Yugoslav Journal of Operations Research
31 (2021), Number 2, 273–283
DOI: https://doi.org/10.2298/YJOR200125011K

AN INVENTORY SYSTEM FOR VARYING DECAYING MEDICINAL PRODUCTS IN HEALTHCARE TRADE

Kamal KUMAR
Department of Mathematics, Baba Mastnath University, Rohtak, India
kamalkumar4maths@gmail.com

Neeraj KUMAR
Department of Mathematics, Delhi-NCR, Sonepat, India
nkneerajkapil@gmail.com

MEENU
Research Scholar, Baba Mastnath University, Rohtak, India
sigrohameenu@gmail.com

Received: January 2020 / Accepted: May 2020

Abstract: This paper introduces an inventory system for declining medicinal products under the effect of inflation with price-sensitive demand. Usually, pharmaceutical inventory systems assume the deteriorating rate as constant, which is insignificant and illogical because healthcare products medicines and drugs deteriorate significantly. Thus, the deteriorating rate of medicinal products is supposed to be time-dependent and to follow three-parameter Weibull distribution. The lack of commodities is permitted with the rate of partial backlogging. This research work develops a model to optimize the total average cost of the items by calculating the ordering quantity and the optimum time intervals. Finally, through a numerical example, with sensitivity analysis, we demonstrate the effect of different parameters.

Keywords: Inventory, Price - dependent Demand Rate, Time - dependent Deterioration Rate, Inflation.

MSC: 90B05.

1. INTRODUCTION

The connotation of term inventory includes that the objects kept under process of product formation are ready to be sold and used during a specific period of time. All healthcare products or pharmaceutical goods decay after a specific period of life time. It is crucial to manage the stock of decaying medicinal products appropriately to meet the effectively demand and to reduce the inventory cost. As after expiration medicines or drugs cannot be used, assuming the rate of deterioration as being based on time may be advantageous for any health care trade. Consequently, the role of decay in medicinal products in healthcare trade cannot be ignored. Demand is an important factor for the success of any trade. Quality and price of the product along with good marketing strategies affect the demand of that product. Thus, it becomes important to focus on selling price factor, which has a direct effect on profit.
Some items in the market start deteriorating after a fixed time during storage, in which case two-parameter Weibull distribution is not feasible to be used. Some medicinal products, e.g., syrups, tablets, capsules do not decline at the receiving time in stock. For this kind of products, considering deterioration rate as three-parameter Weibull distribution is relevant to represent the time of decline. This model has been used by many researches earlier, but less focus is laid by researches to assimilate three-parameter Weibull distribution with partial backlogging. So, we are going to make an attempt to include all the aspects earlier stated into a single problem. Thus, an inventory system for three-parameter Weibull deteriorating products with rate sensitive demand and partial backlogging has been given.

1.1. Literature review

The concept of inflation was firstly proposed by Buzacott [1]. Mandal and Pal [2] developed inventory policies with shortages and constant rate of deterioration. Jain and Kumar [3] proposed an EOQ model with the three-parameter Weibull distribution to show the time of deterioration with the condition of fully backlogged. Kumar and Singh [4] proposed an inventory system for decaying items with stock dependent demand. Shortage of items were permitted with the rate of decline. For healthcare trade, Uthaya Kumar and Karuppasamy[5] explored the policies of inventory under shortages, where demand is based on time and rate of decaying is constant. Saha *et al.* [6] formulated an EPQ model with shortages and price dependent demand rate. In their model, the rate of decaying was constant. Roy [7] recommended a structure for deteriorating goods with cost-based demand, and holding cost of goods are based on time. Chowdhury *et al*[8] offered an order level system with constant holding cost and quadratic rate of demand. Partial backlogged shortages are allowed. A non-instantaneous decaying items production model was given by Tayal *et al* [9] without shortages and time dependent holding cost. An inventory model with partially backlogged shortages and constant holding cost for deteriorating products was suggested by Patel *et al.* [10]. They used the two-parameter Weibull distribution. An inventory system with shortages under credit limit strategic was introduced by Rastogi *et al.* [11] with variable holding cost. An inventory structure for time-based holding cost and fully backlogged shortages were preferred by San-Jose *et al.* [12]. In this structure, demand is a function of price and time. Sundarrajan and Uthayakumar[13] formulated a model for instantaneous decaying products with trade financing. Here, decaying rate and holding cost both are constant. Mondal *et al*[14] discussed on a structure for ameliorating goods with price-based demand. A system with Weibull decaying rate and price dependent demand was discussed by Mukhopadhyay *et al*[15] without shortages. A gain-boosting location system was given by Javid and Hoseinpour [16]. In this model supply network distribution with demand-based price was considered. Sharma *et al* [17] initiated a system with price-based demand for worsen goods with partial backlogging. An inventory structure with cost-based demand for two-warehouse under partial backlogging rate was discussed by Rastogi *et al* [27]. Ouyang *et al* [19] proposed a structure for decaying goods with partial backlogging. Here demand is exponential declining. Singh *et al* [20] investigated a structure for defective items under partial rate of backlogging and multivariate demand. Chukwu [21] explored a structure for ramp type items with shortage of stock under complete backlogging where the effect of three variables Weibull distribution decaying rate was taken. Mishra *et al*[22] proposed a system for deteriorating items by using salvage value. Singh [23] preferred the policy of inventory for decaying products with deficiency and different condition of backlogging. They assumed demand rate is seasonal pattern and stock-based. Kumar *et al*[24] explored an inventory model for deteriorating items with the impact of inflation and trade credit. They used genetic algorithm method to solve the model. Uthayakumar and Karuppasamy [25] introduced an inventory structure for deteriorating pharmaceutical items under the impact of trade credit policy. Here, demand and decaying rate were considered as time-dependent. Shortages were not allowed. Sharma [26] formulated a production model with the effect of inflation for decaying items with two different cases with shortages or without shortages. In later case, occurring shortages were fully backlogged. Rastogi *et al* [27] suggested an EOQ model for non-instantaneous decaying items with shortages. Here demand is a function of price. Rastogi and Singh [28] formulated a structure for deteriorating pharmaceutical items with price-sensitive demand. In this model, they reduced total carrying cost by using variable holding cost and time- dependent deterioration. Panda *et al*[29] formulated a two-warehouse inventory model with stock and price-dependent demand. Shortages were allowed here. Sahoo *et al*[30] investigated about the policies of inventory for selling-price dependent demand under the condition of partially backlogging. Various authors e.g., Malik *et al* [31], Dari Sani[32], Shah *et al* [33] and Shah Naik [34] formulated inventory model for deteriorating items.

Our research derives an inventory system for deteriorating medicinal products under the impact of inflation where demand depends on the price of a product. The main objective of this inventory system is to optimize total average cost by calculating the optimal time interval and best order quantity. The remaining part of the paper is organised as follows: section 2 presents notations and assumptions. Section 3 provides the mathematical formulation of inventory model. Section 4 gives the procedure of solution of the model. Section 5 includes a solved numerical example and provides the sensitivity analysis. Section 6 concludes the paper with Conclusion and some future research directions.

2. NOTATIONS And ASSUMPTIONS

Notations

1. α, β-demand variables
2. P - selling cost
3. γ, δ - deterioration parameter
4. T Ã¢â¬â€œ cycle time
5. t_1- time at which stock level begin to be zero
6. Q_i - initial ordering quantity
7. Q_0 - backordered quantity
8. Q - ordering quantity
9. C - purchasing cost per unit
10. $\theta(\mu)$- rate of backlogging
11. μ- waiting time for the next arrival
12. h_1 - holding cost
13. O- ordering cost
14. S - shortage cost
15. L - lost sale cost
16. R - inflation rate
17. $T.A.C.$ - total average cost

Assumptions

1. Demand is related to selling cost and shown as $D(P) = \dfrac{\alpha}{P^{\beta}}$
2. Lead time is supposed to be zero.
3. There is no replacing system for decaying items.
4. Holding cost is taken as constant and shown as $H(t) = h_1(t)$.
5. Shortages of stock are permitted and partly backlogged. The backlogging rate is a waiting time up to the next arrival μ and $\theta(\mu) = 1 - \dfrac{\mu}{T}, 0 \leq \mu \leq T$.
6. The items decay at three-variable Weibull decaying rate are taken as $\gamma\delta(t-\lambda)^{\delta-1}$ where $\gamma(0 \leq \gamma << 1)$ is the scale variable,$\mu(\mu > 0)$ is the shape variable and $\lambda(\lambda > 0)$ is the location variable.

3. MATHEMATICAL MODEL FORMULATION

The inventory functioning time of the system is drawn in figure 1. At $t = 0$ the distributor receives stock and stock amount reaches $I_1(0) = Q_i$, which is maximal inventory level. During time $[\theta, t_1]$, stock level reduces caused by market demand and deterioration process. At $t = t_1$ the stock level begins to be zero. After this

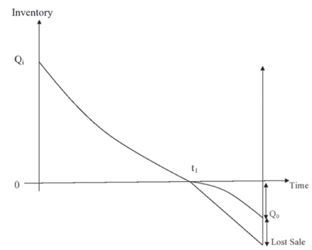

Figure 1: Graphical Representation of the Inventory Model

shortage occurring during the time interval $[t_1, T]$ and shortage of goods is partly backlogged or lost. The backlogging rate is taken as a variable that relates to the waiting time up to the next arrival.

The following differential equations describing the nature of medicinal items in the time $[0, T]$ are

$$\frac{dI_1(t)}{dt} + \gamma\delta(t-\lambda)^{\delta-1} I_1(t) = -\frac{\alpha}{P^\beta}, 0 \le t \le t_1 \tag{1}$$

$$\frac{dI_2(t)}{dt} = -\frac{\alpha}{P^\beta}, t_1 \le t \le T \tag{2}$$

with condition $.I_1(t_1) = 0$ & $I_2(t_1) = 0$.
The solution of equations (1) and (2) subject to the boundary condition are as:

$$I_1(t) = \frac{\alpha}{P^\beta}\left[\begin{array}{l} (t_1-t) + \frac{\gamma}{\delta+1}(t_1-\lambda)^{\delta+1} - \frac{\gamma}{\delta+1}(t-\lambda)^{\delta+1} - \\ \gamma(t_1-t)(t-\lambda)^\delta - \frac{\gamma^2}{(\delta+1)}(t-\lambda)^\delta(t_1-\lambda)^{\delta+1} + \\ \frac{\gamma^2}{(\delta+1)}(t-\lambda)^{2\delta+1} \end{array}\right], 0 \le t \le t_1 \tag{3}$$

$$I_2(t) = \frac{\alpha}{P^\beta}(t_1 - t), t_1 \le t \le T \tag{4}$$

The original size is found from equation (3) by placing $t = 0$ in interval $0 \le t \le t_1$.
From here, the original quantity is $Q_i = I_1(0)$,

$$Q_i = \frac{\alpha}{P^\beta}\left[\begin{array}{l} t_1 + \frac{\gamma}{\delta+1}(t_1-\lambda)^{\delta+1} - \frac{\gamma}{\delta+1}(-\lambda)^{\delta+1} + \frac{\gamma^2}{(\delta+1)}(-\lambda)^{2\delta+1} - \\ \gamma t_1(-\lambda)^\delta - \frac{\gamma^2}{(\delta+1)}(-\lambda)^\delta(t_1-\lambda)^{\delta+1} \end{array}\right] \tag{5}$$

The backordered quantity in $t_1 \le t \le T$

$$Q_o = \int_{t_1}^{T} \frac{\alpha}{P^\beta}\theta(\mu)dt = \frac{\alpha}{2TP^\beta}(T^2 - t_1^2) \tag{6}$$

From here, the total order quantity between $[0, T]$ is $Q = Q_i + Q_o$

$$Q = \left\{ \frac{\alpha}{P^\beta} \left[\begin{array}{l} t_1 + \frac{\gamma}{\delta+1}(t_1-\lambda)^{\delta+1} - \frac{\gamma}{\delta+1}(-\lambda)^{\delta+1} - \\ \gamma t_1(-\lambda)^\delta - \frac{\gamma^2}{(\delta+1)}(-\lambda)^\delta(t_1-\lambda)^{\delta+1} + \\ \frac{\gamma^2}{(\delta+1)}(-\lambda)^{2\delta+1} \end{array} \right] \right\} + \left\{ \frac{\alpha}{2TP^\beta}(T^2 - t_1^2) \right\} \tag{7}$$

$$T.A.C. = \frac{1}{T}[O.C. + S.C. + L.S.C + H.C. + D.C.] \tag{8}$$

$$Now, OrderingCost = O \tag{9}$$

Shortage Cost $= S \int_{t_1}^{T} - I_2(t)e^{-Rt}\, dt$ where $I_2(t) = \frac{\alpha}{P^\beta}(t_1 - t)\, dt,\ t_1 \le t \le T$

$$= \frac{S\alpha}{P^\beta}\left[\frac{e^{-RT}}{R}(T - t_1) + \frac{1}{R^2}\left(e^{-RT} - e^{-Rt_1}\right)\right] \tag{10}$$

Lost Sale Cost=

$$L\int_{t_1}^{T} \frac{\alpha}{P^\beta}\left(1 - \theta(\mu)\right) e^{-Rt}dt = \frac{L\alpha}{TP^\beta}\left[\frac{1}{R}(t_1 e^{-Rt_1} - Te^{-RT}) + \frac{1}{R^2}(e^{-RT} - e^{-Rt_1})\right] \tag{11}$$

Holding Cost$= h_1 \int_0^{t_1} I_1(t)\ e^{-Rt}\, dt$

$$\left[\frac{h_1\alpha}{P^\beta} \left\{ \begin{array}{l} \frac{t_1}{R} + \frac{e^{-Rt_1}}{R} + \frac{\gamma}{(\delta+1)}(t_1-\lambda)^{\delta+1}(1 - e^{-Rt_1}) - \frac{\gamma(t_1)(-\lambda)^\delta}{R} \\ -\frac{\gamma}{(\delta+1)R}\{(-\lambda)^{\delta+1} - (t_1-\lambda)^{\delta+1}e^{-Rt_1}\} + \frac{\gamma^2}{R(\delta+1)}\{(-\lambda)^{2\delta+1} \\ -\frac{\gamma^2}{(\delta+1)R}(t_1-\lambda)^{\delta+1}\left\{(-\lambda)^\delta - (t_1-\lambda)^\delta e^{-Rt_1}\right\} - (t_1-\lambda)^{2\delta+1}e^{-Rt_1}\} \end{array} \right\} \right] \tag{12}$$

In time $[0, t_1]$, demand is $\int_0^{t_1} \frac{\alpha}{P^\beta}dt$ .The number of decaying items $=I_1(0)$-total demand $= I_1(0)$-$\int_0^{t_1} \frac{\alpha}{P^\beta}dt$. From here, deterioration cost is

$$D.C. = C\{I_1(0) - \int_0^{t_1} \frac{\alpha}{P^\beta} dt\}. = C\frac{\alpha}{P^\beta}\begin{bmatrix} \frac{\gamma}{\delta+1}(t_1-\lambda)^{\delta+1} - \frac{\gamma}{\delta+1}(-\lambda)^{\delta+1} - \\ \gamma t_1(-\lambda)^\delta - \frac{\gamma^2}{(\delta+1)}(-\lambda)^\delta (t_1-\lambda)^{\delta+1} + \\ \frac{\gamma^2}{(\delta+1)}(-\lambda)^{2\delta+1} \end{bmatrix} \tag{13}$$

$$T.A.C. = \frac{1}{T}\begin{bmatrix} o + \frac{S\alpha}{P^\beta}\left[\frac{e^{-RT}}{R}(T-t_1) + \frac{1}{R^2}\left(e^{-RT} - e^{-Rt_1}\right)\right] + \\ \frac{L\alpha}{TP^\beta}\left[\frac{1}{R}(t_1 e^{-Rt_1} - Te^{-RT}) + \frac{1}{R^2}(e^{-RT} - e^{-Rt_1})\right] \\ +\frac{h_1\alpha}{P^\beta}\left\{\begin{array}{l} \frac{t_1}{R} + \frac{e^{-Rt_1}}{R} + \frac{\gamma}{(\delta+1)}(t_1-\lambda)^{\delta+1}(1-e^{-Rt_1}) - \frac{\gamma}{(\delta+1)R}\{(-\lambda)^{\delta+1} \\ -(t_1-\lambda)^{\delta+1}e^{-Rt_1}\} - \frac{\gamma^2}{(\delta+1)R}(t_1-\lambda)^{\delta+1}\left\{(-\lambda)^\delta - (t_1-\lambda)^\delta e^{-Rt_1}\right\} \\ -\frac{\gamma(t_1)(-\lambda)^\delta}{R} + \frac{\gamma^2}{R(\delta+1)}\{(-\lambda)^{2\delta+1} - (t_1-\lambda)^{2\delta+1}e^{-Rt_1}\} \end{array}\right\} \\ +C\frac{\alpha}{P^\beta}\left\{\begin{array}{l} \frac{\gamma}{\delta+1}(t_1-\lambda)^{\delta+1} - \frac{\gamma}{\delta+1}(-\lambda)^{\delta+1} - \gamma t_1(-\lambda)^\delta - \frac{\gamma^2}{(\delta+1)}(-\lambda)^\delta(t_1-\lambda)^{\delta+1} + \\ \frac{\gamma^2}{(\delta+1)}(-\lambda)^{2\delta+1} \end{array}\right\} \end{bmatrix} \tag{14}$$

4. SOLUTION PROCESS

The aim is to obtain the best result of t_1 to reduce $T.A.C.$ of the structure.

$$\frac{dT.A.C.}{dt_1} = \frac{1}{T}\begin{bmatrix} \frac{S\alpha}{RP^\beta}\left[(e^{-Rt_1} - e^{-RT})\right] + \frac{L}{TP^\beta}\left[-\alpha t_1 e^{-Rt_1}\right] + C\frac{\alpha}{P^\beta}\left\{\begin{array}{l} \gamma(t_1-\lambda)^\delta - \gamma(-\lambda)^\delta \\ -\gamma^2(-\lambda)^\delta(t_1-\lambda)^\delta \end{array}\right\} \\ +\frac{h_1\alpha}{P^\beta}\left\{\begin{array}{l} \frac{1}{R} - e^{-Rt_1} - \frac{\gamma}{R}(-\lambda)^\delta + \frac{\gamma^2}{R}(-\lambda)^\delta(t_1-\lambda)^\delta + \frac{\gamma^2}{R}(-\lambda)^\delta(t_1-\lambda)^\delta + \\ \frac{\gamma}{(\delta+1)}(t_1-\lambda)^\delta(1-e^{-Rt_1})\{(\delta+1) - R(t_1-\lambda)\} \\ +\frac{\gamma}{(\delta+1)R}(t_1-\lambda)^\delta e^{-Rt_1}\{(\delta+1) - R(t_1-\lambda)\} \end{array}\right\} \end{bmatrix} = 0$$

$$\frac{d^2T.A.C.}{dt_1^2} = \frac{1}{T}\begin{bmatrix} \frac{S\alpha}{P^\beta}(-e^{-Rt_1}) + \frac{L\alpha}{TP^\beta}e^{-Rt_1}(\mathrm{R}t_1 - 3) + \frac{C\alpha\gamma\delta}{P^\beta}(t_1-\lambda)^{\delta-1}\left\{1-\gamma(-\lambda)^\delta\right\} \\ +\frac{h_1}{P^\beta}\left\{\begin{array}{l} Re^{-Rt_1} + \gamma\delta(t_1-\lambda)^{\delta-1}(1-e^{-Rt_1}) - \frac{\gamma R}{(\delta+1)}e^{-Rt_1}(t_1-\lambda)^{\delta+1} \\ +\gamma R(t_1-\lambda)^\delta(e^{-Rt_1}) + \gamma R(t_1-\lambda)^\delta(1+e^{-Rt_1}) - \frac{\gamma^2\delta}{R}(-\lambda)^\delta(t_1-\lambda)^{\delta-1} \\ -\frac{\gamma R^2}{(\delta+1)}e^{-Rt_1}(t_1-\lambda)^{\delta+1} + \frac{\gamma\delta}{R}e^{-Rt_1}(t_1-\lambda)^{\delta-1} \end{array}\right\} \end{bmatrix} > 0$$

Algorithm

Step 1: Allot a value for parameters $\alpha, \beta, \gamma, \delta, \lambda, R, T, P, S, L, C$

Step 2: Put the value in $\frac{dT.A.C.}{dt_1} = 0$ and get value of t_1 & Analyse $\frac{d^2T.A.C.}{d^2t_1}$

Step 3: If $\frac{d^2T.A.C.}{d^2t_1} > 0$,then acquired value of t_1 will be best result and according to this result T.A.C.found using equation for (3) be best total average cost or go to (2) again and took other set of results of variables.

Step 4: Redo (2) to (4) until t_1 & $T.A.C.$ are acquired.

5. NUMERICAL EXAMPLE

Let $\alpha = 100, \beta = 1, h = 0.4, O = 500, T = 50, P = 20, \delta = 1, \lambda = 1, \gamma = 0.01, L = 7, S = 5, C = 12, R = 0.5.$Then $t_1 = 17.63, T.A.C. = 13.35$ and $Q = 205.448$.

6. SENSITIVITY ANALYSIS & OBSERVATIONS

We study the sensitivity for the optimal solution by changing one variable at a time and keeping other variables fixed at their original values. We observed that if demand variable α increases, t_1 also changed accordingly, however $T.A.C.$ and Q also increase with respect to demand variable. As the demand variable α decreases, t_1 also changes while $T.A.C.$ and Q also decrease with respect to demand variable. We observed that an increase in selling cost (p) shows a reverse impact of decrement in $T.A.C.$ and Q. Thus, the increase in selling cost of goods forces the seller to order small quantities due to increase investment or deterioration. When demand variable β extends, t_1 stays unchanged while $T.A.C.$ and Q extend accordingly. While demand variable β decreases, t_1 stays unchanged while $T.A.C.$ and Q also decrease. As the impact of inflation decreases, $T.A.C.$ and Q increase while t_1 decreases but if the impact of inflation increases, $T.A.C.$, Q decrease while t_1 increases.

Parameter	Change in %	Value of parameter	t_1	$T.A.C.$	Q
α	-20	80	16.49	12.55	159.246
α	-10	90	16.49	12.87	179.152
α	0	0	17.63	13.35	205.448
α	10	110	24.17	15.74	218.964
α	20	120	24.17	16.25	238.870
P	-20	16	17.63	14.20	256.810
P	-10	18	17.63	13.73	228.275
P	0	20	17.63	13.35	205.448
P	10	22	17.63	13.05	186.771
P	20	24	17.63	12.80	171.206
β	-20	0.8	17.63	16.12	374.222
β	-10	0.9	17.63	14.53	279.258
β	0	1	17.63	13.35	205.448
β	10	1.1	17.63	12.49	152.295
β	20	1.2	17.63	11.84	112.852
R	-20	0.4	3.13	15.05	1263.64
R	-10	0.45	4.49	10.50	1262.90
R	0	0.5	17.63	13.35	205.448
R	10	0.55	6.27	10.27	1268.51
R	20	0.6	25.09	14.94	1077.80

Table 1: Effect of different paraemters on optimal result

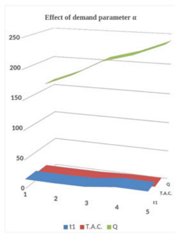

Figure 2: Effect of demand parameter α on $T.A.C.$

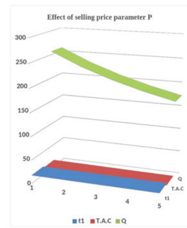

Figure 3: Effect of selling price Parameter P on $T.A.C.$

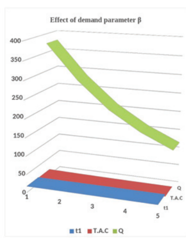

Figure 4: Effect of demand parameter β on $T.A.C.$

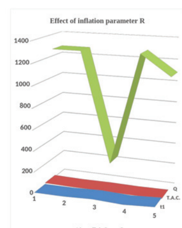

Figure 5: Effect of inflation R on $T.A.C.$

7. CONCLUSION

This study is explored for medicinal products and focused on realistic features like price-sensitive demand, three- parameter Weibull distribution, and inflation. The Weibull distribution is significant for such products whose deterioration rate increases with time and the location parameter ÃŽÂ, we use it here to describe the productÃ¢â¬â¢s lifespan; necessary feature of deteriorating medicinal products. Shortages are allowed with partially backlogged condition. In real life, backlogging always depends on the waiting time till the next arrival, but here the backlogging rate is considered as a function of waiting time. Thus, these realistic aspects make the proposed model more distinguished and reasonable. The purpose of this article is to optimise total average cost of the system and calculate the optimal order in quantity and time interval. The applicability of the proposed model is in Healthcare trade to inventory management of deteriorating medicinal products where the deterioration rate is time-dependent and shortages are partially backlogged. A numerical example and sensitivity analysis regarding various parameters are presented to illustrate the study. In the observed data, total average cost is highly responsive to the variation in the value of demand parameters, selling cost, and cycle time. Ordering quantity is also highly responsive to the variation in demand variable, deterioration rate, and selling price. The future extension for this model includes stochastic demand pattern, fuzzy environment, different preservation technology, and time-dependent ordering cost.

REFERENCES

[1] Buzacott, J. A., "Economics order Quantity with inflation", *Operational Research Quarterly* , 26 (3) (1975) 553–558.

[2] Mandal, B., and Pal, A. K., "Order level inventory system with ramp type demand rate for deteriorating items", *Journal of Interdisciplinary Mathematics*, 1 (1998) 49–66.

[3] Jain, S., and Kumar, M., "An EOQ inventory model for items with ramp type demand, three-parameter Weibull distribution deterioration and starting with shortage ", *Yugoslav Journal of Operational Research*, 20 (2010) 249–259.

[4] Kumar, S., and Singh, A. K., "Optimal time policy for deteriorating items of two-warehouse inventory system with time and stock dependent demand and partial backlogging ", *Sadhana*, 41(2016) 541–548.

[5] Uthayakumar, R., and Karuppasamy, S. K., "A pharmaceutical inventory model for healthcare industries with quadratic demand, linear holding cost and shortages ", *International Journal of Pure and Applied Mathematics*, 106 (2016) 73–83.

[6] Panda, S., Saha, S., Modak, N. M., and Sana, S. S., "A volume Flexible deteriorating inventory model with price sensitive demand ", *TE ´KHNE- Rev. Appl. Manag. Stud.* , 15 (2017) 117–123.

[7] Roy, A., "An inventory model for deteriorating items with price dependent demand and time varying holding cost ", *Modelling Optimization*, 10 (2008) 25–37.

[8] Chowdhury R. R., Ghosh S. K., and Chaudhuri K. S., "An order-level inventory model for a deteriorating item with time-quadratic demand and time-dependent partial backlogging with shortages in all cycles", *American Journal of Mathematical and Management Sciences* , 33 (2014) 75–97.

[9] Tayal, S., Singh, S. R., Sharma, R., and Singh, A. P., " An EPQ model for non-instantaneous deteriorating item with time dependent holding cost and exponential demand rate", *International Journalof Operational Research*, 23 (2015) 145–162.

[10] Kumar, S., Singh, A. K., and Patel, M. K., "Optimization of Weibull deteriorating items inventory model under the effect of price and time dependent demand with partial backlogging ", *Sadhana*, 41 (2016) 977–984.

[11] Rastogi, M., Singh, S. R., Kushwah, P., and Tayal, S., "An EOQ model with variable holding cost and partial backlogging under credit limit policy and cash discount", *Uncertain Supply Chain Management* , 5 (2017) 27–42.

[12] San-Jose, L. A., Sicilia, J., and Alcaide-Lo, P.D., , "An inventory system with demand dependent on both time and price assuming backlogged shortages", *European Journal of Operational Research*, 270 (2018) 889–897.

[13] Sundararajan, R., and Uthayakumar, R., "Optimal pricing and replenishment policies for instantaneous deteriorating items with backlogging and permissible delay in payment under inflation", *American Journal of Mathematical and Management Sciences*, 37 (2018) 1–17.

[14] Mondal, B., Bhunia, A. K., and Maiti, M., "An inventory system of ameliorating items for price dependent demand rate", *Computational Industrial Engineering*, 45 (2003) 443–456.

[15] Mukhopadhyay, S., Mukherjee, R. N., and Chaudhuri, K. S., "An EOQ model with two parameter Weibull distribution deterioration and price-dependent demand", *International Journal of Mathematical Education in Science and Technology* , 36 (2005) 25–33.

[16] Ahmadi-Javid A., and Hoseinpour, P., "Location-inventory-pricing model in a supply chain distribution network with price-sensitive demands and inventory-capacity constraints", *Transportation Research Part E: Logistics and Transportation Review*, 82 (2015) 238–255.

[17] Sharma, S., Singh, S. R., and Ram, M., "An EPQ model for deteriorating items with price sensitive demand and shortages", ck *International Journal of Operational Research*, 23 (2015) 245–255.

[18] Rastogi, M., Singh, S. R., Kushwah, P., and Tayal, S., "Two warehouse inventory policy with price dependent demand and deterioration under partial backlogging", *Decision Science Letters*, 6 (2017) 11–22.

[19] Ouyang, L. Y., Wu, K. S., and Cheng, M. C., "An inventory model for deteriorating items with exponential declining demand and partial backlogging", *Yugoslav Journal of Operational Research*, 15 (2005) 277–288.

[20] Singh, S. R., Singhal, S., and Gupta, P. K. , "A volume flexible inventory inventory model for defective items with multi variate demand and partial backlogging", *International Journal of Operational Research Optimization*, 1 (2010) 55–69.

[21] Sanni, S. S., and Chukwu, W. I. E. , "An economic order quantity model for items with three parameter Weibull distribution deterioration, ramp-type demand and shortages", *Applied Mathematical Modelling*, 37 (2013) 9698–9706.

[22] Mishra, U., and Tripathi, C. K., "An inventory model for Weibull deteriorating items with salvage value", *International Journal of Logistics Systems and Management*, 22 (2015) 67–76.

[23] Singh, S. R., Rastogi, M., and Tayal, S., "An inventory model for deteriorating items having seasonal and stock-dependent demand with allowable shortages", Advances in Intelligent Systems and Computing, 437 (2016) 501–513.

[24] Kumar, S., and Kumar, N., "An inventory model for deteriorating items under inflation and permissible delay in payments by genetic algorithm", *Cogent Business and Management*, 3 (2016) 2331-1975.

[25] Uthayakumar, R., and Karuppasamy, S. K., "An inventory model for variable deteriorating pharmaceutical items with time dependent demand and time dependent holding cost under trade credit in healthcare industries", *Communications in Applied Analysis*, 21 (2017) 533–549.

[26] Singh S. R. and Sharma S., "A production reliable model for deteriorating products with random demand and inflation", *International Journal of Systems Science Operations and Logistics*, 4 (2017) 330–338.

[27] Rastogi M., Singh S. R., and Kushwah P., "An inventory model for non-instantaneous deteriorating products having price sensitive demand and partial backlogging of occurring shortages", *International Journal of Operations and Quantitative Management*, 24 (2018) 59–73.

[28] Rastogi M. and Singh S. R, "An inventory system for varying deteriorating pharmaceutical items with price sensitive demand and variable holding cost under partial backlogging healthcare industries", *Sadhana*, (2019) 44–95.

[29] Panda G. C., Khan A. A., and Shaikh A. A., "A credit policy approach in a two-warehouse inventory model for deteriorating items with price and stock dependent under partial backlogging", *International Journal of Industrial Engineering*, 15 (2019) 147–170.

[30] Sahoo A. K., Inderjitsingha S. K., Misra U. K., and Samanta P. N., "Selling price dependent demand with allowable shortages model under partially backlogged deteriorating items ", *International Journal of Applied and Computational Mathematics*, 5 (4) (2019) 104–110.

[31] Malik A.K. and Sharma A., " An inventory model for deteriorating items with multivariate demand and partial backlogging under inflation ", *Interdisciplinary Journal of History*, 6 (2) (2020) 401–407.

[32] Dari S. and Sani B., "An EPQ model for delayed deteriorating items with quadratic demand and linear holding cost ", *OPSEARCH*, 57 (1) (2020) 46–72.

[33] Shah N. H., Chaudhari U., and Cardenas-Barron L. E., " Integrated credit and replenishment policies for deteriorating items under quadratic demand in a three-echelon supply chain", *International Journal of Systems Sciences, Operations Logistics*, 7 (1) (2020) 34–45.

[34] Shah N.H. and Naik M.K., "Inventory policies for deteriorating items with time-price backlog dependent demand ", *International Journal of Systems Sciences, Operations Logistics*, 7 (1) (2020) 76–89.