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ON STRONGLY REGULAR GRAPHS WITH $m_2 = qm_3$ AND $m_3 = qm_2$ FOR $q = \frac{7}{2}, \frac{7}{3}, \frac{7}{4}, \frac{7}{5}, \frac{7}{6}$

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Abstract: We say that a regular graph G of order n and degree $r \geq 1$ (which is not the complete graph) is strongly regular if there exist non-negative integers τ and θ such that $|S_i \cap S_j| = \tau$ for any two adjacent vertices i and j, and $|S_i \cap S_j| = \theta$ for any two distinct non-adjacent vertices i and j, where S_k denotes the neighborhood of the vertex k. Let $\lambda_1 = r$, λ_2 and λ_3 be the distinct eigenvalues of a connected strongly regular graph. Let $m_1 = 1$, m_2 and m_3 denote the multiplicity of r, λ_2 and λ_3, respectively. We here describe the parameters n, r, τ and θ for strongly regular graphs with $m_2 = qm_3$ and $m_3 = qm_2$ for $q = \frac{7}{2}, \frac{7}{3}, \frac{7}{4}, \frac{7}{5}, \frac{7}{6}$.

Keywords: Strongly Regular Graph, Conference Graph, Integral Graph.

MSC: 05C50.

1. INTRODUCTION

Let G be a simple graph of order n with vertex set $V(G) = \{1, 2, \ldots, n\}$. The spectrum of G consists of the eigenvalues $\lambda_1 \geq \lambda_2 \geq \cdots \geq \lambda_n$ of its (0,1) adjacency matrix A and is denoted by $\sigma(G)$. We say that a regular graph G of order n and degree $r \geq 1$ (which is not the complete graph K_n) is strongly regular if there exist non-negative integers τ and θ such that $|S_i \cap S_j| = \tau$ for any two adjacent vertices i and j, and $|S_i \cap S_j| = \theta$ for any two distinct non-adjacent vertices i and j, where $S_k \subseteq V(G)$ denotes the neighborhood of the vertex k. We know that a regular connected graph G is strongly regular if and only if it has exactly three distinct eigenvalues [1] (see also [2]). Let $\lambda_1 = r$, λ_2 and λ_3 denote the distinct eigenvalues of a connected strongly regular graph G. Let $m_1 = 1$, m_2 and m_3 denote the multiplicity of r, λ_2 and λ_3. Further, let $\overline{r} = (n-1) - r$, $\overline{\lambda}_2 = -\lambda_3 - 1$ and $\overline{\lambda}_3 = -\lambda_2 - 1$ denote the distinct eigenvalues of the strongly regular graph $\overline{G}$, where $\overline{G}$ denotes the complement of G. Then $\overline{\tau} = n-2r-2+\theta$ and $\overline{\theta} = n-2r+\tau$ where $\overline{\tau} = \tau(\overline{G})$ and $\overline{\theta} = \theta(\overline{G})$. Next, according to [1] and [2] we have the following two remarks.

Remark 1. (i) *if G is a disconnected strongly regular graph of degree r then $G = mK_{r+1}$, where mH denotes the m-fold union of the graph H and* (ii) *G is a disconnected strongly regular graph if and only if $\theta = 0$.*

Remark 2. (i) *a strongly regular graph G of order $n = 4k+1$ and degree $r = 2k$ with $\tau = k-1$ and $\theta = k$ is called a conference graph;* (ii) *a strongly regular graph is a conference graph if and only if $m_2 = m_3$ and* (iii) *if $m_2 \neq m_3$ then G is an integral graph.*

We have recently started to investigate strongly regular graphs with $m_2 = qm_3$ and $m_3 = qm_2$, where q is a positive integer [3]. In the same work we have described the parameters n, r, τ and θ for strongly regular graphs with $m_2 = qm_3$ and $m_3 = qm_2$ for $q = 2, 3, 4$. Besides, (i) we have described in [4] the parameters n, r, τ and θ for strongly regular graphs with $m_2 = qm_3$ and $m_3 = qm_2$ for $q = 5, 6, 7, 8$; (ii) we have described in [5] the parameters n, r, τ and θ for strongly regular graphs with $m_2 = qm_3$ and $m_3 = qm_2$ for $q = 9, 10$ and (iii) we have described in [6] the parameters n, r, τ and θ for strongly regular graphs with $m_2 = qm_3$ and $m_3 = qm_2$ for $q = 11, 12$. In particular, we have recently started to investigate strongly regular graphs with $m_2 = qm_3$ and $m_3 = qm_2$, where q is a positive rational number [7]. In the same work we have described the parameters n, r, τ and θ for strongly regular graphs with $m_2 = qm_3$ and $m_3 = qm_2$ for $q = \frac{3}{2}, \frac{4}{3}, \frac{5}{2}, \frac{5}{3}, \frac{5}{4}, \frac{6}{5}$. We now proceed to describe the parameters of strongly regular graphs with $m_2 = qm_3$ and $m_3 = qm_2$ for $q = \frac{7}{2}, \frac{7}{3}, \frac{7}{4}, \frac{7}{5}, \frac{7}{6}$, as follows. First,

Theorem 3 (Lepović [7]). *Let G be a connected strongly regular graph of order n and degree r with $m_2 = ap$ and $m_3 = bp$, where $a, b, p \in \mathbb{N}$ so that $(a, b) = 1$ and $a > b$. Then:*

(1^0) $n = (a+b)p + 1$ *;*

(2^0) $r = pt$ *;*

(3^0) $\tau = \left(pt - \dfrac{ak^2 + kt}{b}\right) - \dfrac{(a-b)k + t}{b}$ *;*

(4^0) $\theta = pt - \dfrac{ak^2 + kt}{b}$ *;*

(5^0) $\lambda_2 = k$ *;*

We say that a connected or disconnected graph G is integral if its spectrum $\sigma(G)$ consists only of integral values.

(6^0) $\lambda_3 = -\dfrac{ak+t}{b}$;

(7^0) $\delta = \dfrac{(a+b)k+t}{b}$;

(8^0) $(bp+1)t^2 - b((a+b)p+1)t + a(a+b)k^2 + 2akt = 0$,

for $k \in \mathbb{N}$ *and* $t = 1, 2, \ldots, a+b-1$, *where* $\delta = \lambda_2 - \lambda_3$.

Theorem 4 (Lepović [7]). *Let* G *be a connected strongly regular graph of order* n *and degree* r *with* $m_2 = bp$ *and* $m_3 = ap$, *where* $a, b, p \in \mathbb{N}$ *so that* $(a, b) = 1$ *and* $a > b$. *Then:*

(1^0) $n = (a+b)p + 1$;

(2^0) $r = pt$;

(3^0) $\tau = \Big(pt - \dfrac{ak^2-kt}{b}\Big) + \dfrac{(a-b)k-t}{b}$;

(4^0) $\theta = pt - \dfrac{ak^2-kt}{b}$;

(5^0) $\lambda_2 = \dfrac{ak-t}{b}$;

(6^0) $\lambda_3 = -k$;

(7^0) $\delta = \dfrac{(a+b)k-t}{b}$;

(8^0) $(bp+1)t^2 - b((a+b)p+1)t + a(a+b)k^2 - 2akt = 0$,

for $k \in \mathbb{N}$ *and* $t = 1, 2, \ldots, a+b-1$, *where* $\delta = \lambda_2 - \lambda_3$.

2. MAIN RESULTS

Remark 5. *Since* $m_2(\overline{G}) = m_3(G)$ *and* $m_3(\overline{G}) = m_2(G)$ *we note that if* $m_2(G) = qm_3(G)$ *then* $m_3(\overline{G}) = qm_2(\overline{G})$.

Remark 6. *In Theorems 11 –15 the complements of strongly regular graphs appear in pairs in* (k^0) *and* $(\overline{k}^0)$ *classes, where* k *denotes the corresponding number of a class.*

Remark 7. $\overline{\alpha K_\beta}$ *is a strongly regular graph of order* $n = \alpha\beta$ *and degree* $r = (\alpha-1)\beta$ *with* $\tau = (\alpha-2)\beta$ *and* $\theta = (\alpha-1)\beta$. *Its eigenvalues are* $\lambda_2 = 0$ *and* $\lambda_3 = -\beta$ *with* $m_2 = \alpha(\beta-1)$ *and* $m_3 = \alpha - 1$.

In order to demonstrate a method which is applied for describing the parameters n, r, τ and θ for strongly regular graphs with $m_2 = qm_3$ and $m_3 = qm_2$ for $q = \frac{7}{2}, \frac{7}{3}, \frac{7}{4}, \frac{7}{5}, \frac{7}{6}$, we shall here establish the parameters of strongly regular graphs with $m_2 = (\frac{7}{2})m_3$ and $m_3 = (\frac{7}{2})m_2$. In a similar way, we can establish Theorems 12, 13, 14 and 15.

Proposition 8. *Let G be a connected strongly regular graph of order n and degree r with $m_2 = (\frac{7}{2})m_3$. Then G belongs to the class $(\overline{2}^0)$ or (3^0) or (4^0) or $(\overline{5}^0)$ or $(\overline{6}^0)$ or (7^0) or $(\overline{8}^0)$ or (9^0) represented in Theorem 11.*

Proof. Let $m_2 = 7p$, $m_3 = 2p$ and $n = 9p + 1$ where $p \in \mathbb{N}$. Let $\lambda_2 = k$ where k is a positive integer. Then according to Theorem 3 we have (i) $\lambda_3 = -\frac{7k+t}{2}$; (ii) $\tau - \theta = -\frac{5k+t}{2}$; (iii) $\delta = \frac{9k+t}{2}$; (iv) $r = pt$ and (v) $\theta = pt - \frac{7k^2+kt}{2}$, where $t = 1, 2, \ldots, 8$. In this case we can easily see that Theorem 3 (8^0) is reduced to

$$(2p + 1)t^2 - 2(9p + 1)t + 63k^2 + 14kt = 0\,. \tag{1}$$

Case 1. ($t = 1$). Using (i)–(v) we find that $\lambda_2 = k$ and $\lambda_3 = -\frac{7k+1}{2}$, $\tau - \theta = -\frac{5k+1}{2}$, $\delta = \frac{9k+1}{2}$, $r = p$ and $\theta = p - \frac{7k^2+k}{2}$. Using (1) we find that $16p + 1 = 7k(9k + 2)$. Replacing k with $4k - 1$ we arrive at $p = 63k^2 - 28k + 3$. So we obtain that G is a strongly regular graph of order $n = 7(9k - 2)^2$ and degree $r = 63k^2 - 28k + 3$ with $\tau = 7k^2 - 12k + 2$ and $\theta = k(7k - 2)$.

Case 2. ($t = 2$). Using (i)–(v) we find that $\lambda_2 = k$ and $\lambda_3 = -\frac{7k+2}{2}$, $\tau - \theta = -\frac{5k+2}{2}$, $\delta = \frac{9k+2}{2}$, $r = 2p$ and $\theta = 2p - \frac{7k^2+2k}{2}$. Using (1) we find that $4p = k(9k+4)$. Replacing k with $2k$ we arrive at $p = k(9k+2)$. So we obtain that G is a strongly regular graph of order $n = (9k + 1)^2$ and degree $r = 2k(9k + 2)$ with $\tau = 4k^2 - 3k - 1$ and $\theta = 2k(2k + 1)$.

Case 3. ($t = 3$). Using (i)–(v) we find that $\lambda_2 = k$ and $\lambda_3 = -\frac{7k+3}{2}$, $\tau - \theta = -\frac{5k+3}{2}$, $\delta = \frac{9k+3}{2}$, $r = 3p$ and $\theta = 3p - \frac{7k^2+3k}{2}$. Using (1) we find that $12p - 1 = 7k(3k + 2)$. Replacing k with $6k + 1$ we arrive at $p = 63k^2 + 28k + 3$. So we obtain that G is a strongly regular graph of order $n = 7(9k + 2)^2$ and degree $r = 3(63k^2 + 28k + 3)$ with $\tau = 9k(7k + 2)$ and $\theta = (3k + 1)(21k + 4)$.

Case 4. ($t = 4$). Using (i)–(v) we find that $\lambda_2 = k$ and $\lambda_3 = -\frac{7k+4}{2}$, $\tau - \theta = -\frac{5k+4}{2}$, $\delta = \frac{9k+4}{2}$, $r = 4p$ and $\theta = 4p - \frac{7k^2+4k}{2}$. Using (1) we find that $40p - 8 = 7k(9k + 8)$. Replacing k with $20k + 4$ we arrive at $p = 630k^2 + 280k + 31$. So we obtain that G is a strongly regular graph of order $n = 70(9k+2)^2$ and degree $r = 4(630k^2 + 280k + 31)$ with $\tau = 2(560k^2 + 235k + 24)$ and $\theta = 20(4k+1)(14k+3)$.

Case 5. ($t = 5$). Using (i)–(v) we find that $\lambda_2 = k$ and $\lambda_3 = -\frac{7k+5}{2}$, $\tau - \theta = -\frac{5k+5}{2}$, $\delta = \frac{9k+5}{2}$, $r = 5p$ and $\theta = 5p - \frac{7k^2+5k}{2}$. Using (1) we find that $40p - 15 = 7k(9k + 10)$. Replacing k with $20k - 5$ we arrive at $p = 630k^2 - 280k + 31$. So we obtain that G is a strongly regular graph of order $n = 70(9k - 2)^2$ and degree $r = 5(630k^2 - 280k + 31)$ with $\tau = 10(5k - 1)(35k - 9)$ and $\theta = 10(5k - 1)(35k - 8)$.

Case 6. ($t = 6$). Using (i)–(v) we find that $\lambda_2 = k$ and $\lambda_3 = -\frac{7k+6}{2}$, $\tau - \theta = -\frac{5k+6}{2}$, $\delta = \frac{9k+6}{2}$, $r = 6p$ and $\theta = 6p - \frac{7k^2+6k}{2}$. Using (1) we find that $12p - 8 = 7k(3k+4)$. Replacing k with $6k-2$ we arrive at $p = 63k^2 - 28k + 3$. So we obtain that G is a strongly regular graph of order $n = 7(9k-2)^2$ and degree $r = 6(63k^2 - 28k + 3)$ with $\tau = 3(84k^2 - 39k + 4)$ and $\theta = 2(6k-1)(21k-5)$.

Case 7. ($t = 7$). Using (i)–(v) we find that $\lambda_2 = k$ and $\lambda_3 = -\frac{7k+7}{2}$, $\tau - \theta = -\frac{5k+7}{2}$, $\delta = \frac{9k+7}{2}$, $r = 7p$ and $\theta = 7p - \frac{7k^2+7k}{2}$. Using (1) we find that $4p - 5 = k(9k+14)$. Replacing k with $2k-1$ we arrive at $p = k(9k-2)$. So we obtain that G is a strongly regular graph of order $n = (9k-1)^2$ and degree $r = 7k(9k-2)$ with $\tau = 49k^2 - 12k - 1$ and $\theta = 7k(7k-1)$.

Case 8. ($t = 8$). Using (i)–(v) we find that $\lambda_2 = k$ and $\lambda_3 = -\frac{7k+8}{2}$, $\tau - \theta = -\frac{5k+8}{2}$, $\delta = \frac{9k+8}{2}$, $r = 8p$ and $\theta = 8p - \frac{7k^2+8k}{2}$. Using (1) we find that $16p - 48 = 7k(9k+16)$. Replacing k with $4k$ we arrive at $p = 63k^2 + 28k + 3$. So we obtain that G is a strongly regular graph of order $n = 7(9k+2)^2$ and degree $r = 8(63k^2 + 28k + 3)$ with $\tau = 2(7k+2)(32k+5)$ and $\theta = 8(4k+1)(14k+3)$. □

Proposition 9. *Let G be a connected strongly regular graph of order n and degree r with $m_3 = (\frac{7}{2})m_2$. Then G belongs to the class (2^0) or $(\overline{3}^0)$ or $(\overline{4}^0)$ or (5^0) or (6^0) or $(\overline{7}^0)$ or (8^0) or $(\overline{9}^0)$ represented in Theorem* 11.

Proof. Let $m_2 = 2p$, $m_3 = 7p$ and $n = 9p + 1$ where $p \in \mathbb{N}$. Let $\lambda_3 = -k$ where k is a positive integer. Then according to Theorem 4 we have (i) $\lambda_2 = \frac{7k-t}{2}$; (ii) $\tau - \theta = \frac{5k-t}{2}$; (iii) $\delta = \frac{9k-t}{2}$; (iv) $r = pt$ and (v) $\theta = pt - \frac{7k^2-kt}{2}$, where $t = 1, 2, \ldots, 8$. In this case we can easily see that Theorem 4 (8^0) is reduced to

$$(2p+1)t^2 - 2(9p+1)t + 63k^2 - 14kt = 0\,. \tag{2}$$

Case 1. ($t = 1$). Using (i)–(v) we find that $\lambda_2 = \frac{7k-1}{2}$ and $\lambda_3 = -k$, $\tau - \theta = \frac{5k-1}{2}$, $\delta = \frac{9k-1}{2}$, $r = p$ and $\theta = p - \frac{7k^2-k}{2}$. Using (2) we find that $16p + 1 = 7k(9k-2)$. Replacing k with $4k+1$ we arrive at $p = 63k^2 + 28k + 3$. So we obtain that G is a strongly regular graph of order $n = 7(9k+2)^2$ and degree $r = 63k^2 + 28k + 3$ with $\tau = 7k^2 + 12k + 2$ and $\theta = k(7k+2)$.

Case 2. ($t = 2$). Using (i)–(v) we find that $\lambda_2 = \frac{7k-2}{2}$ and $\lambda_3 = -k$, $\tau - \theta = \frac{5k-2}{2}$, $\delta = \frac{9k-2}{2}$, $r = 2p$ and $\theta = 2p - \frac{7k^2-2k}{2}$. Using (2) we find that $4p = k(9k-4)$. Replacing k with $2k$ we arrive at $p = k(9k-2)$. So we obtain that G is a strongly regular graph of order $n = (9k-1)^2$ and degree $r = 2k(9k-2)$ with $\tau = 4k^2 + 3k - 1$ and $\theta = 2k(2k-1)$.

Case 3. ($t = 3$). Using (i)–(v) we find that $\lambda_2 = \frac{7k-3}{2}$ and $\lambda_3 = -k$, $\tau - \theta = \frac{5k-3}{2}$, $\delta = \frac{9k-3}{2}$, $r = 3p$ and $\theta = 3p - \frac{7k^2-3k}{2}$. Using (2) we find that $12p - 1 = 7k(3k-2)$. Replacing k with $6k-1$ we arrive at $p = 63k^2 - 28k + 3$. So we obtain that G is a strongly regular graph of order $n = 7(9k-2)^2$ and degree $r = 3(63k^2 - 28k + 3)$ with $\tau = 9k(7k-2)$ and $\theta = (3k-1)(21k-4)$.

Case 4. ($t = 4$). Using (i)–(v) we find that $\lambda_2 = \frac{7k-4}{2}$ and $\lambda_3 = -k$, $\tau - \theta = \frac{5k-4}{2}$, $\delta = \frac{9k-4}{2}$, $r = 4p$ and $\theta = 4p - \frac{7k^2-4k}{2}$. Using (2) we find that $40p - 8 = 7k(9k-8)$. Replacing k with $20k-4$ we arrive at $p = 630k^2 - 280k + 31$. So we obtain that G is a strongly regular graph of order $n = 70(9k-2)^2$ and degree $r = 4(630k^2 - 280k + 31)$ with $\tau = 2(560k^2 - 235k + 24)$ and $\theta = 20(4k-1)(14k-3)$.

Case 5. ($t = 5$). Using (i)–(v) we find that $\lambda_2 = \frac{7k-5}{2}$ and $\lambda_3 = -k$, $\tau - \theta = \frac{5k-5}{2}$, $\delta = \frac{9k-5}{2}$, $r = 5p$ and $\theta = 5p - \frac{7k^2-5k}{2}$. Using (2) we find that $40p - 15 = 7k(9k-10)$. Replacing k with $20k+5$ we arrive at $p = 630k^2 + 280k + 31$. So we obtain that G is a strongly regular graph of order $n = 70(9k+2)^2$ and degree $r = 5(630k^2 + 280k + 31)$ with $\tau = 10(5k+1)(35k+9)$ and $\theta = 10(5k+1)(35k+8)$.

Case 6. ($t = 6$). Using (i)–(v) we find that $\lambda_2 = \frac{7k-6}{2}$ and $\lambda_3 = -k$, $\tau - \theta = \frac{5k-6}{2}$, $\delta = \frac{9k-6}{2}$, $r = 6p$ and $\theta = 6p - \frac{7k^2-6k}{2}$. Using (2) we find that $12p - 8 = 7k(3k-4)$. Replacing k with $6k+2$ we arrive at $p = 63k^2 + 28k + 3$. So we obtain that G is a strongly regular graph of order $n = 7(9k+2)^2$ and degree $r = 6(63k^2 + 28k + 3)$ with $\tau = 3(84k^2 + 39k + 4)$ and $\theta = 2(6k+1)(21k+5)$.

Case 7. ($t = 7$). Using (i)–(v) we find that $\lambda_2 = \frac{7k-7}{2}$ and $\lambda_3 = -k$, $\tau - \theta = \frac{5k-7}{2}$, $\delta = \frac{9k-7}{2}$, $r = 7p$ and $\theta = 7p - \frac{7k^2-7k}{2}$. Using (2) we find that $4p - 5 = k(9k-14)$. Replacing k with $2k+1$ we arrive at $p = k(9k+2)$. So we obtain that G is a strongly regular graph of order $n = (9k+1)^2$ and degree $r = 7k(9k+2)$ with $\tau = 49k^2 + 12k - 1$ and $\theta = 7k(7k+1)$.

Case 8. ($t = 8$). Using (i)–(v) we find that $\lambda_2 = \frac{7k-8}{2}$ and $\lambda_3 = -k$, $\tau - \theta = \frac{5k-8}{2}$, $\delta = \frac{9k-8}{2}$, $r = 8p$ and $\theta = 8p - \frac{7k^2-8k}{2}$. Using (2) we find that $16p - 48 = 7k(9k-16)$. Replacing k with $4k$ we arrive at $p = 63k^2 - 28k + 3$. So we obtain that G is a strongly regular graph of order $n = 7(9k-2)^2$ and degree $r = 8(63k^2 - 28k + 3)$ with $\tau = 2(7k-2)(32k-5)$ and $\theta = 8(4k-1)(14k-3)$. □

Remark 10. *We note that* $\overline{7K_4}$ *is a strongly regular graph with* $m_2 = (\frac{7}{2})m_3$. *It is obtained from the class Theorem 11* ($\overline{6}^0$) *for* $k = 0$.

Theorem 11. *Let* G *be a connected strongly regular graph of order* n *and degree* r *with* $m_2 = (\frac{7}{2})m_3$ *or* $m_3 = (\frac{7}{2})m_2$. *Then* G *is one of the following strongly regular graphs:*

(1^0) *G is the strongly regular graph $\overline{7K_4}$ of order $n = 28$ and degree $r = 24$ with $\tau = 20$ and $\theta = 24$. Its eigenvalues are $\lambda_2 = 0$ and $\lambda_3 = -4$ with $m_2 = 21$ and $m_3 = 6$;*

(2^0) *G is a strongly regular graph of order $n = (9k-1)^2$ and degree $r = 2k(9k-2)$ with $\tau = 4k^2 + 3k - 1$ and $\theta = 2k(2k-1)$, where $k \in \mathbb{N}$. Its eigenvalues are $\lambda_2 = 7k - 1$ and $\lambda_3 = -2k$ with $m_2 = 2k(9k-2)$ and $m_3 = 7k(9k-2)$;*

($\overline{2}^0$) *G is a strongly regular graph of order $n = (9k-1)^2$ and degree $r = 7k(9k-2)$ with $\tau = 49k^2 - 12k - 1$ and $\theta = 7k(7k-1)$, where $k \in \mathbb{N}$. Its eigenvalues are $\lambda_2 = 2k - 1$ and $\lambda_3 = -7k$ with $m_2 = 7k(9k-2)$ and $m_3 = 2k(9k-2)$;*

(3^0) *G is a strongly regular graph of order* $n = (9k+1)^2$ *and degree* $r = 2k(9k+2)$ *with* $\tau = 4k^2 - 3k - 1$ *and* $\theta = 2k(2k+1)$*, where* $k \in \mathbb{N}$*. Its eigenvalues are* $\lambda_2 = 2k$ *and* $\lambda_3 = -(7k+1)$ *with* $m_2 = 7k(9k+2)$ *and* $m_3 = 2k(9k+2)$*;*

($\overline{3}^0$) *G is a strongly regular graph of order* $n = (9k+1)^2$ *and degree* $r = 7k(9k+2)$ *with* $\tau = 49k^2 + 12k - 1$ *and* $\theta = 7k(7k+1)$*, where* $k \in \mathbb{N}$*. Its eigenvalues are* $\lambda_2 = 7k$ *and* $\lambda_3 = -(2k+1)$ *with* $m_2 = 2k(9k+2)$ *and* $m_3 = 7k(9k+2)$*;*

(4^0) *G is a strongly regular graph of order* $n = 7(9k-2)^2$ *and degree* $r = 63k^2 - 28k+3$ *with* $\tau = 7k^2 - 12k + 2$ *and* $\theta = k(7k-2)$*, where* $k \geq 2$*. Its eigenvalues are* $\lambda_2 = 4k-1$ *and* $\lambda_3 = -(14k-3)$ *with* $m_2 = 7(63k^2 - 28k + 3)$ *and* $m_3 = 2(63k^2 - 28k + 3)$*;*

($\overline{4}^0$) *G is a strongly regular graph of order* $n = 7(9k-2)^2$ *and degree* $r = 8(63k^2 - 28k+3)$ *with* $\tau = 2(7k-2)(32k-5)$ *and* $\theta = 8(4k-1)(14k-3)$*, where* $k \geq 2$*. Its eigenvalues are* $\lambda_2 = 14k-4$ *and* $\lambda_3 = -4k$ *with* $m_2 = 2(63k^2 - 28k+3)$ *and* $m_3 = 7(63k^2 - 28k + 3)$*;*

(5^0) *G is a strongly regular graph of order* $n = 7(9k-2)^2$ *and degree* $r = 3(63k^2 - 28k+3)$ *with* $\tau = 9k(7k-2)$ *and* $\theta = (3k-1)(21k-4)$*, where* $k \in \mathbb{N}$*. Its eigenvalues are* $\lambda_2 = 21k-5$ *and* $\lambda_3 = -(6k-1)$ *with* $m_2 = 2(63k^2 - 28k+3)$ *and* $m_3 = 7(63k^2 - 28k + 3)$*;*

($\overline{5}^0$) *G is a strongly regular graph of order* $n = 7(9k-2)^2$ *and degree* $r = 6(63k^2 - 28k+3)$ *with* $\tau = 3(84k^2 - 39k + 4)$ *and* $\theta = 2(6k-1)(21k-5)$*, where* $k \in \mathbb{N}$*. Its eigenvalues are* $\lambda_2 = 6k-2$ *and* $\lambda_3 = -(21k-4)$ *with* $m_2 = 7(63k^2 - 28k + 3)$ *and* $m_3 = 2(63k^2 - 28k + 3)$*;*

(6^0) *G is a strongly regular graph of order* $n = 7(9k+2)^2$ *and degree* $r = 63k^2 + 28k+3$ *with* $\tau = 7k^2 + 12k + 2$ *and* $\theta = k(7k+2)$*, where* $k \in \mathbb{N}$*. Its eigenvalues are* $\lambda_2 = 14k+3$ *and* $\lambda_3 = -(4k+1)$ *with* $m_2 = 2(63k^2 + 28k + 3)$ *and* $m_3 = 7(63k^2 + 28k + 3)$*;*

($\overline{6}^0$) *G is a strongly regular graph of order* $n = 7(9k+2)^2$ *and degree* $r = 8(63k^2 + 28k+3)$ *with* $\tau = 2(7k+2)(32k+5)$ *and* $\theta = 8(4k+1)(14k+3)$*, where* $k \in \mathbb{N}$*. Its eigenvalues are* $\lambda_2 = 4k$ *and* $\lambda_3 = -(14k+4)$ *with* $m_2 = 7(63k^2 + 28k+3)$ *and* $m_3 = 2(63k^2 + 28k + 3)$*;*

(7^0) *G is a strongly regular graph of order* $n = 7(9k+2)^2$ *and degree* $r = 3(63k^2 + 28k+3)$ *with* $\tau = 9k(7k+2)$ *and* $\theta = (3k+1)(21k+4)$*, where* $k \geq 0$*. Its eigenvalues are* $\lambda_2 = 6k+1$ *and* $\lambda_3 = -(21k+5)$ *with* $m_2 = 7(63k^2 + 28k+3)$ *and* $m_3 = 2(63k^2 + 28k + 3)$*;*

($\overline{7}^0$) *G is a strongly regular graph of order* $n = 7(9k+2)^2$ *and degree* $r = 6(63k^2 + 28k+3)$ *with* $\tau = 3(84k^2 + 39k + 4)$ *and* $\theta = 2(6k+1)(21k+5)$*, where* $k \geq 0$*. Its eigenvalues are* $\lambda_2 = 21k+4$ *and* $\lambda_3 = -(6k+2)$ *with* $m_2 = 2(63k^2 + 28k+3)$ *and* $m_3 = 7(63k^2 + 28k + 3)$*;*

(8^0) *G is a strongly regular graph of order $n = 70(9k-2)^2$ and degree $r = 4(630k^2-280k+31)$ with $\tau = 2(560k^2-235k+24)$ and $\theta = 20(4k-1)(14k-3)$, where $k \in \mathbb{N}$. Its eigenvalues are $\lambda_2 = 70k-16$ and $\lambda_3 = -(20k-4)$ with $m_2 = 2(630k^2-280k+31)$ and $m_3 = 7(630k^2-280k+31)$;*

($\overline{8}^0$) *G is a strongly regular graph of order $n = 70(9k-2)^2$ and degree $r = 5(630k^2-280k+31)$ with $\tau = 10(5k-1)(35k-9)$ and $\theta = 10(5k-1)(35k-8)$, where $k \in \mathbb{N}$. Its eigenvalues are $\lambda_2 = 20k-5$ and $\lambda_3 = -(70k-15)$ with $m_2 = 7(630k^2-280k+31)$ and $m_3 = 2(630k^2-280k+31)$;*

(9^0) *G is a strongly regular graph of order $n = 70(9k+2)^2$ and degree $r = 4(630k^2+280k+31)$ with $\tau = 2(560k^2+235k+24)$ and $\theta = 20(4k+1)(14k+3)$, where $k \geq 0$. Its eigenvalues are $\lambda_2 = 20k+4$ and $\lambda_3 = -(70k+16)$ with $m_2 = 7(630k^2+280k+31)$ and $m_3 = 2(630k^2+280k+31)$;*

($\overline{9}^0$) *G is a strongly regular graph of order $n = 70(9k+2)^2$ and degree $r = 5(630k^2+280k+31)$ with $\tau = 10(5k+1)(35k+9)$ and $\theta = 10(5k+1)(35k+8)$, where $k \geq 0$. Its eigenvalues are $\lambda_2 = 70k+15$ and $\lambda_3 = -(20k+5)$ with $m_2 = 2(630k^2+280k+31)$ and $m_3 = 7(630k^2+280k+31)$.*

Proof. First, according to Remark 7 we have $2\alpha(\beta-1) = 7(\alpha-1)$, from which we find that $\alpha = 7$, $\beta = 4$. In view of this we obtain the strongly regular graph represented in Theorem 11 (1^0). Next, according to Proposition 8 it turns out that G belongs to the class ($\overline{2}^0$) or (3^0) or (4^0) or ($\overline{5}^0$) or ($\overline{6}^0$) or (7^0) or ($\overline{8}^0$) or (9^0) if $m_2 = (\frac{7}{2})m_3$. According to Proposition 9 it turns out that G belongs to the class (2^0) or ($\overline{3}^0$) or ($\overline{4}^0$) or (5^0) or (6^0) or ($\overline{7}^0$) or (8^0) or ($\overline{9}^0$) if $m_3 = (\frac{7}{2})m_2$. □

Theorem 12. *Let G be a connected strongly regular graph of order n and degree r with $m_2 = (\frac{7}{3})m_3$ or $m_3 = (\frac{7}{3})m_2$. Then G is one of the following strongly regular graphs:*

(1^0) *G is the strongly regular graph $\overline{7K_3}$ of order $n = 21$ and degree $r = 18$ with $\tau = 15$ and $\theta = 18$. Its eigenvalues are $\lambda_2 = 0$ and $\lambda_3 = -3$ with $m_2 = 14$ and $m_3 = 6$;*

(2^0) *G is a strongly regular graph of order $n = (10k-1)^2$ and degree $r = 6k(5k-1)$ with $\tau = 9k^2+k-1$ and $\theta = 3k(3k-1)$, where $k \in \mathbb{N}$. Its eigenvalues are $\lambda_2 = 7k-1$ and $\lambda_3 = -3k$ with $m_2 = 6k(5k-1)$ and $m_3 = 14k(5k-1)$;*

($\overline{2}^0$) *G is a strongly regular graph of order $n = (10k-1)^2$ and degree $r = 14k(5k-1)$ with $\tau = 49k^2-11k-1$ and $\theta = 7k(7k-1)$, where $k \in \mathbb{N}$. Its eigenvalues are $\lambda_2 = 3k-1$ and $\lambda_3 = -7k$ with $m_2 = 14k(5k-1)$ and $m_3 = 6k(5k-1)$;*

(3^0) *G is a strongly regular graph of order $n = (10k+1)^2$ and degree $r = 6k(5k+1)$ with $\tau = 9k^2-k-1$ and $\theta = 3k(3k+1)$, where $k \in \mathbb{N}$. Its eigenvalues are $\lambda_2 = 3k$ and $\lambda_3 = -(7k+1)$ with $m_2 = 14k(5k+1)$ and $m_3 = 6k(5k+1)$;*

($\overline{3}^0$) *G is a strongly regular graph of order* $n = (10k+1)^2$ *and degree* $r = 14k(5k+1)$ *with* $\tau = 49k^2 + 11k - 1$ *and* $\theta = 7k(7k+1)$, *where* $k \in \mathbb{N}$. *Its eigenvalues are* $\lambda_2 = 7k$ *and* $\lambda_3 = -(3k+1)$ *with* $m_2 = 6k(5k+1)$ *and* $m_3 = 14k(5k+1)$ *;*

(4^0) *G is a strongly regular graph of order* $n = 21(10k-1)^2$ *and degree* $r = 210k^2 - 42k + 2$ *with* $\tau = 21k^2 - 15k + 1$ *and* $\theta = 3k(7k-1)$, *where* $k \in \mathbb{N}$. *Its eigenvalues are* $\lambda_2 = 9k-1$ *and* $\lambda_3 = -(21k-2)$ *with* $m_2 = 7(210k^2-42k+2)$ *and* $m_3 = 3(210k^2 - 42k + 2)$ *;*

($\overline{4}^0$) *G is a strongly regular graph of order* $n = 21(10k-1)^2$ *and degree* $r = 9(210k^2 - 42k + 2)$ *with* $\tau = 3(567k^2 - 113k + 5)$ *and* $\theta = 9(9k-1)(21k-2)$, *where* $k \in \mathbb{N}$. *Its eigenvalues are* $\lambda_2 = 21k - 3$ *and* $\lambda_3 = -9k$ *with* $m_2 = 3(210k^2 - 42k + 2)$ *and* $m_3 = 7(210k^2 - 42k + 2)$ *;*

(5^0) *G is a strongly regular graph of order* $n = 21(10k-1)^2$ *and degree* $r = 2(210k^2 - 42k + 2)$ *with* $\tau = 84k^2 - 4k - 1$ *and* $\theta = (6k-1)(14k-1)$, *where* $k \in \mathbb{N}$. *Its eigenvalues are* $\lambda_2 = 28k - 3$ *and* $\lambda_3 = -(12k-1)$ *with* $m_2 = 3(210k^2 - 42k + 2)$ *and* $m_3 = 7(210k^2 - 42k + 2)$ *;*

($\overline{5}^0$) *G is a strongly regular graph of order* $n = 21(10k-1)^2$ *and degree* $r = 8(210k^2 - 42k + 2)$ *with* $\tau = 4(336k^2 - 68k + 3)$ *and* $\theta = 4(12k-1)(28k-3)$, *where* $k \in \mathbb{N}$. *Its eigenvalues are* $\lambda_2 = 12k - 2$ *and* $\lambda_3 = -(28k-2)$ *with* $m_2 = 7(210k^2 - 42k + 2)$ *and* $m_3 = 3(210k^2 - 42k + 2)$ *;*

(6^0) *G is a strongly regular graph of order* $n = 21(10k-1)^2$ *and degree* $r = 5(210k^2 - 42k + 2)$ *with* $\tau = 525k^2 - 115k + 5$ *and* $\theta = (15k-1)(35k-4)$, *where* $k \in \mathbb{N}$. *Its eigenvalues are* $\lambda_2 = 15k - 2$ *and* $\lambda_3 = -(35k-3)$ *with* $m_2 = 7(210k^2 - 42k + 2)$ *and* $m_3 = 3(210k^2 - 42k + 2)$ *;*

($\overline{6}^0$) *G is a strongly regular graph of order* $n = 21(10k-1)^2$ *and degree* $r = 5(210k^2 - 42k + 2)$ *with* $\tau = 525k^2 - 95k + 3$ *and* $\theta = (15k-2)(35k-3)$, *where* $k \in \mathbb{N}$. *Its eigenvalues are* $\lambda_2 = 35k - 4$ *and* $\lambda_3 = -(15k-1)$ *with* $m_2 = 3(210k^2 - 42k + 2)$ *and* $m_3 = 7(210k^2 - 42k + 2)$ *;*

(7^0) *G is a strongly regular graph of order* $n = 21(10k+1)^2$ *and degree* $r = 210k^2 + 42k + 2$ *with* $\tau = 21k^2 + 15k + 1$ *and* $\theta = 3k(7k+1)$, *where* $k \in \mathbb{N}$. *Its eigenvalues are* $\lambda_2 = 21k+2$ *and* $\lambda_3 = -(9k+1)$ *with* $m_2 = 3(210k^2+42k+2)$ *and* $m_3 = 7(210k^2 + 42k + 2)$ *;*

($\overline{7}^0$) *G is a strongly regular graph of order* $n = 21(10k+1)^2$ *and degree* $r = 9(210k^2 + 42k + 2)$ *with* $\tau = 3(567k^2 + 113k + 5)$ *and* $\theta = 9(9k+1)(21k+2)$, *where* $k \in \mathbb{N}$. *Its eigenvalues are* $\lambda_2 = 9k$ *and* $\lambda_3 = -(21k+3)$ *with* $m_2 = 7(210k^2 + 42k + 2)$ *and* $m_3 = 3(210k^2 + 42k + 2)$ *;*

(8^0) *G is a strongly regular graph of order* $n = 21(10k+1)^2$ *and degree* $r = 2(210k^2 + 42k + 2)$ *with* $\tau = 84k^2 + 4k - 1$ *and* $\theta = (6k+1)(14k+1)$, *where* $k \in \mathbb{N}$. *Its eigenvalues are* $\lambda_2 = 12k + 1$ *and* $\lambda_3 = -(28k+3)$ *with* $m_2 = 7(210k^2 + 42k + 2)$ *and* $m_3 = 3(210k^2 + 42k + 2)$ *;*

($\overline{8}^0$) *G is a strongly regular graph of order $n = 21(10k+1)^2$ and degree $r = 8(210k^2+42k+2)$ with $\tau = 4(336k^2+68k+3)$ and $\theta = 4(12k+1)(28k+3)$, where $k \in \mathbb{N}$. Its eigenvalues are $\lambda_2 = 28k+2$ and $\lambda_3 = -(12k+2)$ with $m_2 = 3(210k^2+42k+2)$ and $m_3 = 7(210k^2+42k+2)$;*

(9^0) *G is a strongly regular graph of order $n = 21(10k+1)^2$ and degree $r = 5(210k^2+42k+2)$ with $\tau = 525k^2+95k+3$ and $\theta = (15k+2)(35k+3)$, where $k \geq 0$. Its eigenvalues are $\lambda_2 = 15k+1$ and $\lambda_3 = -(35k+4)$ with $m_2 = 7(210k^2+42k+2)$ and $m_3 = 3(210k^2+42k+2)$;*

($\overline{9}^0$) *G is a strongly regular graph of order $n = 21(10k+1)^2$ and degree $r = 5(210k^2+42k+2)$ with $\tau = 525k^2+115k+5$ and $\theta = (15k+1)(35k+4)$, where $k \geq 0$. Its eigenvalues are $\lambda_2 = 35k+3$ and $\lambda_3 = -(15k+2)$ with $m_2 = 3(210k^2+42k+2)$ and $m_3 = 7(210k^2+42k+2)$.*

Theorem 13. *Let G be a connected strongly regular graph of order n and degree r with $m_2 = (\frac{7}{4})m_3$ or $m_3 = (\frac{7}{4})m_2$. Then G is one of the following strongly regular graphs:*

(1^0) *G is a strongly regular graph of order $n = (11k-1)^2$ and degree $r = 4k(11k-2)$ with $\tau = 16k^2-k-1$ and $\theta = 4k(4k-1)$, where $k \in \mathbb{N}$. Its eigenvalues are $\lambda_2 = 7k-1$ and $\lambda_3 = -4k$ with $m_2 = 4k(11k-2)$ and $m_3 = 7k(11k-2)$;*

($\overline{1}^0$) *G is a strongly regular graph of order $n = (11k-1)^2$ and degree $r = 7k(11k-2)$ with $\tau = 49k^2-10k-1$ and $\theta = 7k(7k-1)$, where $k \in \mathbb{N}$. Its eigenvalues are $\lambda_2 = 4k-1$ and $\lambda_3 = -7k$ with $m_2 = 7k(11k-2)$ and $m_3 = 4k(11k-2)$;*

(2^0) *G is a strongly regular graph of order $n = (11k+1)^2$ and degree $r = 4k(11k+2)$ with $\tau = 16k^2+k-1$ and $\theta = 4k(4k+1)$, where $k \in \mathbb{N}$. Its eigenvalues are $\lambda_2 = 4k$ and $\lambda_3 = -(7k+1)$ with $m_2 = 7k(11k+2)$ and $m_3 = 4k(11k+2)$;*

($\overline{2}^0$) *G is a strongly regular graph of order $n = (11k+1)^2$ and degree $r = 7k(11k+2)$ with $\tau = 49k^2+10k-1$ and $\theta = 7k(7k+1)$, where $k \in \mathbb{N}$. Its eigenvalues are $\lambda_2 = 7k$ and $\lambda_3 = -(4k+1)$ with $m_2 = 4k(11k+2)$ and $m_3 = 7k(11k+2)$;*

(3^0) *G is a strongly regular graph of order $n = 14(11k-2)^2$ and degree $r = 2(154k^2-56k+5)$ with $\tau = k(56k-13)$ and $\theta = 2(4k-1)(7k-1)$, where $k \in \mathbb{N}$. Its eigenvalues are $\lambda_2 = 21k-4$ and $\lambda_3 = -(12k-2)$ with $m_2 = 4(154k^2-56k+5)$ and $m_3 = 7(154k^2-56k+5)$;*

($\overline{3}^0$) *G is a strongly regular graph of order $n = 14(11k-2)^2$ and degree $r = 9(154k^2-56k+5)$ with $\tau = 18(7k-1)(9k-2)$ and $\theta = 9(6k-1)(21k-4)$, where $k \in \mathbb{N}$. Its eigenvalues are $\lambda_2 = 12k-3$ and $\lambda_3 = -(21k-3)$ with $m_2 = 7(154k^2-56k+5)$ and $m_3 = 4(154k^2-56k+5)$;*

(4^0) *G is a strongly regular graph of order* $n = 14(11k+2)^2$ *and degree* $r = 2(154k^2+56k+5)$ *with* $\tau = k(56k+13)$ *and* $\theta = 2(4k+1)(7k+1)$, *where* $k \geq 0$. *Its eigenvalues are* $\lambda_2 = 12k+2$ *and* $\lambda_3 = -(21k+4)$ *with* $m_2 = 7(154k^2+56k+5)$ *and* $m_3 = 4(154k^2+56k+5)$ *;*

($\overline{4}^0$) *G is a strongly regular graph of order* $n = 14(11k+2)^2$ *and degree* $r = 9(154k^2+56k+5)$ *with* $\tau = 18(7k+1)(9k+2)$ *and* $\theta = 9(6k+1)(21k+4)$, *where* $k \geq 0$. *Its eigenvalues are* $\lambda_2 = 21k+3$ *and* $\lambda_3 = -(12k+3)$ *with* $m_2 = 4(154k^2+56k+5)$ *and* $m_3 = 7(154k^2+56k+5)$ *;*

(5^0) *G is a strongly regular graph of order* $n = 42(11k-4)^2$ *and degree* $r = 3(462k^2-336k+61)$ *with* $\tau = 18(3k-1)(7k-3)$ *and* $\theta = 6(3k-1)(21k-8)$, *where* $k \in \mathbb{N}$. *Its eigenvalues are* $\lambda_2 = 24k-9$ *and* $\lambda_3 = -(42k-15)$ *with* $m_2 = 7(462k^2-336k+61)$ *and* $m_3 = 4(462k^2-336k+61)$ *;*

($\overline{5}^0$) *G is a strongly regular graph of order* $n = 42(11k-4)^2$ *and degree* $r = 8(462k^2-336k+61)$ *with* $\tau = 2(1344k^2-975k+176)$ *and* $\theta = 24(8k-3)(14k-5)$, *where* $k \in \mathbb{N}$. *Its eigenvalues are* $\lambda_2 = 42k-16$ *and* $\lambda_3 = -(24k-8)$ *with* $m_2 = 4(462k^2-336k+61)$ *and* $m_3 = 7(462k^2-336k+61)$ *;*

(6^0) *G is a strongly regular graph of order* $n = 42(11k+4)^2$ *and degree* $r = 3(462k^2+336k+61)$ *with* $\tau = 18(3k+1)(7k+3)$ *and* $\theta = 6(3k+1)(21k+8)$, *where* $k \geq 0$. *Its eigenvalues are* $\lambda_2 = 42k+15$ *and* $\lambda_3 = -(24k+9)$ *with* $m_2 = 4(462k^2+336k+61)$ *and* $m_3 = 7(462k^2+336k+61)$ *;*

($\overline{6}^0$) *G is a strongly regular graph of order* $n = 42(11k+4)^2$ *and degree* $r = 8(462k^2+336k+61)$ *with* $\tau = 2(1344k^2+975k+176)$ *and* $\theta = 24(8k+3)(14k+5)$, *where* $k \geq 0$. *Its eigenvalues are* $\lambda_2 = 24k+8$ *and* $\lambda_3 = -(42k+16)$ *with* $m_2 = 7(462k^2+336k+61)$ *and* $m_3 = 4(462k^2+336k+61)$ *;*

(7^0) *G is a strongly regular graph of order* $n = 70(11k-5)^2$ *and degree* $r = 770k^2-700k+159$ *with* $\tau = 2(35k^2-25k+4)$ *and* $\theta = 5(2k-1)(7k-3)$, *where* $k \in \mathbb{N}$. *Its eigenvalues are* $\lambda_2 = 35k-16$ *and* $\lambda_3 = -(20k-9)$ *with* $m_2 = 4(770k^2-700k+159)$ *and* $m_3 = 7(770k^2-700k+159)$ *;*

($\overline{7}^0$) *G is a strongly regular graph of order* $n = 70(11k-5)^2$ *and degree* $r = 10(770k^2-700k+159)$ *with* $\tau = 5(1400k^2-1273k+289)$ *and* $\theta = 10(20k-9)(35k-16)$, *where* $k \in \mathbb{N}$. *Its eigenvalues are* $\lambda_2 = 20k-10$ *and* $\lambda_3 = -(35k-15)$ *with* $m_2 = 7(770k^2-700k+159)$ *and* $m_3 = 4(770k^2-700k+159)$ *;*

(8^0) *G is a strongly regular graph of order* $n = 70(11k+5)^2$ *and degree* $r = 770k^2+700k+159$ *with* $\tau = 2(35k^2+25k+4)$ *and* $\theta = 5(2k+1)(7k+3)$, *where* $k \geq 0$. *Its eigenvalues are* $\lambda_2 = 20k+9$ *and* $\lambda_3 = -(35k+16)$ *with* $m_2 = 7(770k^2+700k+159)$ *and* $m_3 = 4(770k^2+700k+159)$ *;*

($\overline{8}^0$) *G is a strongly regular graph of order* $n = 70(11k+5)^2$ *and degree* $r = 10(770k^2+700k+159)$ *with* $\tau = 5(1400k^2+1273k+289)$ *and* $\theta = 10(20k+$

9)(35k + 16), where $k \geq 0$. Its eigenvalues are $\lambda_2 = 35k + 15$ and $\lambda_3 = -(20k+10)$ with $m_2 = 4(770k^2+700k+159)$ and $m_3 = 7(770k^2+700k+159)$;

(9^0) *G is a strongly regular graph of order $n = 210(11k-1)^2$ and degree $r = 5(2310k^2-420k+19)$ with $\tau = 10(525k^2-93k+4)$ and $\theta = 15(10k-1)(35k-3)$, where $k \in \mathbb{N}$. Its eigenvalues are $\lambda_2 = 105k - 10$ and $\lambda_3 = -(60k-5)$ with $m_2 = 4(2310k^2 - 420k + 19)$ and $m_3 = 7(2310k^2 - 420k + 19)$;*

($\overline{9}^0$) *G is a strongly regular graph of order $n = 210(11k-1)^2$ and degree $r = 6(2310k^2-420k+19)$ with $\tau = 9(840k^2-155k+7)$ and $\theta = 30(12k-1)(21k-2)$, where $k \in \mathbb{N}$. Its eigenvalues are $\lambda_2 = 60k - 6$ and $\lambda_3 = -(105k-9)$ with $m_2 = 7(2310k^2 - 420k + 19)$ and $m_3 = 4(2310k^2 - 420k + 19)$;*

(10^0) *G is a strongly regular graph of order $n = 210(11k+1)^2$ and degree $r = 5(2310k^2+420k+19)$ with $\tau = 10(525k^2+93k+4)$ and $\theta = 15(10k+1)(35k+3)$, where $k \geq 0$. Its eigenvalues are $\lambda_2 = 60k + 5$ and $\lambda_3 = -(105k+10)$ with $m_2 = 7(2310k^2 + 420k + 19)$ and $m_3 = 4(2310k^2 + 420k + 19)$;*

($\overline{10}^0$) *G is a strongly regular graph of order $n = 210(11k+1)^2$ and degree $r = 6(2310k^2+420k+19)$ with $\tau = 9(840k^2+155k+7)$ and $\theta = 30(12k+1)(21k+2)$, where $k \geq 0$. Its eigenvalues are $\lambda_2 = 105k + 9$ and $\lambda_3 = -(60k+6)$ with $m_2 = 4(2310k^2 + 420k + 19)$ and $m_3 = 7(2310k^2 + 420k + 19)$.*

Theorem 14. *Let G be a connected strongly regular graph of order n and degree r with $m_2 = (\frac{7}{5})m_3$ or $m_3 = (\frac{7}{5})m_2$. Then G is one of the following strongly regular graphs:*

(1^0) *G is a strongly regular graph of order $n = (12k-1)^2$ and degree $r = 10k(6k-1)$ with $\tau = 25k^2 - 3k - 1$ and $\theta = 5k(5k-1)$, where $k \in \mathbb{N}$. Its eigenvalues are $\lambda_2 = 7k-1$ and $\lambda_3 = -5k$ with $m_2 = 10k(6k-1)$ and $m_3 = 14k(6k-1)$;*

($\overline{1}^0$) *G is a strongly regular graph of order $n = (12k-1)^2$ and degree $r = 14k(6k-1)$ with $\tau = 49k^2 - 9k - 1$ and $\theta = 7k(7k-1)$. where $k \in \mathbb{N}$. Its eigenvalues are $\lambda_2 = 5k-1$ and $\lambda_3 = -7k$ with $m_2 = 14k(6k-1)$ and $m_3 = 10k(6k-1)$;*

(2^0) *G is a strongly regular graph of order $n = (12k+1)^2$ and degree $r = 10k(6k+1)$ with $\tau = 25k^2+3k-1$ and $\theta = 5k(5k+1)$, where $k \in \mathbb{N}$. Its eigenvalues are $\lambda_2 = 5k$ and $\lambda_3 = -(7k+1)$ with $m_2 = 14k(6k+1)$ and $m_3 = 10k(6k+1)$;*

($\overline{2}^0$) *G is a strongly regular graph of order $n = (12k+1)^2$ and degree $r = 14k(6k+1)$ with $\tau = 49k^2+9k-1$ and $\theta = 7k(7k+1)$, where $k \in \mathbb{N}$. Its eigenvalues are $\lambda_2 = 7k$ and $\lambda_3 = -(5k+1)$ with $m_2 = 10k(6k+1)$ and $m_3 = 14k(6k+1)$;*

(3^0) *G is a strongly regular graph of order $n = 385(12k-5)^2$ and degree $r = 4620k^2-3850k+802$ with $\tau = 385k^2-341k+75$ and $\theta = 11(5k-2)(7k-3)$, where $k \in \mathbb{N}$. Its eigenvalues are $\lambda_2 = 55k - 23$ and $\lambda_3 = -(77k-32)$ with $m_2 = 7(4620k^2 - 3850k + 802)$ and $m_3 = 5(4620k^2 - 3850k + 802)$;*

($\overline{3}^0$) *G is a strongly regular graph of order* $n = 385(12k-5)^2$ *and degree* $r = 11(4620k^2 - 3850k + 802)$ *with* $\tau = 11(4235k^2 - 3529k + 735)$ *and* $\theta = 11(55k-23)(77k-32)$, *where* $k \in \mathbb{N}$. *Its eigenvalues are* $\lambda_2 = 77k - 33$ *and* $\lambda_3 = -(55k-22)$ *with* $m_2 = 5(4620k^2 - 3850k + 802)$ *and* $m_3 = 7(4620k^2 - 3850k + 802)$ *;*

(4^0) *G is a strongly regular graph of order* $n = 385(12k+5)^2$ *and degree* $r = 4620k^2 + 3850k + 802$ *with* $\tau = 385k^2 + 341k + 75$ *and* $\theta = 11(5k+2)(7k+3)$, *where* $k \geq 0$. *Its eigenvalues are* $\lambda_2 = 77k + 32$ *and* $\lambda_3 = -(55k+23)$ *with* $m_2 = 5(4620k^2 + 3850k + 802)$ *and* $m_3 = 7(4620k^2 + 3850k + 802)$ *;*

($\overline{4}^0$) *G is a strongly regular graph of order* $n = 385(12k+5)^2$ *and degree* $r = 11(4620k^2 + 3850k + 802)$ *with* $\tau = 11(4235k^2 + 3529k + 735)$ *and* $\theta = 11(55k+23)(77k+32)$, *where* $k \geq 0$. *Its eigenvalues are* $\lambda_2 = 55k + 22$ *and* $\lambda_3 = -(77k+33)$ *with* $m_2 = 7(4620k^2 + 3850k + 802)$ *and* $m_3 = 5(4620k^2 + 3850k + 802)$ *.*

Theorem 15. *Let G be a connected strongly regular graph of order n and degree r with* $m_2 = (\frac{7}{6})m_3$ *or* $m_3 = (\frac{7}{6})m_2$. *Then G is one of the following strongly regular graphs:*

(1^0) *G is the strongly regular graph* $\overline{7K_2}$ *of order* $n = 14$ *and degree* $r = 12$ *with* $\tau = 10$ *and* $\theta = 12$. *Its eigenvalues are* $\lambda_2 = 0$ *and* $\lambda_3 = -2$ *with* $m_2 = 7$ *and* $m_3 = 6$ *;*

(2^0) *G is a strongly regular graph of order* $n = (13k-1)^2$ *and degree* $r = 6k(13k-2)$ *with* $\tau = 36k^2 - 5k - 1$ *and* $\theta = 6k(6k-1)$, *where* $k \in \mathbb{N}$. *Its eigenvalues are* $\lambda_2 = 7k-1$ *and* $\lambda_3 = -6k$ *with* $m_2 = 6k(13k-2)$ *and* $m_3 = 7k(13k-2)$ *;*

($\overline{2}^0$) *G is a strongly regular graph of order* $n = (13k-1)^2$ *and degree* $r = 7k(13k-2)$ *with* $\tau = 49k^2 - 8k - 1$ *and* $\theta = 7k(7k-1)$, *where* $k \in \mathbb{N}$. *Its eigenvalues are* $\lambda_2 = 6k-1$ *and* $\lambda_3 = -7k$ *with* $m_2 = 7k(13k-2)$ *and* $m_3 = 6k(13k-2)$ *;*

(3^0) *G is a strongly regular graph of order* $n = (13k+1)^2$ *and degree* $r = 6k(13k+2)$ *with* $\tau = 36k^2+5k-1$ *and* $\theta = 6k(6k+1)$, *where* $k \in \mathbb{N}$. *Its eigenvalues are* $\lambda_2 = 6k$ *and* $\lambda_3 = -(7k+1)$ *with* $m_2 = 7k(13k+2)$ *and* $m_3 = 6k(13k+2)$ *;*

($\overline{3}^0$) *G is a strongly regular graph of order* $n = (13k+1)^2$ *and degree* $r = 7k(13k+2)$ *with* $\tau = 49k^2+8k-1$ *and* $\theta = 7k(7k+1)$, *where* $k \in \mathbb{N}$. *Its eigenvalues are* $\lambda_2 = 7k$ *and* $\lambda_3 = -(6k+1)$ *with* $m_2 = 6k(13k+2)$ *and* $m_3 = 7k(13k+2)$ *;*

(4^0) *G is a strongly regular graph of order* $n = 14(13k-1)^2$ *and degree* $r = 182k^2 - 28k + 1$ *with* $\tau = 2k(7k-2)$ *and* $\theta = 2k(7k-1)$, *where* $k \in \mathbb{N}$. *Its eigenvalues are* $\lambda_2 = 12k-1$ *and* $\lambda_3 = -(14k-1)$ *with* $m_2 = 7(182k^2 - 28k + 1)$ *and* $m_3 = 6(182k^2 - 28k + 1)$ *;*

($\overline{4}^0$) *G is a strongly regular graph of order* $n = 14(13k-1)^2$ *and degree* $r = 12(182k^2-28k+1)$ *with* $\tau = 2(1008k^2-155k+5)$ *and* $\theta = 12(12k-1)(14k-1)$, *where* $k \in \mathbb{N}$. *Its eigenvalues are* $\lambda_2 = 14k-2$ *and* $\lambda_3 = -12k$ *with* $m_2 = 6(182k^2-28k+1)$ *and* $m_3 = 7(182k^2-28k+1)$ *;*

(5^0) *G is a strongly regular graph of order* $n = 14(13k+1)^2$ *and degree* $r = 182k^2+28k+1$ *with* $\tau = 2k(7k+2)$ *and* $\theta = 2k(7k+1)$, *where* $k \in \mathbb{N}$. *Its eigenvalues are* $\lambda_2 = 14k+1$ *and* $\lambda_3 = -(12k+1)$ *with* $m_2 = 6(182k^2+28k+1)$ *and* $m_3 = 7(182k^2+28k+1)$ *;*

($\overline{5}^0$) *G is a strongly regular graph of order* $n = 14(13k+1)^2$ *and degree* $r = 12(182k^2+28k+1)$ *with* $\tau = 2(1008k^2+155k+5)$ *and* $\theta = 12(12k+1)(14k+1)$, *where* $k \in \mathbb{N}$. *Its eigenvalues are* $\lambda_2 = 12k$ *and* $\lambda_3 = -(14k+2)$ *with* $m_2 = 7(182k^2+28k+1)$ *and* $m_3 = 6(182k^2+28k+1)$ *;*

(6^0) *G is a strongly regular graph of order* $n = 35(13k-4)^2$ *and degree* $r = 3(455k^2-280k+43)$ *with* $\tau = 315k^2-190k+28$ *and* $\theta = 15(3k-1)(7k-2)$, *where* $k \in \mathbb{N}$. *Its eigenvalues are* $\lambda_2 = 35k-11$ *and* $\lambda_3 = -(30k-9)$ *with* $m_2 = 6(455k^2-280k+43)$ *and* $m_3 = 7(455k^2-280k+43)$ *;*

($\overline{6}^0$) *G is a strongly regular graph of order* $n = 35(13k-4)^2$ *and degree* $r = 10(455k^2-280k+43)$ *with* $\tau = 5(7k-2)(100k-33)$ *and* $\theta = 10(10k-3)(35k-11)$, *where* $k \in \mathbb{N}$. *Its eigenvalues are* $\lambda_2 = 30k-10$ *and* $\lambda_3 = -(35k-10)$ *with* $m_2 = 7(455k^2-280k+43)$ *and* $m_3 = 6(455k^2-280k+43)$ *;*

(7^0) *G is a strongly regular graph of order* $n = 35(13k+4)^2$ *and degree* $r = 3(455k^2+280k+43)$ *with* $\tau = 315k^2+190k+28$ *and* $\theta = 15(3k+1)(7k+2)$, *where* $k \geq 0$. *Its eigenvalues are* $\lambda_2 = 30k+9$ *and* $\lambda_3 = -(35k+11)$ *with* $m_2 = 7(455k^2+280k+43)$ *and* $m_3 = 6(455k^2+280k+43)$ *;*

($\overline{7}^0$) *G is a strongly regular graph of order* $n = 35(13k+4)^2$ *and degree* $r = 10(455k^2+280k+43)$ *with* $\tau = 5(7k+2)(100k+33)$ *and* $\theta = 10(10k+3)(35k+11)$, *where* $k \geq 0$. *Its eigenvalues are* $\lambda_2 = 35k+10$ *and* $\lambda_3 = -(30k+10)$ *with* $m_2 = 6(455k^2+280k+43)$ *and* $m_3 = 7(455k^2+280k+43)$ *;*

(8^0) *G is a strongly regular graph of order* $n = 42(13k-3)^2$ *and degree* $r = 4(546k^2-252k+29)$ *with* $\tau = 2(336k^2-153k+17)$ *and* $\theta = 12(4k-1)(14k-3)$, *where* $k \in \mathbb{N}$. *Its eigenvalues are* $\lambda_2 = 42k-10$ *and* $\lambda_3 = -(36k-8)$ *with* $m_2 = 6(546k^2-252k+29)$ *and* $m_3 = 7(546k^2-252k+29)$ *;*

($\overline{8}^0$) *G is a strongly regular graph of order* $n = 42(13k-3)^2$ *and degree* $r = 9(546k^2-252k+29)$ *with* $\tau = 6(567k^2-262k+30)$ *and* $\theta = 18(9k-2)(21k-5)$, *where* $k \in \mathbb{N}$. *Its eigenvalues are* $\lambda_2 = 36k-9$ *and* $\lambda_3 = -(42k-9)$ *with* $m_2 = 7(546k^2-252k+29)$ *and* $m_3 = 6(546k^2-252k+29)$ *;*

(9^0) *G is a strongly regular graph of order* $n = 42(13k+3)^2$ *and degree* $r = 4(546k^2+252k+29)$ *with* $\tau = 2(336k^2+153k+17)$ *and* $\theta = 12(4k+1)(14k+3)$, *where $k \geq 0$. Its eigenvalues are $\lambda_2 = 36k + 8$ and $\lambda_3 = -(42k + 10)$ with $m_2 = 7(546k^2 + 252k + 29)$ and $m_3 = 6(546k^2 + 252k + 29)$;*

($\overline{9}^0$) *G is a strongly regular graph of order $n = 42(13k + 3)^2$ and degree $r = 9(546k^2+252k+29)$ with $\tau = 6(567k^2+262k+30)$ and $\theta = 18(9k+2)(21k+5)$, where $k \geq 0$. Its eigenvalues are $\lambda_2 = 42k + 9$ and $\lambda_3 = -(36k + 9)$ with $m_2 = 6(546k^2 + 252k + 29)$ and $m_3 = 7(546k^2 + 252k + 29)$;*

(10^0) *G is a strongly regular graph of order $n = 105(13k - 1)^2$ and degree $r = 5(1365k^2-210k+8)$ with $\tau = 5(525k^2-82k+3)$ and $\theta = 5(15k-1)(35k-3)$, where $k \in \mathbb{N}$. Its eigenvalues are $\lambda_2 = 60k - 5$ and $\lambda_3 = -(70k - 5)$ with $m_2 = 7(1365k^2 - 210k + 8)$ and $m_3 = 6(1365k^2 - 210k + 8)$;*

($\overline{10}^0$) *G is a strongly regular graph of order $n = 105(13k - 1)^2$ and degree $r = 8(1365k^2-210k+8)$ with $\tau = 2(3360k^2-515k+19)$ and $\theta = 40(12k-1)(14k-1)$, where $k \in \mathbb{N}$. Its eigenvalues are $\lambda_2 = 70k-6$ and $\lambda_3 = -(60k-4)$ with $m_2 = 6(1365k^2 - 210k + 8)$ and $m_3 = 7(1365k^2 - 210k + 8)$;*

(11^0) *G is a strongly regular graph of order $n = 105(13k + 1)^2$ and degree $r = 5(1365k^2+210k+8)$ with $\tau = 5(525k^2+82k+3)$ and $\theta = 5(15k+1)(35k+3)$, where $k \geq 0$. Its eigenvalues are $\lambda_2 = 70k + 5$ and $\lambda_3 = -(60k + 5)$ with $m_2 = 6(1365k^2 + 210k + 8)$ and $m_3 = 7(1365k^2 + 210k + 8)$;*

($\overline{11}^0$) *G is a strongly regular graph of order $n = 105(13k + 1)^2$ and degree $r = 8(1365k^2+210k+8)$ with $\tau = 2(3360k^2+515k+19)$ and $\theta = 40(12k+1)(14k+1)$, where $k \geq 0$. Its eigenvalues are $\lambda_2 = 60k+4$ and $\lambda_3 = -(70k+6)$ with $m_2 = 7(1365k^2 + 210k + 8)$ and $m_3 = 6(1365k^2 + 210k + 8)$;*

(12^0) *G is a strongly regular graph of order $n = 231(13k - 2)^2$ and degree $r = 2(3003k^2-924k+71)$ with $\tau = 924k^2-275k+20$ and $\theta = 22(6k-1)(7k-1)$, where $k \in \mathbb{N}$. Its eigenvalues are $\lambda_2 = 77k - 12$ and $\lambda_3 = -(66k - 10)$ with $m_2 = 6(3003k^2 - 924k + 71)$ and $m_3 = 7(3003k^2 - 924k + 71)$;*

($\overline{12}^0$) *G is a strongly regular graph of order $n = 231(13k - 2)^2$ and degree $r = 11(3003k^2 - 924k + 71)$ with $\tau = 11(2541k^2 - 782k + 60)$ and $\theta = 11(33k - 5)(77k - 12)$, where $k \in \mathbb{N}$. Its eigenvalues are $\lambda_2 = 66k - 11$ and $\lambda_3 = -(77k-11)$ with $m_2 = 7(3003k^2-924k+71)$ and $m_3 = 6(3003k^2-924k+71)$;*

(13^0) *G is a strongly regular graph of order $n = 231(13k + 2)^2$ and degree $r = 2(3003k^2+924k+71)$ with $\tau = 924k^2+275k+20$ and $\theta = 22(6k+1)(7k+1)$, where $k \geq 0$. Its eigenvalues are $\lambda_2 = 66k + 10$ and $\lambda_3 = -(77k + 12)$ with $m_2 = 7(3003k^2 + 924k + 71)$ and $m_3 = 6(3003k^2 + 924k + 71)$;*

($\overline{13}^0$) *G is a strongly regular graph of order $n = 231(13k + 2)^2$ and degree $r = 11(3003k^2 + 924k + 71)$ with $\tau = 11(2541k^2 + 782k + 60)$ and $\theta = 11(33k + 5)(77k + 12)$, where $k \geq 0$. Its eigenvalues are $\lambda_2 = 77k + 11$ and $\lambda_3 = -(66k+11)$ with $m_2 = 6(3003k^2+924k+71)$ and $m_3 = 7(3003k^2+924k+71)$.*

3. CONCLUDING REMARKS

Using Theorems 3 and 4, it is possible to describe the parameters n, r, τ and θ, for any connected strongly regular graph by using only one parameter k. In the forthcoming paper we shall describe the parameters n, r, τ and θ, for strongly regular graphs with $m_2 = qm_3$ and $m_3 = qm_2$ for $q = \frac{8}{3}, \frac{8}{5}, \frac{8}{7}$.

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The all results presented in this work are verified by using a computer program `srgpar.exe`, which has been written by the author in the programming language `Borland C++ Builder 5.5`.