

AN ECONOMIC PRODUCTION QUANTITY (EPQ) MODEL FOR A DETERIORATING ITEM WITH PARTIAL TRADE CREDIT POLICY FOR PRICE DEPENDENT DEMAND UNDER INFLATION AND RELIABILITY

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Abstract: It is well-known that the production-inventory problem for deteriorating items in the supply chain is a challenge when deciding on how many products to manufacture to obtain a maximum total profit. This research work develops an economic production quantity model for a deteriorating item under partial trade credit policy considering inflation, the effect of reliability factor of a production system, and the demand depending on the price of a product whose selling price is optimized. The production-inventory model is formulated as a nonlinearly constrained optimization problem by analyzing different cases. Finally, through a numerical example, a sensitivity analysis is

performed to study the effect of different parameters, changing one parameter at a time and keeping others fixed at their original values.

Keywords: Economic Production Quantity (EPQ) Model, Partial Trade Credit Policy, Inflation, Reliability, Deterioration.

MSC: 90B05, 90B30.

1. INTRODUCTION

The production and demand rate are considered constant in classical economic production (EPQ) models, but the demand depends on many factors (price, stock, time), which in a real situation, affect it. In the actual competitive marketing situation, manufacturing firms try to fabricate the product with minimum defects. So, they have taken some most useful policies to improve the production quality of their manufacturing systems as well as to fabricate more products at minimum cost and time. Generally, a manufacturing firm strictly maintains (i) how to purchase raw materials, (ii) how to use effectively and efficiently the raw materials in the production system, (iii) how to use manpower in the production system, (iv) how to reduce the defective items, (v) how to decrease the production time and fabricate an item with high-quality level, etc. Deterioration of goods is a natural phenomenon but a vital issue in inventory management. So, inventory management can not avoid the effect of deterioration. Also, the aim of inventory management is to determine the production quantity of the product which maximizes the profit. Sanni and Chigbu [12] investigated optimal replenishment policy for deteriorating items where deterioration rate follows a three-parameter Weibull distribution. They also assumed stock dependent demand rate and partial backlogged shortages. Mahata et al. [10] discussed an ordering policy for deteriorating items with price-dependent demand. Also, in their inventory model, a permissible delay in payment is allowed. Khanna and Pritam [9] described an inventory system that optimizes preservation strategies for deteriorating items, considering stock-level dependent demand and time-dependent holding costs. Das et al. [7] developed an inventory model with partial backlogged shortages and price-dependent demand and investigated the preservation technology to preserve deteriorating items. Shaikh and Cárdenas-Barrón [13] studied an economic order quantity inventory model for non-instantaneous deteriorating items with order quantity dependent trade credit policy. They assumed that the demand rate depends on both price and advertisement. Khan et al. [8] suggested a purchase price discount facility on an inventory model for deteriorating items. They considered a selling price and stock dependent demand rate, and partial backlogged shortages.

Usually, when a retailer purchases some goods, the purchased items are payed to the supplier immediately. In a competitive business strategy as well as highly competitive marketing situations, a supplier gives different payment offers to his retailer so to make him purchase more products. According to the literature in inventory research, there are several kinds of payment policies proposed by researchers. The credit policy approach, one of the well-known policies in inventory

research, provides a supplier some time to pay to the retailer (permissible delay in payment). This type of concept is generally known as a single level-credit policy. If the retailer also offers some delay in payment to its customers, this type of credit policy is known as a two-level trade credit policy. Recently, Taleizadeh et al. [16] quantified the impact of ordering policies for a mixed sale of products under the partial trade credit scheme. Moreover, their inventory model includes complete backorder, multiple prepayments, and inspection policy. Cárdenas-Barrón et al. [5] discussed an inventory model in which holding cost and demand depend on inventory level. They also considered an allowable delay in payment. Table 1 provides a comparison of some inventory models.

Table 1: A comparison of some inventory models

Author(s)	Deterioration	Demand rate	Inflation	Level of permissible delay in payments	Reliability	EOQ/EPQ
Bhunia and Shaikh [1]	Yes	Selling price dependent	No	Single	No	EOQ
Bhunia et al. [2]	Yes	Linearly time dependent	No	No	No	EOQ
Bhunia et al. [3]	Yes	Stock dependent	No	Single	No	EOQ
Bhunia et al. [4]	Yes	Linearly time dependent	No	Single	No	EOQ
Chen et al. [6]	Yes	Constant	No	Two-level	No	EOQ
Shah and Cárdenas-Barrón [14]	Yes	Constant	No	Two-level	No	EOQ
Sharma and Chaudhary [15]	Yes	Constant	No	No	No	EOQ
Wu et al. [17]	Yes	Constant	No	Two-level	No	EOQ
This paper	Yes	Selling price dependent demand	Yes	Two level partial credit policy	Yes	EPQ

Generally, in a manufacturing process some items with defects are produced along with the well-produced ones. If the items with defects are found in the inspection process, the manufacturer repairs them to be in a useable condition. So, the defective products could be repaired, reworked, or rejected. Manna et al. [11] investigated the inspection errors effect on an imperfect production inventory model by considering that a part of imperfect items is converted into perfect items by the rework process.

This research derives an economic production quantity model for a deteriorating item under partial trade credit policy considering inflation, the effect of the reliability factor of a production system, supposing that demand depends on the price of a product. The related production-inventory model is formulated as a nonlinearly constrained optimization problem.

The rest of our paper is organized as follows: Section 2 presents the assumptions and notation. Section 3 gives the mathematical formulation of the inventory model. Section 4 solves a numerical example. Section 5 provides a sensitivity analysis. Finally, Section 6 presents the conclusions and some futures research directions.

2. ASSUMPTIONS and NOTATION

2.1. Assumptions

We propose a mathematical model based on the following assumptions: 1) produced items are reliable and repairable; 2) the lead time is negligible, and the

replenishment rate is instantaneous; 3) demand function D depends on selling price (s) of the product at any time t , i.e., $D = a - bs$; 4) the inventory system has an infinite planning horizon; 5) shortages are not permitted; 6) the inventory model is formulated under partial trade credit policy, and; 7) the inflation effect has the rate r .

2.2. Notation

The following nomenclature is used in this work:

Notation	Units	Description
Parameters:		
$I(t)$	units	Inventory level at any time t , where $0 \leq t \leq T$
P	units/unit time	Production rate
D	units/unit time	Demand rate
M	unit time	The retailer's trade credit time provided by the supplier
N	unit time	The customer's trade credit time provided by the retailer
c_0	\$/cycle	Setup cost
h	\$/unit/unit time	Holding cost
θ	$0 \leq \theta \leq 1$	Deterioration rate
c	\$/unit	Regular production cost
c_d	\$/unit	Repairable cost
r	%	Inflation rate
r_e	%	Reliability rate to produce a perfect item
α	$0 \leq \alpha \leq 1$	The portion of the payment that the customer needs to pay to its retailer when he or she places the order $0 \leq \alpha \leq 1$
I_e	\$/\$/ unit time	The interest earned by the retailer
I_p	\$/\$/ unit time	The interest paid by the retailer
$\pi(s, t_1)$	\$/ unit time	Total profit per unit time
Decision variables:		
s	\$/unit	Selling price
t_1	unit time	Production time
Dependent variable:		
T	unit time	The length of the replenishment cycle

3. MATHEMATICAL FORMULATION OF THE INVENTORY MODEL

Initially, zero stock is assumed. At time $t = 0$, the manufacturing firm starts the production. The items are manufactured at a rate of P units per unit time, and simultaneously, the customer's demand occurs at a rate of D units per unit

of time. The production rate is greater than the demand rate therefore, products in excess are stored. At time $t = t_1$, a manufacturing firm makes the decision of stopping the production and fulfills the customer's demand from stored items up to time $t = T$.

Consequently, the inventory level at any time t over the period $(0, T)$ is described by the following differential equations:

$$\frac{dI(t)}{dt} + \theta I(t) = r_e P - D, \quad 0 < t \leq t_1 \tag{1}$$

$$\frac{dI(t)}{dt} + \theta I(t) = -D, \quad t_1 < t \leq T \tag{2}$$

with the boundary conditions $I(0) = 0 = I(T)$, and considering that is $I(t)$ continuous at $t = t_1$. By solving the differential equations (1) and (2) using the boundary conditions,

$$I(t) = \frac{r_e P - D}{\theta} (1 - e^{-\theta t}), \quad 0 < t \leq t_1 \tag{3}$$

$$I(t) = \frac{D}{\theta} \{ e^{\theta(T-t)} - 1 \}, \quad t_1 < t \leq T \tag{4}$$

By using the continuity condition at $t = t_1 \Rightarrow \frac{r_e P - D}{\theta} (1 - e^{-\theta t_1}) = \frac{D}{\theta} \{ e^{\theta(T-t_1)} - 1 \}$ then

$$T = t_1 + \frac{1}{\theta} \log \left[1 + \frac{r_e P - D}{\theta} (1 - e^{-\theta t_1}) \right] \tag{5}$$

The production cost is determined by $PC = cP \int_0^{t_1} e^{-rt} dt = \frac{c}{r} P (1 - e^{-rt_1})$.

The inventory holding cost is computed with

$$C_{hol} = h \left[\int_0^{t_1} e^{-rt} I(t) dt + \int_{t_1}^T e^{-rt} I(t) dt \right] = h \left[\frac{r_e P - D}{\theta} \left\{ \frac{1}{r} (1 - e^{-rt_1}) + \frac{1}{r + \theta} (e^{-(r+\theta)t_1} - 1) \right\} + \frac{D}{\theta} \left\{ \frac{e^{\theta T}}{r + \theta} (e^{-(r+\theta)t_1} - e^{-(r+\theta)T}) + \frac{1}{r} (e^{-rT} - e^{-rt_1}) \right\} \right].$$

The repair cost of defective items is calculated with $RC = c_d(1 - r_e)DT$. The setup cost is given by $OC = c_0 e^{-rT}$. The total sales revenue is given by $SR = sD \int_0^T e^{-rt} dt = \frac{s}{r} D (1 - e^{-rT})$.

Considering the credit periods M and N then the following two scenarios occur: Scenario 1: $N < M$ and Scenario 2: $M < N$, discussed in detail below.

Scenario 1: In this situation, the customer's credit period (N) is shorter than the retailer's credit time (M). In this scenario the following four cases happen: Case 1: $M \leq t_1$, Case 2: $t_1 < M \leq T$, Case 3: $N < T \leq M$, and Case 4: $T \leq N < M$.

Case 1. $M \leq t_1$

In Case 1 the payable interest and earned interest are calculated as follows:

The interest payable is calculated with

$$\begin{aligned} IP_{11} &= cI_p \left[\int_M^{t_1} e^{-rt} I(t) dt + \int_{t_1}^T e^{-rt} I(t) dt \right] \\ &= cI_p \left[\frac{(r_e P - D)}{\theta} \left\{ \frac{1}{r} (e^{-rM} - e^{-rt_1}) + \frac{1}{r + \theta} (e^{-(r+\theta)t_1} - e^{-(r+\theta)M}) \right\} \right. \\ &\quad \left. + \frac{D}{\theta} \left\{ \frac{e^{\theta T}}{r + \theta} (e^{-(r+\theta)t_1} - e^{-(r+\theta)T}) + \frac{1}{r} (e^{-rT} - e^{-rt_1}) \right\} \right]. \end{aligned}$$

The interest earned is computed with

$$\begin{aligned} IE_{11} &= sI_e \left[\alpha D \int_0^N e^{-rt} t dt + D \int_N^M e^{-rt} t dt \right] \\ &= \frac{s}{r^2} I_e D \left[\alpha + (Nr + 1)(1 - \alpha)e^{-rN} - (Mr + 1)e^{-rM} \right]. \end{aligned}$$

Therefore, the total profit per unit time of the inventory system is given by

$$\begin{aligned} \pi_1(s, t_1) &= \frac{1}{T} \left[\langle \text{Sales revenue} \rangle + \langle \text{interest earned} \rangle - \langle \text{production cost} \rangle - \langle \text{holding cost} \rangle \right. \\ &\quad \left. - \langle \text{repair cost} \rangle - \langle \text{interest paid} \rangle - \langle \text{setup cost} \rangle \right] \\ &= \frac{1}{T} \left[SR + IE_{11} - PC - C_{hol} - RC - IP_{11} - OC \right] \end{aligned}$$

Problem 1.

Maximize $\pi_1(s, t_1)$

subject to $s > 0$, $T > t_1 > 0$, $0 < M \leq t_1$ (6)

Case 2: $t_1 < M \leq T$

In Case 2 the payable interest and the earned interest are computed as follows:

The interest payable is determined with

$$\begin{aligned} IP_{12} &= cI_p \int_M^T e^{-rt} I(t) dt \\ &= cI_p \left[\frac{D}{\theta} \left\{ \frac{e^{\theta T}}{r + \theta} (e^{-(r+\theta)M} - e^{-(r+\theta)T}) + \frac{1}{r} (e^{-rT} - e^{-rM}) \right\} \right]. \end{aligned}$$

The interest earned is obtained with

$$\begin{aligned} IE_{12} &= sI_e \left[\alpha D \int_0^N e^{-rt} t dt + D \int_N^M e^{-rt} t dt \right] \\ &= \frac{s}{r^2} I_e D \left[\alpha + (Nr + 1)(1 - \alpha)e^{-rN} - (Mr + 1)e^{-rM} \right]. \end{aligned}$$

So, the total profit per unit time of the inventory system is given by

$$\pi_2(s, t_1) = \frac{1}{T} \left[SR + IE_{12} - PC - C_{hol} - RC - IP_{12} - OC \right]$$

Problem 2.

$$\begin{aligned} &\text{Maximize } \pi_2(s, t_1) \\ &\text{subject to } s > 0, T > t_1 > 0, 0 < t_1 < M \leq T \end{aligned} \tag{7}$$

Case 3: $N < T \leq M$ In this case, the retailer’s credit period (M) is longer or equal than the cycle length (T). So, the retailer does not need to pay the interest. Thus, $IP_{13} = 0$. The retailer has earned interest from the customer under partial trade credit policy.

The interest earned is given by

$$\begin{aligned} IE_{13} &= sI_e \left[\alpha D \int_0^N e^{-rt} dt + D \int_N^T e^{-rt} dt + TD \int_T^M e^{-rt} dt \right] \\ &= \frac{s}{r^2} I_e D \left[\alpha + (Nr + 1)(1 - \alpha)e^{-rN} - rTe^{-rM} - e^{-rT} \right] \end{aligned}$$

Consequently, the total profit per unit time of the inventory system is given by

$$\pi_3(s, t_1) = \frac{1}{T} \left[SR + IE_{13} - PC - C_{hol} - RC - IP_{13} - OC \right]$$

Problem 3.

$$\begin{aligned} &\text{Maximize } \pi_3(s, t_1) \\ &\text{subject to } s > 0, T > t_1 > 0, 0 < N < T \leq M \end{aligned} \tag{8}$$

Case 4: $T \leq N < M$

Notice that in this case the cycle length (T) is less or equal than the customer’s credit period (N) as well as the retailer’s credit period (M). So, the retailer does not need to pay the interest. Thus, $IP_{14} = 0$. The retailer has earned the interest from customer under partial trade credit policy and this is calculated as follows:

The interest earned is calculated with

$$\begin{aligned} IE_{14} &= sI_e \left[\alpha D \int_0^T e^{-rt} dt + \alpha DT \int_T^N e^{-rt} dt + DT \int_N^M e^{-rt} dt \right] \\ &= \frac{s}{r^2} I_e D \left[\alpha - \alpha N(r + 1)e^{-rN} + \alpha rT(e^{-rT} - e^{-rN}) + rT(e^{-rN} - e^{-rM}) \right]. \end{aligned}$$

Thus, the total profit per unit time of the inventory system is given by

$$\pi_4(s, t_1) = \frac{1}{T} \left[SR + IE_{14} - PC - C_{hol} - RC - IP_{14} - OC \right]$$

Problem 4.

$$\begin{aligned} & \text{Maximize } \pi_4(s, t_1) \\ & \text{subject to } s > 0, T > t_1 > 0, 0 < T \leq N < M \end{aligned} \quad (9)$$

Scenario 2: $M < N$

Now, the details of the **Scenario 2** are discussed as follows. In this situation, the customer's credit period (N) from retailer is greater than the retailer's credit time (M) from supplier. In this scenario the following three cases take place: **Case 5:** $M < N \leq t_1$, **Case 6:** $t_1 < M < N \leq T$, and **Case 7:** $T \leq M < N$.

Case 5: $M < N \leq t_1$

In Case 5 the payable interest and earned interest are obtained as follows: The interest payable is determined with

$$\begin{aligned} IP_{25} &= cI_p \left[\int_M^{t_1} e^{-rt} I(t) dt + \int_{t_1}^T e^{-rt} I(t) dt \right] \\ &= cI_p \left[\frac{(r_e P - D)}{\theta} \left\{ \frac{1}{r} (e^{-rM} - e^{-rt_1}) + \frac{1}{r + \theta} (e^{-(r+\theta)t_1} - e^{-(r+\theta)M}) \right\} \right. \\ &\quad \left. + \frac{D}{\theta} \left\{ \frac{e^{\theta T}}{r + \theta} (e^{-(r+\theta)t_1} - e^{-(r+\theta)T}) + \frac{1}{r} (e^{-rT} - e^{-rt_1}) \right\} \right]. \end{aligned}$$

The interest earned is obtained with

$$IE_{25} = sI_e \alpha D \int_0^M e^{-rt} t dt = \frac{s}{r^2} I_e \alpha D \left[1 - (Mr + 1)e^{-rM} \right].$$

Hence, the total profit per unit time of the inventory system is given by

$$\pi_5(s, t_1) = \frac{1}{T} \left[SR + IE_{25} - PC - C_{hol} - RC - IP_{25} - OC \right]$$

Problem 5.

$$\begin{aligned} & \text{Maximize } \pi_5(s, t_1) \\ & \text{subject to } s > 0, T > t_1 > 0, 0 < M < N \leq t_1 \end{aligned} \quad (10)$$

Case 6: $t_1 < M < N \leq T$

In Case 6 the payable interest and the earned interest are calculated as follows:

The interest payable is given by

$$\begin{aligned} IP_{26} &= cI_p \int_M^T e^{-rt} I(t) dt \\ &= cI_p \left[\frac{D}{\theta} \left\{ \frac{e^{\theta T}}{r + \theta} (e^{-(r+\theta)M} - e^{-(r+\theta)T}) + \frac{1}{r} (e^{-rT} - e^{-rM}) \right\} \right]. \end{aligned}$$

The interest earned is calculated with

$$IE_{26} = sI_e \alpha D \int_0^M e^{-rt} t dt = \frac{s}{r^2} I_e \alpha D \left[1 - (Mr + 1)e^{-rM} \right].$$

As a result, the total profit per unit time of the inventory system is given by

$$\pi_6(s, t_1) = \frac{1}{T} \left[SR + IE_{26} - PC - C_{hol} - RC - IP_{26} - OC \right]$$

Problem 6.

$$\begin{aligned} &\text{Maximize } \pi_6(s, t_1) \\ &\text{subject to } s > 0, T > t_1 > 0, 0 < t_1 < M < N \leq T \end{aligned} \quad (11)$$

Case 7: $T \leq M < N$

In this case the retailer’s credit period (M) is greater or equal than the cycle length (T). As a result, the retailer does not need to pay the interest. Therefore, $IP_{27} = 0$. The retailer earns the interest from the customer under partial trade credit policy.

The interest earned is computed as follows:

$$\begin{aligned} IE_{27} &= sI_e \left[\alpha D \int_0^T e^{-rt} dt + DT \int_T^M e^{-rt} dt \right] \\ &= \frac{s}{r^2} I_e D \left[\alpha \left\{ 1 - (Tr + 1)e^{-rT} \right\} + rT(e^{-rT} - e^{-rM}) \right] \end{aligned}$$

So, the total profit per unit time of the inventory system is given by

$$\pi_7(s, t_1) = \frac{1}{T} \left[SR + IE_{27} - PC - C_{hol} - RC - IP_{27} - OC \right]$$

Problem 7.

$$\begin{aligned} &\text{Maximize } \pi_7(s, t_1) \\ &\text{subject to } s > 0, T > t_1 > 0, 0 < T \leq M < N \end{aligned} \quad (12)$$

4. NUMERICAL EXAMPLE

This section presents a numerical example in order to illustrate the seven cases that can occur. The parametric values are as follows:

$P = 130 \text{ units/year}$, $a = 130 \text{ units/year}$, $b = 0.5$, $\theta = 0.05$, $c_0 = \$200$, $h = \$5/\text{unit/year}$, $c = \$50/\text{unit}$, $c_d = \$35/\text{unit}$, $r = 0.06$, $r_e = 0.95$, $\alpha = 0.05$, $\theta = 0.05$, $I_e = \$0.10/\$/\text{year}$, $I_p = \$0.15/\$/\text{year}$. The optimal solutions are presents in Table 2.

Table 2: The optimal solution of the numerical example

Case & M, N (in year)	Selling price (s)	Production time (t_1)	Cycle length (T^*)	Total profit per unit time ($\pi_1(s, t_1)$)
Case-1: $M = 0.10, N = 0.05$	177.0756	0.2813149	0.6163726	7215.416
Case-2: $M = 0.30, N = 0.28$	177.2759	0.2655840	0.5832213	7253.488
Case-3: $M = 0.60, N = 0.50$	177.5270	0.2652629	0.5838045	7288.957
Case-4: $M = 0.70, N = 0.65$	176.9488	0.2964952	0.6485981	7342.392
Case-5: $M = 0.20, N = 0.25$	177.1786	0.2781891	0.6101337	7221.679
Case-6: $M = 0.30, N = 0.35$	177.3136	0.2692388	0.5913720	7243.459
Case-7: $M = 0.40, N = 0.45$	176.9303	0.1697021	0.3726945	7199.779

From Table 2, it is clearly observed that if the credit facility increases then total profit per unit time increases.

5. SENSITIVITY ANALYSIS

The numerical example is used to study the effect of under or overestimation of system parameters on the optimal values of the selling price (s), production period (t_1), cycle length (T), and total profit per unit time of inventory system ($\pi_1(s, t_1)$). The percentage changes in the above-mentioned optimal values are taken as measures of sensitivity. The results of the sensitivity analysis are given in Table 3.

Table 3: Sensitivity analysis

Parameters	% of change parameters	% change in optimal values			
		$\pi_1(s, t_1)$	s	t_1	T
c_0	-20	0.92	0	-11.16	-11.08
	-10	0.45	0	-5.43	-5.38
	+10	-0.42	0	5.17	5.12
	+20	-0.82	0	10.10	10.01
h	-20	0.13	0	1.54	1.53
	-10	0.07	0	0.76	0.76
	+10	-0.42	0	-0.75	-0.74
	+20	-	-	-	-
a	-20	-44.1	-17.13	-14.96	14.17
	-10	-23.58	-8.58	-7.34	6.33
	+10	-	-	-	-
	+20	56.29	17.24	14.15	-9.68
b	-20	37.32	21.23	-3.09	-7.41
	-10	16.52	9.44	-1.25	-3.5
	+10	-13.42	-7.72	0.77	3.16
	+20	-24.52	-14.16	1.13	6.04
P	-20	-0.35	0.25	37.16	7.82
	-10	-	-	-	-
	+10	0.14	-0.10	-11.9	-2.49
	+20	0.25	-0.18	-21.27	-4.44
θ	-20	4.84	-0.40	-12.25	-0.87
	-10	2.31	-0.15	-5.88	-0.38
	+10	-2.11	0.07	5.45	0.3
	+20	-4.05	0.08	10.53	0.54
c	-20	8.53	-2.93	7.18	2.46
	-10	4.22	-1.46	3.53	1.21
	+10	-4.12	1.45	-3.41	-1.16
	+20	-8.15	2.9	-6.69	-2.27
M	-20	-0.06	0	0.74	0.73
	-10	-0.03	0	0.39	0.38
	+10	0.04	0	-0.43	-0.43
	+20	0.08	0	-0.90	-0.89
N	-20	0.01	0	-0.12	-0.12
	-10	0.01	0	-0.06	-0.06
	+10	-0.01	0	0.07	0.07
	+20	-0.01	0	0.15	0.15
r	-20	-	-	-	-
	-10	-2.07	0.07	9.37	3.54
	+10	1.84	-0.13	-8.04	-3.25
	+20	3.47	-0.29	-14.99	-6.22
c_d	-20	0.30	-0.10	0.15	0
	-10	0.15	-0.05	0.08	0
	+10	-0.15	0.05	-0.08	0
	+20	-0.30	0.10	-0.15	0

From Table 3, it is observed the following results:

1. The total profit per unit time ($\pi_1(s, t_1)$) is highly sensitive with respect to the demand parameters a (direct effect) and b (reverse effect); moderately sensitive reversely respect to per unit production cost (c); and less sensitive with respect to the rest of the parameters $h, \theta, M, N, c_0, c_d$ (reverse effect) and P, r (direct effect).
2. The selling price (s) is equally sensitive with respect to the demand parameters a (direct effect) and b (reverse effect); less sensitive with respect to the parameters θ, c, c_d, r (direct effect) and P (reverse effect); insensitive with respect to h, M, N, c_0 .
3. The production period (t_1) is equally sensitive reversely with respect to the parameter P ; moderately sensitive with respect to r, c (reverse effect) and c_0, a, θ (direct effect); less sensitive respect to the rest of the parameters h, c_d, M (reverse effect) and N (direct effect).
4. The cycle length (T) is moderately sensitive with respect to the parameters P, a (reverse effect) and c_0, b (direct effect); less sensitive with respect to the parameters h, M, r, c (reverse effect) and θ, N (direct effect); and insensitive with respect to c_d .

6. CONCLUSION

We developed a production-inventory model for deteriorating items considering inflation, reliability, and partial trade credit policy, where demand for a product depends on the selling price. It is observed that if credit facility time increases up to a certain period then profit per unit time of the system increases. For future research directions, this production-inventory model can be extended considering fuzzy-valued inventory cost, interval-valued inventory costs, price, and stock dependent demand, displayed stock dependent demand, advertisement dependent demand, fully backlogging, among others.

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