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ON (λ, μ, ζ)-ZWEIER IDEAL CONVERGENCE IN INTUITIONISTIC FUZZY NORMED SPACES

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Abstract: In this paper, we introduce and study a new type of convergence which is namely (λ, μ, ζ)-Zweier convergence and (λ, μ, ζ)-Zweier ideal convergence of triple sequences $x = (x_{ijk})$ in intuitionistic fuzzy normed spaces (IFNS), where $\lambda = (\lambda_n), \mu = (\mu_m)$ and $\zeta = (\zeta_p)$ are three non-decreasing sequences of positive real numbers such that each tend to infinity. Besides, we define and study (λ, μ, ζ)-Zweier Cauchy and (λ, μ, ζ)-Zweier ideal Cauchy sequences on the said space and establish some relations among them.problem.

Keywords: Ideal convergence, Zweier operator, (α, μ, ζ)-convergence, Intuitionistic Fuzzy Normed Spaces.

MSC: 40C05, 40J05, 46A45.

1. Introduction

The notion of statistical convergence for sequences of real numbers was initiated by Fast [7] and Steinhaus [37] independently. There has been an immense interest of researchers in finding the analogues and applications of statistical convergence in classical theory (see, [9], [32], [36], [38], [25], [13]). In 2000, Mursaleen [31] introduced the notion of λ-statistical convergence as an extension of the $[V, \lambda]$

summability, given by Leindler [30]. Subsequently, Sahiner and Tripathy [35] extended this concept to triple sequences. One of the most important generalization of the statistical convergence was introduced by Kostyrko et al.[29] by using the ideal I as a subset of the set of natural number $\mathbb{N}$, which they called I-convergence. Afterwards, it was investigated from the sequence space point of view by Salát et al. [39], Tripathy et al.[42], and Khan et al.[23]. Sengonul [40] initiated Zweier sequence space and various researchers extended this concept in different areas (see [6], [19], [20], [5], [11], [12], [26], [27]). In the crisp situation, we often come across triple sequences, i.e., sequences of matrices and certainly, there are situations where either the idea of ordinary convergence does not work or the underlying space does not serve our purpose. Therefore, to deal with such situations, we have to introduce some new type of measures in the fuzzy set theory, which can provide a better tool and a suitable framework. Fuzzy set theory is a powerful handset for modelling uncertainty and vagueness in various problems arising in the field of science and engineering. It has a wide range of applications in various fields such that population dynamics [2], chaos control [8], computer programming [10], non-linear dynamical systems [22] and so on.

On the other hand, Zadeh [43] introduced the concept of fuzzy sets. It has extensive applications in various fields within and beyond mathematics which also includes various types of real word problems. In 1986, Atanassov [1] generalized the fuzzy sets and introduced the notion of intuitionistic fuzzy sets. Later on, Park [33] studied the notion of intuitionistic fuzzy metric space and Saadati et al.[34] extended this concepts to topological spaces. The convergence of sequence in an IFNS is vital to fuzzy functional analysis, and we believe that I-convergence in an IFNS would yield a more comprehensive foundation of this field. Sengonul [41] defined Zweier Sequence Spaces of fuzzy numbers, and Hazarika et al. [21] further extended the work carried out by Sengonul to statistical convergence. Battor and Elaf [4] further generalized it to intuitionistic fuzzy Zweier I-convergent in triple sequence spaces. For more notions related to sequences spaces, we refer the reader to [14, 15, 16, 17, 18].

This paper is organized in the following sections: In section 2, we state some basic definitions and notions, i.e., ideal, filter, Zweier operator, (λ, μ, ζ)-convergence and intuitionistic fuzzy normed spaces. In section 3, we introduce and define (λ, μ, ζ)-Zweier convergence, and (λ, μ, ζ)-Zweier ideal convergence of triple sequences $x = (x_{ijk})$ in IFNS. We also define (λ, μ, ζ)-Zweier Cauchy and (λ, μ, ζ)-Zweier ideal Cauchy sequences and give some interesting results.

2. Preliminaries

In this section, we show some well-known notions and definitions which are needed for the development of this paper.

Definition 1. *[29] A family of subsets of the power set X, i.e., $I \subseteq P(X)$ is said to be an ideal in X if it satisfies the following conditions:*

1. $\emptyset \in I$,

2. *for each* $A, B \in I$, $A \cup B \in I$,
3. *for each* $A \in I$ *and* $B \subseteq A$, $B \in I$.

Remark 2. *[29] If* $I \neq P(X)$*, then ideal* I *is said to be non-trivial. A non-trivial ideal* I *is called admissible if* $\{\{x\} : x \in X\} \subseteq I$.

Definition 3. *[29] Suppose* $I \subseteq P(X)$ *be a non-trivial ideal, then the class* $F(I) = \{A \subset X : A^c \in I\}$ *is a filter on* X*, called the filter associated with the ideal* I.

Definition 4. *Let* $\lambda = (\lambda_n), \mu = (\mu_m)$ *and* $\zeta = (\zeta_p)$ *be three non-decreasing sequences of positive real numbers in such a way that* $\lambda_n, \mu_m, \zeta_p \to \infty$ *as* $n, m, p \to \infty$.

$$\begin{gathered}\lambda_{n+1} \leq \lambda_n + 1, \lambda_1 = 0, \\ \mu_{n+1} \leq \mu_n + 1, \mu_1 = 0, \\ \zeta_{n+1} \leq \zeta_n + 1, \zeta_1 = 0.\end{gathered}$$

Let $J_n = [n - \lambda_n + 1, n], J_m = [m - \mu_m + 1, m]$ *and* $J_p = [p - \zeta_p + 1, p]$*. Then , the number*

$$\delta_{\lambda,\mu,\zeta}(K) = \lim_{n,m,p\to\infty} \frac{1}{\lambda_n \mu_m \zeta_p} |\{(i, j, k) \in I_n \times I_m \times I_p : (i, j, k) \in K\}|$$

is said to be (λ, μ, ζ)*-density of the set* $K \subseteq \mathbb{N} \times \mathbb{N} \times \mathbb{N}$*, provided that the limit exists. In case* $\lambda_n = n, \mu_m = m, \zeta_p = p$*, the* (λ, μ, ζ)*-density reduces to the natural triple density.*

Now, the generalized triple Valée-Pousin mean is

$$t_{n,m,p}(x) = \tfrac{1}{\lambda_n \mu_m \zeta_p} \sum_{i\in J_n} \sum_{j\in J_m} \sum_{k\in J_p} x_{ijk}$$

where $J_n = [n - \lambda_n + 1, n], J_m = [m - \mu_m + 1, m]$ *and* $J_p = [p - \zeta_p + 1, p]$*. If* $\lambda_n = n$ *for all* n*,* $\mu_m = m$ *for all* m *and* $\zeta_p = p$ *for all* p*, then* (V, λ, μ, ζ)*-summablility is reduced to* $[C, 1, 1, 1]$*-summablility.*

Definition 5. *A triple sequence* $x = (x_{ijk})$ *of numbers is said to be* (λ, μ, ζ)*-statistical convergent to* L *if* $\delta_{(\lambda,\mu,\zeta)}(E) = 0$*, where* $E = \{i \in I_n, j \in I_m, k \in I_p : |x_{ijk} - L| \geq \epsilon\}$*, i.e., if for every* $\epsilon > 0$,

$$\lim_{n,m,p} \frac{1}{\lambda_n \mu_m \zeta_p} |\{i \in J_n, j \in J_m, k \in J_p : |x_{ijk} - L| \geq \epsilon\}| = 0$$

If $\lambda_n = n$ *for all* n*,* $\mu_m = m$ *for all* m *and* $\zeta_p = p$ *for all* p*, then* (λ, μ, ζ)*-triple statistical convergence is reduced to triple statistical convergence.*

Definition 6. *[28] A binary operation* $* : [0, 1] \times [0, 1] \to [0, 1]$ *is said to be t-norm if it satisfies the following conditions:*

1. $*$ *is commutative and associative,*

2. $x * y \leq x * z$ *whenever* $y \leq z$ *for all* $x, y, z \in [0, 1]$,
3. $x * 1 = x$.

Example 7. *[28] The following are important examples for t-norms:*

1. *Minimum* $*_m(x, y) = \min\{x, y\}$,
2. *Product*$*_p(x, y) = x.y$,
3. *Lukasiewicz t-norm* $*_L = \max(x + y - 1, 0)$.

Definition 8. *[28] A binary operation* $\diamond : [0, 1] \times [0, 1] \to [0, 1]$ *is said to be continuous t-conorm if it satisfies the following conditions:*

1. $\diamond$ *is commutative and associative,*
2. $x \diamond y \leq x \diamond z$ *whenever* $y \leq z$ *for all* $x, y, z \in [0, 1]$,
3. $x \diamond 0 = x$

Example 9. *[28] The following are important examples for t-conorms:*

1. *Maximum* $\diamond_m(x, y) = \max\{x, y\}$,
2. *Probabilistic sum* $\diamond_p(x, y) = x + y - x.y$,
3. *Lukasiewicz t-conorm* $\diamond_L = \min(x + y, 1)$.

Remark 10. *A binary operation* $\diamond : [0, 1] \times [0, 1] \to [0, 1]$ *is a t-conorm if and only if there exists a t-norm* $*$ *such that for all* $(x, y) \in [0, 1]^2$ *either one of the following of the two equivalent equalities holds:*

$$x \diamond y = 1 - (1 - x) * (1 - y) \tag{1}$$

$$x * y = 1 - (1 - x) \diamond (1 - y) \tag{2}$$

The t-conorm given by (1) is called the dual t-conorm of $*$ *and, analogously, the t-norm given by (2) is said to be the dual t-norm of* $\diamond$. *Obviously,* $(*_m, \diamond_m)$, $(*_p, \diamond_p)$ *and* $(*_L, \diamond_L)$ *are pairs of t-norms and t-conorms which are mutually dual to each other.*

The duality expressed in (1) allows us to translate many properties of t-norms into the corresponding properties of t-conorms, including the n ary and infinitary extensions of a t-conorm. The duality changes the order: If for some t-norms t_1 *and* t_2 *we have* $t_1 \leq t_2$, *and if* s_1 *and* s_2 *are the dual t-conorms of* t_1 *and* t_2, *respectively. Then, we obtain* $s_1 \geq s_2$.

A t-norm $* : [0, 1] \times [0, 1] \to [0, 1]$ *is continuous if for all convergent sequences* $(x_n), (y_n) \in [0, 1]^{\mathbb{N}}$, *we have* $\lim_{n\to\infty} x_n * \lim_{n\to\infty} y_n = \lim_{n\to\infty} x_n * y_n$. *Obviously, the continuity of a t-conorm* $\diamond$ *is equivalent to the continuity of the dual t-norm.*

Definition 11. *[34] The five-tuple* $(X, \phi, \psi, *, \diamond)$ *is said to be an instuitionistic fuzzy normed space (simply, IFNS) if* X *is a vector space,* ϕ *and* ψ *are fuzzy sets on* $X \times (0, \infty)$, $*$ *is a continuous t-norm and* $\diamond$ *is a continuous t-conorm satisfying the following conditions, for every* $s, t > 0; x, y \in X$*:*

1. $\phi(x,t) > 0$,
2. $\phi(x,t) + \phi(x,t) \leq 1$,
3. $\phi(ax,t) = \phi(x, \frac{t}{|a|})$ *for each* $a \neq 0$, $a \in \mathbb{R}$,
4. $\phi(x,t) = 1$ *if and only if* $x = 0$,
5. $\phi(x,t) * \phi(y,s) \leq \phi(x+y, t+s)$,
6. $\phi(x,.) : (0,\infty) \to [0,1]$ *is continuous,*
7. $\lim_{t\to\infty} \phi(x,t) = 1$ *and* $\lim_{t\to 0} \phi(x,t) = 0$,
8. $\psi(x,t) < 1$,
9. $\psi(x,t) = 0$ *if and only if* $x = 0$,
10. $\psi(x,t) \diamond \psi(y,s) \geq \psi(x+y, t+s)$,
11. $\psi(ax,t) = \psi(x, \frac{t}{|a|})$ *for each* $a \neq 0$, $a \in \mathbb{R}$,
12. $\psi(x,.) : (0,\infty) \to [0,1]$ *is continuous,*
13. $\lim_{t\to\infty} \psi(x,t) = 0$ *and* $\lim_{t\to 0} \psi(x,t) = 1$.

In this case, (ϕ, ψ) *is said to be an intuitionistic fuzzy norm.*

Remark 12. *Sengonul [40] introduced the notion of the sequence* $u = (u_n)$ *that is frequently well used as the* Z^p *transformation of the sequence* $x = (x_n)$*, this means that* $u_n = px_n + (1-p)x_{n-1}$*, where* $x_{n-1} \neq 0$, $p \neq 1$, $1 < p < \infty$ *and* Z^p *denotes the matrix* $Z^p = (z_{nm})$ *which is defined as:*

$$z_{nm} = \begin{cases} p, & if \quad n = m \\ 1-p, & if \quad n-1 = m;\ n,m \in \mathbb{N} \\ 0, & otherwise \end{cases}$$

Taking into account those notions, Basar and Altay [3], and Sengonul [40] introduced the Zweier sequence spaces $\mathcal{Z}$ *and* $\mathcal{Z}_0$ *as:*

$$\mathcal{Z} = \{x = (x_n) \in \omega : Z^p x \in c\}$$
$$\mathcal{Z}_0 = \{x = (x_n) \in \omega : \mathcal{Z}^p x \in c_0\}$$

3. Results

Throughout the article, for the sake of convenience, we denote $Z^q x = Z^q x_{ijk}$ for the sequence $x = (x_{ijk}) \in X$. We also take I^3 as a non-trivial admissible ideal of $\mathbb{N} \times \mathbb{N} \times \mathbb{N}$.

Definition 13. *Let* $(X, \phi, \psi, *, \diamond)$ *be an IFNS and* I^3 *is an admissible ideal. A triple sequence* $x = (x_{ijk})$ *in* X *is said to be* (λ, μ, ζ)*-Zweier ideal convergent to* L *with respect to the intuitionistic fuzzy norm* (ϕ, ψ)*, if for every* $\epsilon > 0$ *and for all* $s > 0$*, the set*

$$\{(n,m,p)\in\mathbb{N}\times\mathbb{N}\times\mathbb{N}:\frac{1}{\lambda_n\mu_m\zeta_p}\sum_{i\in J_n}\sum_{j\in J_m}\sum_{k\in J_p}\phi(Z^q x_{ijk}-L,s)\leq 1-\epsilon \text{ or } \frac{1}{\lambda_n\mu_m\zeta_p}\sum_{i\in J_n}\sum_{j\in J_m}\sum_{k\in J_p}\psi(Z^q x_{ijk}-L,s)\geq\epsilon\}\in I^3.$$

We write (ϕ,ψ)-$I^3_{(\lambda,\mu,\zeta)}$-$\lim_{i,j,k\to\infty} Z^q x_{ijk}=L$.

Lemma 14. *Let I^3 be an admissible ideal, and $(X,\phi,\psi,*,\diamond)$ be IFNS, and $x=(x_{ijk})$ be a triple sequence in X. Then, for each $s>0$ and $\epsilon>0$, the following statements are equivalent:*

1. (ϕ,ψ)-$I^3_{(\lambda,\mu,\zeta)}$-$\lim_{i,j,k\to\infty} Z^q x_{ijk}=L$.
2. $\{(n,m,p)\in\mathbb{N}\times\mathbb{N}\times\mathbb{N}:\frac{1}{\lambda_n\mu_m\zeta_p}\sum_{i\in J_n}\sum_{j\in J_m}\sum_{k\in J_p}\phi(Z^q x_{ijk}-L,s)\leq 1-\epsilon\}\in I^3$ *and* $\{(n,m,p)\in\mathbb{N}\times\mathbb{N}\times\mathbb{N}:\frac{1}{\lambda_n\mu_m\zeta_p}\sum_{i\in J_n}\sum_{j\in J_m}\sum_{k\in J_p}\psi(Z^q x_{ijk}-L,s)\geq\epsilon\}\in I^3$.
3. $\{(n,m,p)\in\mathbb{N}\times\mathbb{N}\times\mathbb{N}:\frac{1}{\lambda_n\mu_m\zeta_p}\sum_{i\in J_n}\sum_{j\in J_m}\sum_{k\in J_p}\phi(Z^q x_{ijk}-L,s)> 1-\epsilon\}\in F(I^3)$ *and* $\{(n,m,p)\in\mathbb{N}\times\mathbb{N}\times\mathbb{N}:\frac{1}{\lambda_n\mu_m\zeta_p}\sum_{i\in J_n}\sum_{j\in J_m}\sum_{k\in J_p}\psi(Z^q x_{ijk}-L,s)<\epsilon\}\in F(I^3)$.
4. $\{(n,m,p)\in\mathbb{N}\times\mathbb{N}\times\mathbb{N}:\frac{1}{\lambda_n\mu_m\zeta_p}\sum_{i\in J_n}\sum_{j\in J_m}\sum_{k\in J_p}\phi(Z^q x_{ijk}-L,s)\leq 1-\epsilon$ *or* $\frac{1}{\lambda_n\mu_m\zeta_p}\sum_{i\in J_n}\sum_{j\in J_m}\sum_{k\in J_p}\psi(Z^q x_{ijk}-L,s)\geq\epsilon\}\in I^3$.
5. (ϕ,ψ)-$I^3_{(\lambda,\mu,\zeta)}$-$\lim_{i,j,k\to\infty}\phi(Z^q x_{ijk}-L,s)=1$ *and* (ϕ,ψ)-$I^3_{(\lambda,\mu,\zeta)}$-$\lim_{i,j,k\to\infty}\psi(Z^q x_{ijk}-L,s)=0$.

Proof. The proof is followed directly by using Definition 13. □

Theorem 15. *Let $(X,\phi,\psi,*,\diamond)$ be an IFNS and $I^3\subset 2^{\mathbb{N}\times\mathbb{N}\times\mathbb{N}}$ be an ideal. A triple sequence $x=(x_{ijk})$ in X is (λ,μ,ζ)-Zweier ideal convergent with respect to the intuitionistic fuzzy norm (ϕ,ψ), then its limit is unique.*

Proof. Let (ϕ,ψ)-$I^3_{(\lambda,\mu,\zeta)}$-$\lim_{i,j,k\to\infty} Z^q x_{ijk}=L_1$ and (ϕ,ψ)-$I^3_{(\lambda,\mu,\zeta)}$-$\lim_{i,j,k\to\infty} Z^q x_{ijk}=L_2$. For a given $\epsilon>0$, take $\alpha>0$ in such a way that $(1-\alpha)*(1-\alpha)>1-\epsilon$ and $\alpha\diamond\alpha<\epsilon$. Then, for any $\alpha>0$, define

$$P_{\phi,1}(\alpha,s)=\{(n,m,p)\in\mathbb{N}\times\mathbb{N}\times\mathbb{N}:\frac{1}{\lambda_n\mu_m\zeta_p}\sum_{i\in J_n}\sum_{j\in J_m}\sum_{p\in J_p}\phi(Z^q x_{ijk}-L_1,\frac{s}{2})\leq 1-\alpha\}$$

$$P_{\phi,2}(\alpha,s)=\{(n,m,p)\in\mathbb{N}\times\mathbb{N}\times\mathbb{N}:\frac{1}{\lambda_n\mu_m\zeta_p}\sum_{i\in J_n}\sum_{j\in J_m}\sum_{p\in J_p}\phi(Z^qx_{ijk}-L_2,\frac{s}{2})\leq 1-\alpha\}$$

$$P_{\psi,1}(\alpha,s)=\{(n,m,p)\in\mathbb{N}\times\mathbb{N}\times\mathbb{N}:\frac{1}{\lambda_n\mu_m\zeta_p}\sum_{i\in J_n}\sum_{j\in J_m}\sum_{p\in J_p}\psi(Z^qx_{ijk}-L_1,\frac{s}{2})\geq\alpha\}$$

$$P_{\psi,2}(\alpha,s)=\{(n,m,p)\in\mathbb{N}\times\mathbb{N}\times\mathbb{N}:\frac{1}{\lambda_n\mu_m\zeta_p}\sum_{i\in J_n}\sum_{j\in J_m}\sum_{p\in J_p}\psi(Z^qx_{ijk}-L_2,\frac{s}{2})\geq\alpha\}$$

Since (ϕ,ψ)-$I^3_{(\lambda,\mu,\zeta)}$-$\lim_{i,j,k\to\infty}Z^qx_{ijk}=L_1$, we have $P_{\phi,1}(\alpha,s)$ and $P_{\psi,1}(\alpha,s)\in I^3$. Besides, using (ϕ,ψ)-$I^3_{(\lambda,\mu,\zeta)}$-$\lim_{i,j,k\to\infty}Z^qx_{ijk}=L_2$, we obtain $P_{\psi,2}(\alpha,s)$ and $P_{\psi,2}(\alpha,s)\in I^3$. Now, consider that

$$P_{\phi,\psi}(\alpha,s)=[P_{\phi,1}(\alpha,s)\cup P_{\phi,2}(\alpha,s)]\cap[P_{\psi,1}(\alpha,s)\cup P_{\psi,2}(\alpha,s)].$$

Thus, $P_{\phi,\psi}(\alpha,s)\in I^3$, this implies that $P^c_{\phi,\psi}(\alpha,s)$ is non-empty set in $F(I^3)$. If $(n,m,p)\in P^c_{\phi,\psi}(\alpha,s)$, then two possibilities arises: Either $(n,m,p)\in P^c_{\phi,1}(\alpha,s)\cap P_{\phi,2}(\alpha,s)$ or $(n,m,p)\in P^c_{\psi,1}(\alpha,s)\cap P_{\psi,2}(\alpha,s)$.

First all at, we consider that $(n,m,p)\in P^c_{\phi,1}(\alpha,s)\cap P_{\phi,2}(\alpha,s)$. Then, we obtain,

$$\frac{1}{\lambda_n\mu_m\zeta_p}\sum_{i\in J_n}\sum_{j\in J_m}\sum_{p\in J_p}\phi(Z^qx_{ijk}-L_1,\frac{s}{2})>1-\alpha$$

and

$$\frac{1}{\lambda_n\mu_m\zeta_p}\sum_{i\in J_n}\sum_{j\in J_m}\sum_{p\in J_p}\phi(Z^qx_{ijk}-L_2,\frac{s}{2})>1-\alpha$$

Now, taking $(q,w,e)\in\mathbb{N}\times\mathbb{N}\times\mathbb{N}$ such that

$$\phi(Z^qx_{qwe}-L_1,\tfrac{s}{2})>\frac{1}{\lambda_n\mu_m\zeta_p}\sum_{i\in J_n}\sum_{j\in J_m}\sum_{p\in J_p}\phi(Z^qx_{ijk}-L_1,\frac{s}{2})>1-\alpha$$

and

$$\phi(Z^qx_{qwe}-L_2,\tfrac{s}{2})>\frac{1}{\lambda_n\mu_m\zeta_p}\sum_{i\in J_n}\sum_{j\in J_m}\sum_{p\in J_p}\phi(Z^qx_{ijk}-L_2,\frac{s}{2})>1-\alpha$$

e.g., take into account, $\max\{\phi(Z^qx_{ijk}-L_1,\frac{s}{2}),\psi(Z^qx_{ijk}-L_2,\frac{s}{2}):i\in J_n;j\in J_m;k\in J_p\}$ and choose $(i,j,k)as(q,w,e)$ for which maximum occurs. Then, we have that

$$\phi(L_1-L_2,s)\geq\phi(Z^qx_{qwe}-L_1,\tfrac{s}{2})*\phi(Z^qx_{qwe}-L_2,\tfrac{s}{2})>(1-\alpha)*(1-\alpha)>1-\epsilon$$

Since $\epsilon>0$ was arbitrary, for every $\alpha>0$, we have $\phi(L_1-L_2,\alpha)=1$, which provides that $L_1=L_2$- Under other conditions, if $(n,m,p)\in P^c_{\psi,1}(\alpha,s)\cap P^c_{\psi,2}(\alpha,s)$. Then, on similar manner we can prove that $\psi(L_1-L_2,s)<\epsilon$ for all $\alpha>0$. Hence, we have $\psi(L_1-L_2,s)=0$ for all $\alpha>0$, which yields $L_1=L_2$. Therefore, we have proved that $I^3_{(\lambda,\mu,\zeta)}$-limit is unique.

□

Definition 16. *Let $(X,\phi,\psi,*,\diamond)$ be an IFNS. A triple sequence $x=(x_{ijk})$ in X is said to be (λ,μ,ζ)-Zweier convergent to L with respect to the intutitionistic fuzzy norm (ϕ,ψ), if for every $\epsilon,s>0$, there exits $n_0\in\mathbb{N}$ such that*

$$\frac{1}{\lambda_n\mu_m\zeta_p}\sum_{i\in J_n}\sum_{j\in J_m}\sum_{k\in J_p}\phi(Z^qx_{ijk}-L,s)>1-\epsilon \text{ and}$$
$$\frac{1}{\lambda_n\mu_m\zeta_p}\sum_{i\in J_n}\sum_{j\in J_m}\sum_{k\in J_p}\psi(Z^qx_{ijk}-L,s)<\epsilon \text{ for all } m,n,p\geq n_0.$$

We write (λ,μ,ζ)-$\lim_{i,j,k\to\infty}Z^qx_{ijk}=L$.

Theorem 17. *Let $(X,\phi,\psi,*,\diamond)$ be an IFNS. A triple sequence $x=(x_{ijk})$ in X is (λ,μ,ζ)-Zweier convergent with respect to the intuitionistic fuzzy norm (ϕ,ψ), then its limit is unique.*

Proof. Let (λ,μ,ζ)-$\lim_{i,j,k\to\infty}Z^qx_{ijk}=L_1$ and (λ,μ,ζ)-$\lim_{i,j,k\to\infty}Z^qx_{ijk}=L_2$. For a given $\epsilon>0$, take $\alpha>0$ in such a way that $(1-\alpha)*(1-\alpha)>1-\epsilon$ and $\alpha\diamond\alpha<\epsilon$. Then, for any $s>0$, there exists $n_2\in\mathbb{N}$ such that

$$\frac{1}{\lambda_n\mu_m\zeta_p}\sum_{i\in J_n}\sum_{j\in J_m}\sum_{p\in J_p}\phi(Z^qx_{ijk}-L_1,s)>1-\epsilon$$

and

$$\frac{1}{\lambda_n\mu_m\zeta_p}\sum_{i\in J_n}\sum_{j\in J_m}\sum_{p\in J_p}\psi(Z^qx_{ijk}-L_1,s)<\epsilon, \text{ for all } m,n,p\geq n_2$$

Also, there exists $n_3\in\mathbb{N}$ such that

$$\frac{1}{\lambda_n\mu_m\zeta_p}\sum_{i\in J_n}\sum_{j\in J_m}\sum_{p\in J_p}\phi(Z^qx_{ijk}-L_2,s)>1-\epsilon$$

and

$$\frac{1}{\lambda_n\mu_m\zeta_p}\sum_{i\in J_n}\sum_{j\in J_m}\sum_{p\in J_p}\psi(Z^qx_{ijk}-L_2,s)<\epsilon, \text{ for all } m,n,p\geq n_3.$$

Now, consider $n_i=\max\{n_2,n_3\}$. Then, for all $n\geq n_i$, we will obtain a $(q,w,e)\in\mathbb{N}\times\mathbb{N}\times\mathbb{N}$ such that

$$\phi(Z^q x_{qwe}-L_1,s) > \frac{1}{\lambda_n\mu_m\zeta_p}\sum_{i\in J_n}\sum_{j\in J_m}\sum_{p\in J_p}\phi(Z^q x_{ijk}-L_1,s) > 1-\alpha$$

and

$$\phi(Z^q x_{qwe}-L_2,s) > \frac{1}{\lambda_n\mu_m\zeta_p}\sum_{i\in J_n}\sum_{j\in J_m}\sum_{p\in J_p}\phi(Z^q x_{ijk}-L_2,s) > 1-\alpha$$

Hence, we have

$$\phi(L_1-L_2,s) \geq \phi(Z^q x_{qwe}-L_1,s) * \phi(Z^q x_{qwe}-L_2,s) > (1-\alpha)*(1-\alpha) > 1-\epsilon.$$

Since $\epsilon > 0$ is arbitrary, we have $\phi(L_1-L_2,s)=1$ for all $s>0$. In the same way, we can prove that $\psi(L_1-L_2,s)=0$ for all $s>0$. Therefore, this implies that $L_1=L_2$.

□

Theorem 18. *Let $(X,\phi,\psi,*,\diamond)$ be an IFNS and $x=(x_{ijk})$ be a triple sequence in X such that (λ,μ,ζ)-$\lim_{i,j,k\to\infty} Z^q x_{ijk}=L$, then (ϕ,ψ)-$I^3_{(\lambda,\mu,\zeta)}$-$\lim_{i,j,k\to\infty} Z^q x_{ijk}=L$.*

Proof. Let (λ,μ,ζ)-$\lim_{i,j,k\to\infty} Z^q z_{ijk}=L$. For each $s>0$ and $\epsilon>0$ there exists a positive integer $N_0\in\mathbb{N}$ such that $\frac{1}{\lambda_n\mu_m\zeta_p}\sum_{i\in J_n}\sum_{j\in J_m}\sum_{p\in J_p}\phi(Z^q x_{ijk}-L,s) > 1-\epsilon$ and $\frac{1}{\lambda_n\mu_m\zeta_p}\sum_{i\in J_n}\sum_{j\in J_m}\sum_{p\in J_p}\psi(Z^q x_{ijk}-L,s) < \epsilon$ for each $n,m,p\geq N_0$. Then, the set

$$Q(\epsilon,s)=\{(n,m,p)\in\mathbb{N}\times\mathbb{N}\times\mathbb{N}: \frac{1}{\lambda_n\mu_m\zeta_p}\sum_{i\in J_n}\sum_{j\in J_m}\sum_{p\in J_p}\phi(Z^q x_{ijk}-L,s)\leq 1-\epsilon$$
$$\text{or } \frac{1}{\lambda_n\mu_m\zeta_p}\sum_{i\in J_n}\sum_{j\in J_m}\sum_{p\in J_p}\psi(Z^q x_{ijk}-L,s)\geq\epsilon\}$$

$Q(\epsilon,s)\subseteq\{(1,1),(1,2),(2,1),(2,2),...,(N_0-1,N_0-1)\}$. This implies that the set $Q(\epsilon,s)$ has at most finitely many terms. Since ideal I^3 being admissible, we have that $Q(\epsilon,s)\in I^3$. This shows that (ϕ,ψ)-$I^3_{(\lambda,\mu,\zeta)}$-$\lim_{i,j,k\to\infty} Z^q x_{ijk}=L$.

□

Theorem 19. *Let $(X,\phi,\psi,*,\diamond)$ be an IFNS. If triple sequence $x=(x_{ijk})$ in X such that (λ,μ,ζ)-$\lim_{i,j,k\to\infty} Z^q x_{ijk}=L$, then there is a subsequence $(x_{i_q j_w k e})$ of sequence $x=(x_{ijk})$ such that (λ,μ,ζ)-$\lim_{i,j,k\to\infty} Z^q x_{i_q j_w k_e}=L$.*

Proof. Let (λ,μ,ζ)-$\lim_{i,j,k\to\infty} Z^q x_{ijk}=L$. Then, for every $s>0,\epsilon>0$ there exists a positive integer $n_0\in\mathbb{N}$ such that $\frac{1}{\lambda_n\mu_m\zeta_p}\sum_{i\in J_n}\sum_{j\in J_m}\sum_{p\in J_p}\phi(Z^q x_{ijk}-L,s) >$

$1-\epsilon$ and $\dfrac{1}{\lambda_n\mu_m\zeta_p}\sum_{i\in J_n}\sum_{j\in J_m}\sum_{p\in J_p}\phi(Z^q x_{ijk}-L,s)<\epsilon$ for each $n,m,p\geq n_0$. We can see that for each $n,m,p\geq n_0$, one can choose $i_q\in J_n, j_w\in J_m$ and $k_e\in J_p$ in such a way that

$$\phi(Z^q x_{i_q j_w e_k}-L,s)>\frac{1}{\lambda_n\mu_m\zeta_p}\sum_{i\in J_n}\sum_{j\in J_m}\sum_{p\in J_p}\phi(Z^q x_{ijk}-L,s)>1-\epsilon$$

and

$$\psi(Z^q x_{i_q j_w e_k}-L,s)>\frac{1}{\lambda_n\mu_m\zeta_p}\sum_{i\in J_n}\sum_{j\in J_m}\sum_{p\in J_p}\psi(Z^q x_{ijk}-L,s)<\epsilon$$

Therefore, (λ,μ,ζ)-$\lim_{i,j,k\to\infty} Z^q x_{i_q j_w k_e}=L$. $\square$

Definition 20. *Let $(X,\phi,\psi,*,\diamond)$ be an IFNS. A triple sequence $x=(x_{ijk})$ in X is said to be (λ,μ,ζ)-Zweier Cauchy sequence with respect to th intuitionistic fuzzy norm (ϕ,ψ), if for every $\epsilon,s>0$, there exists $n_0\in\mathbb{N}$ such that*

$$\frac{1}{\lambda_n\mu_m\zeta_p}\sum_{i,q\in J_n}\sum_{j,w\in J_m}\sum_{k,e\in J_p}\phi(Z^q x_{ijk}-Z^q x_{qwe},s)>1-\epsilon \text{ and}$$
$$\frac{1}{\lambda_n\mu_m\zeta_p}\sum_{i,q\in J_n}\sum_{j,w\in J_m}\sum_{k,e\in J_p}\psi(Z^q x_{ijk}-Z^q x_{qwe},s)<\epsilon \text{ for all } m,n,p\geq n_0$$

Definition 21. *Let $(X,\phi,\psi,*,\diamond)$ be an IFNS and $I^3\subseteq 2^{\mathbb{N}\times\mathbb{N}\times\mathbb{N}}$ be an ideal. A triple sequence $x=(x_{ijk})$ in X is said to be (λ,μ,ζ)-Zweier ideal Cauchy sequence with respect to the intuitionistic fuzzy norm (ϕ,ψ), if for every $\epsilon,t>0$, there exits $n_0\in\mathbb{N}$ such that*

$$\{(n,m,p)\in\mathbb{N}\times\mathbb{N}\times\mathbb{N}:\frac{1}{\lambda_n\mu_m\zeta_p}\sum_{i,q\in J_n}\sum_{j,w\in J_m}\sum_{k,e\in J_p}\phi(Z^q x_{ijk}-Z^q x_{qwe},s)>$$
$$1-\epsilon\}\in F(I^3) \text{ and } \{(n,m,p)\in\mathbb{N}\times\mathbb{N}\times\mathbb{N}:$$
$$\frac{1}{\lambda_n\mu_m\zeta_p}\sum_{i,q\in J_n}\sum_{j,w\in J_m}\sum_{k,e\in J_p}\psi(Z^q x_{ijk}-Z^q x_{qwe},s)<\epsilon\}\in F(I^3)$$

In the following Theorem, we establish a relation between the (λ,μ,ζ)-Zweier ideal convergent sequences and (λ,μ,ζ)-Zweier ideal Cauchy sequences.

Theorem 22. *Let $(X,\phi,\psi,*,\diamond)$ be an IFNS. A triple sequence $x=(x_{ijk})$ in X is (λ,μ,ζ)-Zweier ideal convergent with respect to the intuitionistic fuzzy norm (ϕ,ψ) if and only if it is (λ,μ,ζ)-Zweier ideal Cauchy sequence with respect to the same norm.*

Proof. Consider that the sequence $x=(x_{ijk})$ is (α,μ,ζ)-Zweier ideal Cauchy but not (α,μ,ζ)-Zweier ideal convergent with respect to the intuitionistic fuzzy norm (ϕ,ψ). Then, there exist positive integers q,w,e in such a way that if we take

$$R(\epsilon,s)=\{(n,m,p)\in\mathbb{N}\times\mathbb{N}\times\mathbb{N}:$$
$$\frac{1}{\lambda_n\mu_m\zeta_p}\sum_{i,q\in J_n}\sum_{j,w\in J_m}\sum_{k,e\in J_p}\phi(Z^qx_{ijk}-Z^qx_{qwe},s)\le 1-\epsilon \text{ or}$$
$$:\frac{1}{\lambda_n\mu_m\zeta_p}\sum_{i,q\in J_n}\sum_{j,w\in J_m}\sum_{k,e\in J_p}\psi(Z^qx_{ijk}-Z^qx_{qwe},s)\ge \epsilon\}$$

and

$$S(\epsilon,s)=\{(n,m,p)\in\mathbb{N}\times\mathbb{N}\times\mathbb{N}:\frac{1}{\lambda_n\mu_m\zeta_p}\sum_{i\in J_n}\sum_{j\in J_m}\sum_{k\in J_p}\phi(Z^qx_{ijk}-L,\frac{s}{2})\le 1-\epsilon$$
$$\text{or}:\frac{1}{\lambda_n\mu_m\zeta_p}\sum_{i,q\in J_n}\sum_{j,w\in J_m}\sum_{k,e\in J_p}\psi(Z^qx_{ijk}-L,\frac{s}{2})\ge \epsilon\}$$

Then, $R(\epsilon,s)\in I^3$ implies that $R^c(\epsilon,s)\in F(I^3)$. Since

$$\frac{1}{\lambda_n\mu_m\zeta_p}\sum_{i,q\in J_n}\sum_{j,w\in J_m}\sum_{k,e\in J_p}\phi(Z^qx_{ijk}-Z^qx_{qwe},s)\ge$$
$$\frac{2}{\lambda_n\mu_m\zeta_p}\sum_{i,q\in J_n}\sum_{j,w\in J_m}\sum_{k,e\in J_p}\phi(Z^qx_{ijk}-L,\frac{s}{2})>1-\epsilon$$

and

$$\frac{1}{\lambda_n\mu_m\zeta_p}\sum_{i,q\in J_n}\sum_{j,w\in J_m}\sum_{k,e\in J_p}\psi(Z^qx_{ijk}-Z^qx_{qwe},s)\le$$
$$\frac{2}{\lambda_n\mu_m\zeta_p}\sum_{i,q\in J_n}\sum_{j,w\in J_m}\sum_{k,e\in J_p}\psi(Z^qx_{ijk}-L,\frac{s}{2})<\epsilon.$$

If $\frac{1}{\lambda_n\mu_m\zeta_p}\sum_{i\in J_n}\sum_{j\in J_m}\sum_{k\in J_p}\phi(Z^qx_{ijk}-L,\frac{s}{2})>\frac{1-\epsilon}{2}$ and $\frac{1}{\lambda_n\mu_m\zeta_p}\sum_{i\in J_n}\sum_{j\in J_m}\sum_{k\in J_p}\psi(Z^qx_{ijk}-L,\frac{s}{2})<\frac{\epsilon}{2}$, receptively. Then, we have

$$\delta_{(\lambda,\mu,\zeta)}(\{(n,m,p)\in\mathbb{N}\times\mathbb{N}\times\mathbb{N}:$$
$$\frac{1}{\lambda_n\mu_m\zeta_p}\sum_{i,q\in J_n}\sum_{j,w\in J_m}\sum_{k,e\in J_p}\phi(Z^qx_{ijk}-Z^qx_{qwe},s)>1-\epsilon \text{ and}$$
$$\frac{1}{\lambda_n\mu_m\zeta_p}\sum_{i,q\in J_n}\sum_{j,w\in J_m}\sum_{k,e\in J_p}\phi(Z^qx_{ijk}-Z^qx_{qwe},s)<\epsilon\})=0$$

That is that $R(\epsilon,s)\in F(I^3)$, which contradicts our assumption. Hence, the sequence $x=(x_{ijk})$ is (λ,μ,ζ)-Zweier ideal convergent with respect to the intutitionistic fuzzy norm (ϕ,ψ).

Conversely,consider that (ϕ,ψ)-$I^3_{(\lambda,\mu,\zeta)}$-$\lim_{i,j,k\to\infty} Z^qx_{ijk}=L$. Now, take $\theta>0$ in such a way that $(1-\theta)*(1-\theta)>1-\epsilon$ and $\theta\diamond\theta<\epsilon$. For all $t>0$. Now, define

$$E(\theta,s)=\{(n,m,p)\in\mathbb{N}\times\mathbb{N}\times\mathbb{N}:\frac{1}{\lambda_n\mu_m\zeta_p}\sum_{i\in J_n}\sum_{j\in J_m}\sum_{k\in J_p}\phi(Z^q x_{ijk}-L,\frac{s}{2})\leq 1-\theta$$
$$\text{or }\frac{1}{\lambda_n\mu_m\zeta_p}\sum_{i\in J_n}\sum_{j\in J_m}\sum_{k\in J_p}\psi(Z^q x_{ijk}-L,\frac{s}{2})\geq\theta\}\in I^3.$$

This means $E(\theta,s)^c\in F(I^3)$. Now, assume $(q,w,e)\in E(\theta,s)^c$. Then, we have

$$\frac{1}{\lambda_n\mu_m\zeta_p}\sum_{i\in J_n}\sum_{j\in J_m}\sum_{k\in J_p}\phi(Z^q x_{ijk}-L,\frac{s}{2})>1-\theta$$

and

$$\frac{1}{\lambda_n\mu_m\zeta_p}\sum_{i\in J_n}\sum_{j\in J_m}\sum_{k\in J_p}\psi(Z^q x_{ijk}-L,\frac{s}{2})<\theta$$

For every $\epsilon>0$, we choose

$$H(\epsilon,s)=\{(n,m,p)\in\mathbb{N}\times\mathbb{N}\times\mathbb{N}:$$
$$\frac{1}{\lambda_n\mu_m\zeta_p}\sum_{i,q\in J_n}\sum_{j,w\in J_m}\sum_{k,e\in J_p}\phi(Z^q x_{ijk}-Z^q x_{qwe},s)\leq 1-\epsilon\text{ or}$$
$$\frac{1}{\lambda_n\mu_m\zeta_p}\sum_{i\in J_n}\sum_{j\in J_m}\sum_{k\in J_p}\psi(Z^q x_{ijk}-Z^q x_{qwe},s)\geq\epsilon\}.$$

Now, we ought to show that $H(\epsilon,s)\subset E(\theta,s)$. Let $(a,s,d)\in H(\epsilon,s)$. Then, we have

$$\frac{1}{\lambda_n\mu_m\zeta_p}\sum_{a,q\in J_n}\sum_{s,w\in J_m}\sum_{d,e\in J_p}\phi(Z^q x_{asd}-Z^q x_{qwe},s)\leq 1-\epsilon$$

and

$$\frac{1}{\lambda_n\mu_m\zeta_p}\sum_{a,q\in J_n}\sum_{s,w\in J_m}\sum_{d,e\in J_p}\psi(Z^q x_{asd}-Z^q x_{qwe},s)\geq\epsilon$$

One the basis of above inequality, we distinguish the following two cases as follows:

Case 1: Let $\frac{1}{\lambda_n\mu_m\zeta_p}\sum_{a,q\in J_n}\sum_{s,w\in J_m}\sum_{d,e\in J_p}\phi(Z^q x_{asd}-Z^q x_{qwe},s)\leq 1-\epsilon$. Then, $\frac{1}{\lambda_n\mu_m\zeta_p}\sum_{a\in J_n}\sum_{s\in J_m}\sum_{d\in J_p}\phi(Z^q x_{asd}-L,\frac{s}{2})\leq 1-\theta$, therefore $(a,s,d)\in E(\theta,t)$.

Otherwise, if $\frac{1}{\lambda_n\mu_m\zeta_p}\sum_{a\in J_n}\sum_{s\in J_m}\sum_{d\in J_p}\phi(Z^q x_{asd}-L,\frac{s}{2})\leq 1-\theta$. Then, we have

$$1\text{-}\epsilon \geq \frac{1}{\lambda_n\mu_m\zeta_p}\sum_{a,q\in J_n}\sum_{s,w\in J_m}\sum_{d,e\in J_p}\phi(Z^q x_{asd}-Z^q x_{qwe},s)$$
$$\geq$$
$$\frac{1}{\lambda_n\mu_m\zeta_p}\sum_{a\in J_n}\sum_{s\in J_m}\sum_{d\in J_p}\phi(Z^q x_{asd}-L,\frac{s}{2})*\frac{1}{\lambda_n\mu_m\zeta_p}\sum_{q\in J_n}\sum_{w\in J_m}\sum_{e\in J_p}\phi(Z^q x_{qwe}-L,\frac{s}{2})$$
$$>(1-\theta)*(1-\theta)$$
$$1-\epsilon$$

which is a contradiction. Therefore, $H(\epsilon,s)\subset E(\theta,s)$.

Case 2: Consider $\frac{1}{\lambda_n\mu_m\zeta_p}\sum_{a,q\in J_n}\sum_{s,w\in J_m}\sum_{d,e\in J_p}\psi(Z^q x_{asd}-Z^q x_{qwe},s)\geq\epsilon$. We obtain $\frac{1}{\lambda_n\mu_m\zeta_p}\sum_{a\in J_n}\sum_{s\in J_m}\sum_{d\in J_p}\psi(Z^q x_{asd}-L,\frac{s}{2})\geq\theta$, hence $(a,s,d)\in E(\theta,s)$. Otherwise, if $\frac{1}{\lambda_n\mu_m\zeta_p}\sum_{a\in J_n}\sum_{s\in J_m}\sum_{d\in J_p}\psi(Z^q x_{asd}-L,\frac{s}{2})<\theta$. Then, we have

$$\epsilon \leq \frac{1}{\lambda_n\mu_m\zeta_p}\sum_{a,q\in J_n}\sum_{s,w\in J_m}\sum_{d,e\in J_p}\psi(Z^q x_{asd}-Z^q x_{qwe},s)$$
$$\leq$$
$$\frac{1}{\lambda_n\mu_m\zeta_p}\sum_{a\in J_n}\sum_{s\in J_m}\sum_{d\in J_p}\psi(Z^q x_{asd}-L,\frac{s}{2})\diamond\frac{1}{\lambda_n\mu_m\zeta_p}\sum_{q\in J_n}\sum_{w\in J_m}\sum_{e\in J_p}\psi(Z^q x_{qwe}-L,\frac{s}{2})$$
$$<\theta\diamond\theta$$
$$<\epsilon$$

which is a contradiction. Therefore, $H(\epsilon,s)\subset E(\theta,s)$. Hence, we have concluded that $H(\epsilon,s)\subset E(\theta,s)$. This proves that $H(\epsilon,s)\in I^3$. In conclusion, the triple sequence $x=(x_{ijk})$ is a Zweier ideal Cauchy sequence with respect to the intuisionistic fuzzy norm (ϕ,ψ).

□

If a triple sequence $x=(x_{ijk})$ in X is (λ,μ,ζ)-Zweier ideal convergent with respect to the intuitionistic fuzzy norm $(\phi,\psi)_1$ and $x=(x_{ijk})$ in X is (λ,μ,ζ)-Zweier ideal Cauchy sequence with respect to the intuitionistic fuzzy norm $(\phi,\psi)_2$ where $(\phi,\psi)_1\neq(\phi,\psi)_2$, then Theorem 22 does not satisfy. Therefore, it is necessary that conditions presented in Theorem 22 as can be seen in the following example.

Example 23. *Let $I=I_\delta$ and let $(X,\|\cdot\|)$ be a normed space where $a*b=ab$ and $a\diamond b=\min\{a+b,1\}$, ϕ_1 and ψ_1 are defined as follows*

$$\phi_1(x,t)=\begin{cases}\dfrac{t}{t+\|x\|}, & if \quad t>0\\[2ex] 0, & t\leq 0\end{cases}$$

$$\psi_2(x,t)=\begin{cases}\dfrac{t}{t+\|x\|}, & if \quad t>0\\ \\ 1, & t\leq 0\end{cases}$$

let $(X,\|\cdot\|)$ be a normed space where $*$ is a continuous t-norms and $\diamond$ is a continuous t-conorms, ϕ_2 and ψ_2 are defined as follows

$$\phi_2(x,t)=\begin{cases}0, & if \quad t\leq \|x\|\\ \\ 1, & if \quad t>\|x\|\end{cases}$$

$$\psi_2(x,t)=\begin{cases}1, & if \quad t\leq \|x\|\\ \\ 0, & if \quad t>\|x\|\end{cases}$$

Then, $(\phi,\psi)_1$ and $(\phi,\psi)_2$ are IFNS on X. For $I=I_\delta$ and $(x_{ijk})\in X$ where

$$x_{ijk}=\begin{cases}p, & if \qquad i=j, i=k\\ \\ 1-p, & if \qquad i-1=j, i-1=k;\ i,j,k\in\mathbb{N}\\ \\ 0, & otherwise\end{cases}$$

$x=(x_{ijk})$ in X is (λ,μ,ζ)-Zweier ideal convergent with respect to the intuitionistic fuzzy norm $(\phi,\psi)_1$ and $x=(x_{ijk})$ in X is (λ,μ,ζ)-Zweier ideal Cauchy sequence with respect to the intuitionistic fuzzy norm $(\phi,\psi)_2$, and $(\phi,\psi)_1\neq(\phi,\psi)_2$.

The following results are showed without proofs due to the proofs are followed directly by the previously results.

Theorem 24. *Let $(X,\phi,\psi,*,\diamond)$ be an IFNS and $x=(x_{ijk})$ be a triple sequence in X is (λ,μ,ζ)-Zweier Cauchy sequence with respect to intuitionistic fuzzy norm (ϕ,ψ), then it is (λ,μ,ζ)-Zweier ideal Cauchy sequence with respect to the same norm.*

Theorem 25. *Let $(X,\phi,\psi,*,\diamond)$ be an IFNS. If triple sequence $x=(x_{ijk})$ in X is (λ,μ,ζ)-Zweier Cauchy sequence with respect to intuitionistic fuzzy norm (ϕ,ψ), then there is a subsequence $(x_{i_q j_w k e})$ of sequence $x=(x_{ijk})$ which is a ordinary Zweier Cauchy sequence with respect to the same norm.*

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