Then, we obtain

$$\delta_{(\lambda,\mu)}\Big(\Big\{(n,m)\in\mathbb{N}\times\mathbb{N}: \frac{1}{\lambda_n\mu_m}\sum_{i,k\in J_n}\sum_{j,l\in J_m}\phi(Z^q x_{ij}-Z^q x_{kl},s) > 1-\epsilon \text{ and}$$

$$\frac{1}{\lambda_n\mu_m}\sum_{i,k\in J_n}\sum_{j,l\in J_m}\psi(Z^q x_{ij}-Z^q x_{kl},s) < \epsilon\Big\}\Big) = 0,$$

that is, $R(\epsilon,s)\in\mathcal{F}(\mathcal{I}^2)$, which contadicts our assumption. Therefore, sequence $x=(x_{ij})$ is (λ,μ)- Zweier ideal convergent with respect to the intuitionistic fuzzy norm (ϕ,ψ).

Conversely, Suppose that $(\phi,\psi)-I^2_{(\lambda,\mu)}-\lim_{i,j\to\infty} Z^q x_{ij}=L$. Choose $\gamma>0$, in such a way that $(1-\gamma)*(1-\gamma)>1-\epsilon$ and $\gamma\diamond\gamma<\epsilon$. For all $t>0$, define

$$E(\gamma,s)=\Big\{(n,m)\in\mathbb{N}\times\mathbb{N}: \frac{1}{\lambda_n\mu_m}\sum_{i\in J_n}\sum_{j\in J_m}\phi(Z^q x_{ij}-L,\frac{s}{2})\le 1-\gamma \text{ or}$$

$$\frac{1}{\lambda_n\mu_m}\sum_{i\in J_n}\sum_{j\in J_m}\psi(Z^q x_{ij}-L,\frac{s}{2})\ge\gamma\Big\}\in\mathcal{I}^2.$$

This implies $E(\gamma,s)^C\in\mathcal{F}(\mathcal{I}^2)$. Assume $(k,l)\in E^c(\gamma,s)$. Then, we have

$$\frac{1}{\lambda_n\mu_m}\sum_{k\in J_n}\sum_{l\in J_m}\phi(Z^q x_{ij}-L,\frac{s}{2})>1-\gamma$$

and

$$\frac{1}{\lambda_n\mu_m}\sum_{k\in J_n}\sum_{l\in J_m}\psi(Z^q x_{ij}-L,\frac{s}{2})<\gamma.$$

For every $\epsilon>0$, we take

$$H(\epsilon,s)=\Big\{(n,m)\in\mathbb{N}\times\mathbb{N}: \frac{1}{\lambda_n\mu_m}\sum_{i,k\in J_n}\sum_{j,l\in J_m}\phi(Z^q x_{ij}-Z^q x_{kl},s)\le 1-\epsilon \text{ or}$$

$$\frac{1}{\lambda_n\mu_m}\sum_{i,k\in J_n}\sum_{j,l\in J_m}\psi(Z^q x_{ij}-Z^q x_{kl},s)\ge\epsilon\Big\}.$$

Now, we have to show that $H(\epsilon,s)\subset E(\gamma,s)$. Let $(u,v)\in H(\epsilon,s)$, we have

$$\frac{1}{\lambda_n\mu_m}\sum_{u,k\in J_n}\sum_{v,l\in J_m}\phi(Z^q x_{uv}-Z^q x_{kl},\frac{s}{2})\le 1-\epsilon$$

and

$$\frac{1}{\lambda_n\mu_m}\sum_{u,k\in J_n}\sum_{v,l\in J_m}\psi(Z^q x_{uv}-Z^q x_{kl},\frac{s}{2})\ge\epsilon.$$

On the basis of above inequality, we distinguish the following two cases as follows:

Case 1: Let $\frac{1}{\lambda_n \mu_m} \sum_{u,k \in J_n} \sum_{v,l \in J_m} \phi(Z^q x_{uv} - Z^q x_{kl}, s) \leq 1 - \epsilon$. Then $\frac{1}{\lambda_n \mu_m} \sum_{u \in J_n} \sum_{v \in J_m} \phi(Z^q x_{uv} - L, \frac{s}{2}) \leq 1 - \gamma$, therefore $(u, v) \in E(\gamma, t)$. Otherwise, if $\frac{1}{\lambda_n \mu_m} \sum_{u \in J_n} \sum_{v \in J_m} \phi(Z^q x_{uv} - L, \frac{s}{2}) > 1 - \gamma$. Then, we have

$$\begin{aligned}
1 - \epsilon &\geq \frac{1}{\lambda_n \mu_m} \sum_{u,k \in J_n} \sum_{v,l \in J_m} \phi(Z^q x_{uv} - Z^q x_{kl}, s) \\
&\geq \frac{1}{\lambda_n \mu_m} \sum_{u \in J_n} \sum_{v \in J_m} \phi\left(Z^q x_{uv} - L, \frac{s}{2}\right) * \frac{1}{\lambda_n \mu_m} \sum_{k \in J_n} \sum_{l \in J_m} \phi\left(Z^q x_{kl} - L, \frac{s}{2}\right) \\
&> (1 - \gamma) * (1 - \gamma) \\
&> 1 - \epsilon
\end{aligned}$$

a contradiction. Therefore $H(\epsilon, s) \subset E(\gamma, s)$.

Case 2: Consider $\frac{1}{\lambda_n \mu_m} \sum_{u,p \in J_n} \sum_{v,q \in J_m} \psi(Z^q x_{uv} - Z^q x_{kl}, s) \geq \epsilon$.

We get $\frac{1}{\lambda_n \mu_m} \sum_{u \in J_n} \sum_{v \in J_m} \psi(Z^q x_{uv} - L, \frac{s}{2}) \geq \gamma$, hence $(u, v) \in E(\gamma, s)$. Otherwise, if $\frac{1}{\lambda_n \mu_m} \sum_{u \in J_n} \sum_{v \in J_m} \psi(Z^q x_{uv} - L, \frac{s}{2}) < \gamma$. Then, we obtain

$$\begin{aligned}
\epsilon &\leq \frac{1}{\lambda_n \mu_m} \sum_{u,p \in J_n} \sum_{v,q \in J_m} \psi(Z^q x_{uv} - Z^q x_{kl}, s) \\
&\leq \frac{1}{\lambda_n \mu_m} \sum_{u \in J_n} \sum_{v \in J_m} \psi(Z^q x_{uv} - L, \frac{s}{2}) \diamond \frac{1}{\lambda_n \mu_m} \sum_{k \in J_n} \sum_{l \in J_m} \psi(Z^q x_{kl} - L, \frac{s}{2}) \\
&< \gamma \diamond \gamma \\
&< \epsilon,
\end{aligned}$$

a contradiction. Hence $H(\epsilon, s) \subset E(\gamma, s)$. Therefore, we conclude that $H(\epsilon, t) \subset E(\gamma, s)$. This implies that $H(\epsilon, s) \in \mathcal{I}^2$. Hence double sequence $x = (x_{ij})$ is a Zweier ideal Cauchy sequence with respect to the intuitionistic fuzzy norm (ϕ, ψ).

□

We are stating the following theorems without proof, since the proof is straightforward.

Theorem 22. *Let $(X, \phi, \psi, *, \diamond)$ be IFNS and $x = (x_{ij})$ be a double sequence in X is (λ, μ)- Zweier Cauchy sequence with respect to intuitionistic fuzzy norm (ϕ, ψ), then it is $(\lambda, \mu)-$ Zweier ideal Cauchy sequence with respect to the same norm.*

Theorem 23. *Let $(X, \phi, \psi, *, \diamond)$ be IFNS. If double sequence $x = (x_{ij})$ in X is (λ, μ)- Zweier Cauchy sequence with respect to intuitionistic fuzzy norm (ϕ, ψ), then there is a subsequence $(x_{i_k j_l})$ of sequence $x = (x_{ij})$ which is a ordinary Zweier Cauchy sequence with respect to the same norm.*

Acknowledgement: The authors would like to record their gratitude to the reviewer for her/his careful reading and making some useful corrections which improved the presentation of the paper.

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