Yugoslav Journal of Operations Research
32 (2022), Number 2, 251–278
DOI: https://doi.org/10.2298/YJOR210617003J

A NOVEL TECHNIQUE FOR SOLVING TWO-PERSON ZERO-SUM MATRIX GAMES IN A ROUGH FUZZY ENVIRONMENT

Vinod JANGID
Department of Mathematics, University of Rajasthan, Jaipur-302004, India
vinodjangid124@gmail.com

Ganesh KUMAR
Department of Mathematics, University of Rajasthan, Jaipur-302004, India
Corresponding author: ganeshmandha1988@gmail.com

Received: June 2021 / Accepted: March 2022

Abstract: This study proposes a novel way to deal with uncertainty in a two-person zero-sum matrix game with payoffs expressed as fuzzy rough numbers. Complete and reasonable solutions to these types of games are obtained. In this research we develop two linear programming models with upper and lower approximation intervals of fuzzy rough numbers and handle multi-objective crisp linear programming models by incorporating trapezoidal fuzzy rough numbers as payoffs. To provide each opponent with the optimal strategy and value of the game, the usual simplex approach is applied. Finally, two numerical examples demonstrate the matrix game outcomes using Wolfram Cloud.

Keywords: Fuzzy Rough Number, Zero-Sum Game, Fuzzy Rough Game, Multi-Objective Model.

MSC: 91A05, 93A30.

1. INTRODUCTION

Game theory is a mathematical concept that can be used to simulate the strategic interactions of two or more players because it combines science, engineering, and logic. It has a wide range of applications, including the analysis of diverse industries, firms, and sectors. The concept of game theory arises with the idea of equilibrium strategies in two-person zero-sum games which was invented by [35]. Afterwards, [33] expanded the concept to cooperative and non-cooperative games.

But in the study of traditional game theory, all the data of game have precise values which are known exactly by the players of the game. However, in recent time not all the data known by the players have the precise values. Fuzzy sets refer the imprecise and ambiguous nature and were introduced in [56], [57]. Further [9], [3] and [4] extended the concept of game theory for modelling the conflict situations with vagueness information. By applying the theory of fuzzy games many researchers [7, 10, 22, 29, 34, 25] contributed to develop mathematical programming, ranking approaches, defuzzification techniques etc. These results played a significant role to handle the fuzzy decision-making problems. In recent years fuzzy games have become an interesting theory to analyze the real-life competitive problems. [58] used FLP approach to solve crisp LPPs with several objective functions. [37] outlined a procedure to solve a majority voting game. Succeeding this fuzzy linear programming approach [11] defined such models to solve the two-person zero-sum fuzzy matrix games. Fuzzy multi-objective matrix games were studied in [46] using max-min solution procedure. [36] analyzed the multi-objective conflict resolution problems of games . [28] characterized the equilibrium strategy for two-person zero-sum fuzzy games. A mathematical approach based on defuzzification technique was described in [5]. Further, the concept of duality in linear programming was used in [51] and [42] applying possibility and necessity relations. In [12] was proposed a new approach to analyze two-person zero-sum matrix games (TPZSMG). Several researchers [27, 45, 47] examined the concept of defuzzification to solve TPZSMG. In [39, 40] were extended the interval-valued fuzzy numbers to rough fuzzy sets. [21] investigated rough fuzzy sets. In [53] new approaches for the study of these sets were used. Using the concept of fuzzy rough sets, interval-valued FMG was proposed in [13]. [54] studied TPZSMG with rough payoffs. Also, an FMG in rough scenario using genetic algorithm was explained in [44]. Later [43] investigated a procedure to solve interval-valued FMG. [38] studied transportation problems with RFNs. Two-person zero sum stochastic linear quadratic differential games with deterministic coefficients were explored in [49], and the required condition for the finiteness of open-loop lower and higher values, as well as the presence of an open-loop saddle point, were derived. A new technique for solving multi-objective linear programming problems in neutrosophic environments was developed in [52] by utilising linear membership function (LMF). Multi-objective linear fractional programming problems under a hesitant fuzzy environment can be solved using a solution approach proposed in [23]. To solve the two-person zero-sum matrix games in a fuzzy environment, [24] used the signed distance ranking approach. In [6] were introduced two novel methods for studying fuzzy linear programming problems with triangular fuzzy numbers as uncertainties, based on fuzzy centre, core, and radius. In [41] alpha cut and goal programming approaches were used to create a new fuzzy multi-period multi-objective portfolio optimization issue in an uncertain environment, taking three practical limitations into account: wealth, risk, and liquidity. In [16] was explored a two-person zero sum matrix game with fuzzy payoffs in credibility space and created an expected value model for it. In [15] a new strategy based on lexicographic order rather than the ranking approach was proposed to solve the neutrosophic linear programming problem with trapezoidal neutrosophic numbers as payoffs for greater decision-making flexibility. Some similarity measures for rough interval Pythagorean fuzzy sets with properties like as Cosine, Jaccard, and Dice and built multi attribute decision making algorithms based on these similarity measures were created in [48]. In [1] was developed a mathematical model based on a fuzzy method for a multicriteria decision making problem using the concept of a two-person zero sum matrix game with asymmetrical triangular distribution. A novel technique for tackling the linear programming problem in the Diet problem in a Pythagorean fuzzy environment to deal with managing permeability and insufficient information was presented in [14]. In [30] was used a two-person zero-sum matrix game with probabilistic language information to answer a multicriteria decision-making problem. For solving the two-person zero-sum matrix game with intuitionistic fuzzy goals and payoffs as symmetric triangular intuitionistic fuzzy numbers in [32] was provided a new method. In [50] were developed three distinct duality models to investigate a class of multi-objective fractional programming problems with non-differentiability in terms of higher-order support functions. Wolfe type second-order multi-objective symmetric programming problems with various duality relations along with relevant duality theorems under (F, G_f)-convexity assumptions were investigated in[19]. Later, [20] investigated a second-order non-differentiable symmetric dual model over arbitrary cone constraints and some relevant duality theorems under strongly K-pseudo-convexity assumptions. Some relevant duality theorems under high order K-convexity and K-pseudo-convexity assumptions and introduced a pair of non-differentiable multi-objective Mond-Weir type higher-order symmetric fractional programming problems over arbitrary cone constraints were proved in [17]. In [18] was investigated a class of new type unified non-differentiable higher-order symmetric duality in scalar objective programming over arbitrary cone constraints under generalized assumptions and derived some appropriate duality theorems over arbitrary cones underη-pseudo-convexity/ η-invexity/ C-pseudo-convexity/ C-convexity. More recently, constrained MG with fuzzy rough payoffs were analyzed in [8]. In [2] was proposed a mathematical model for solving fuzzy integer LPP using triangular FRNs.

The present study includes the following novelties:

- In a two-person zero-sum matrix game with payoffs as rough numbers, a novel technique is established to address perception and ambiguity concerns.
- Complete and relatively reasonable solutions are achieved.
- Two linear programming models with upper and lower approximation intervals of fuzzy rough numbers are solved in form of multi-objective crisp linear programming models.
- Introducing the trapezoidal fuzzy rough numbers as payoffs in games.

- New class of games TPZSFRMG are solved by standard simplex method.

The remaining part of this paper is organized as follows: Section 2, presents definitions and preliminaries. In Section 3, fuzzy rough linear programming models are presented. Section 4, gives the proposed solution procedure and is the main part of the paper. In Section 5, numerical examples are provided to justify the solution procedure. Finally, Section 6 concludes the paper.

2. Definitions and Preliminaries

This section deals with certain fundamental definitions and notions of rough and fuzzy theory to be used in this document.

2.1. Trapezoidal Fuzzy Number(TFN)

A fuzzy number defined on the real line as universal set X denoted by the quadruplet $\tilde{A} = (\xi_l, \underline{\xi}, \overline{\xi}, \xi_r)$ is said to be a TFN if $\mu_{\tilde{A}} : X \to [0,1]$ is defined as

$$\mu_{\tilde{A}}(x) = \begin{cases} \frac{x-\xi_l}{\underline{\xi}-\xi_l} & ; \xi_l \leq x \leq \underline{\xi} \\ 1 & ; \underline{\xi} \leq x \leq \overline{\xi} \\ \frac{\xi_r - x}{\xi_r - \overline{\xi}} & ; \overline{\xi} \leq x \leq \xi_r \\ 0 & ; elsewhere \end{cases}$$

The α-cut of a TFN $\tilde{A} = (\xi_l, \underline{\xi}, \overline{\xi}, \xi_r)$ is a closed crisp interval $\tilde{A}_\alpha = [\xi_l + \alpha(\underline{\xi} - \xi_l), \xi_r - \alpha(\xi_r - \overline{\xi})] \equiv [A^L_\alpha, A^U_\alpha]$ for $\alpha \in [0,1]$.

2.2. Fuzzy Rough Number

A fuzzy rough number is denoted by $\tilde{A}^R = [\tilde{A}^L, \tilde{A}^U]$, where $\mu_{\tilde{A}^L} : X \to R$ and $\mu_{\tilde{A}^U} : X \to R$ are piecewise continuous functions and satisfy the condition that $\mu_{\tilde{A}^L}(x) \leq \mu_{\tilde{A}^U}(x) \; \forall x \in X$.
The α-cut of FRN $\tilde{A}^R$ is $\tilde{A}^R_\alpha = [({A_\alpha}^{LL}, {A_\alpha}^{UL}) : ({A_\alpha}^{LU}, {A_\alpha}^{UU})]$ with $({A_\alpha}^{LL}, {A_\alpha}^{UL}) \subseteq ({A_\alpha}^{LU}, {A_\alpha}^{UU})$.

2.3. Trapezoidal Fuzzy Rough Number(TFRN)

A fuzzy rough number $\tilde{A}^R$ is said to be a TFRN denoted by $\tilde{A}^R = [\tilde{A}^L, \tilde{A}^U] \equiv [({\xi_l}^{LL}, \underline{\xi}^M, \overline{\xi}^M, {\xi_r}^{UL}) : ({\xi_l}^{LU}, \underline{\xi}^M, \overline{\xi}^M, {\xi_r}^{UU})]$ such that the real values ${\xi_l}^{LU}$, ${\xi_l}^{LL}$, $\underline{\xi}^M$, $\overline{\xi}^M$, ${\xi_r}^{UL}$, ${\xi_r}^{UU}$ satisfies the condition ${\xi_l}^{LU} \leq {\xi_l}^{LL} \leq \underline{\xi}^M \leq \overline{\xi}^M \leq {\xi_r}^{UL} \leq {\xi_r}^{UU}$ and

$$\mu_{\tilde{A}^R}(x)=\begin{cases}\mu_{\tilde{A}^L}(x)=\begin{cases}\frac{x-{\xi_l}^{LL}}{\underline{\xi}^M-{\xi_l}^{LL}} & ;{\xi_l}^{LL}\le x\le \underline{\xi}^M\\ 1 & ;\underline{\xi}^M\le x\le \overline{\xi}^M\\ \frac{{\xi_r}^{UL}-x}{{\xi_r}^{UL}-\overline{\xi}^M} & ;\overline{\xi}^M\le x\le {\xi_r}^{UL}\\ 0 & ;elsewhere\end{cases}\\ \mu_{\tilde{A}^U}(x)=\begin{cases}\frac{x-{\xi_l}^{LU}}{\underline{\xi}^M-{\xi_l}^{LU}} & ;{\xi_l}^{LU}\le x\le \underline{\xi}^M\\ 1 & ;\underline{\xi}^M\le x\le \overline{\xi}^M\\ \frac{{\xi_r}^{UU}-x}{{\xi_r}^{UU}-\overline{\xi}^M} & ;\overline{\xi}^M\le x\le {\xi_r}^{UU}\\ 0 & ;elsewhere\end{cases}\end{cases}$$

where $\tilde{A}^L=({\xi_l}^{LL},\underline{\xi}^M,\overline{\xi}^M,{\xi_r}^{UL})$ and $\tilde{A}^U=({\xi_l}^{LU},\underline{\xi}^M,\overline{\xi}^M,{\xi_r}^{UU})$ are lower and upper TFNs respectively.

2.4. Arithmetic Operations of TFRNs

Let $\tilde{A}^R=[({\xi_l}^{LL},\underline{\xi}^M,\overline{\xi}^M,{\xi_r}^{UL}):({\xi_l}^{LU},\underline{\xi}^M,\overline{\xi}^M,{\xi_r}^{UU})]$ and $\tilde{B}^R=[({\eta_l}^{LL},\underline{\eta}^M,\overline{\eta}^M,{\eta_r}^{UL}):({\eta_l}^{LU},\underline{\eta}^M,\overline{\eta}^M,{\eta_r}^{UU})]$ be two TFRNs such that $\tilde{A}^R,\tilde{B}^R\ge 0$ then some relevant arithmetic operations on TFRNs are defined as

(i) Addition:

$\tilde{A}^R(+)\tilde{B}^R=[({\xi_l}^{LL}+{\eta_l}^{LL},\underline{\xi}^M+\underline{\eta}^M,\overline{\xi}^M+\overline{\eta}^M,{\xi_r}^{UL}+{\eta_r}^{UL}):({\xi_l}^{LU}+{\eta_l}^{LU},\underline{\xi}^M+\underline{\eta}^M,\overline{\xi}^M+\overline{\eta}^M,{\xi_r}^{UU}+{\eta_r}^{UU})]$

(ii) Symmetric Image:

$-\tilde{A}^R=[(-{\xi_r}^{uL},-\overline{\xi}^M,-\underline{\xi}^M,-{\xi_l}^{LL}):(-{\xi_r}^{UU},-\overline{\xi}^M,-\underline{\xi}^M,-{\xi_l}^{LU})]$

(iii) Subtraction:

$\tilde{A}^R(-)\tilde{B}^R=[({\xi_l}^{LL}-{\eta_r}^{UL},\underline{\xi}^M-\overline{\eta}^M,\overline{\xi}^M-\underline{\eta}^M,{\xi_r}^{UL}-{\eta_l}^{LL}):({\xi_l}^{LU}-{\eta_r}^{UU},\underline{\xi}^M-\overline{\eta}^M,\overline{\xi}^M-\underline{\eta}^M,{\xi_r}^{UU}-{\eta_l}^{LU})]$

(iv) Multiplication:

$\tilde{A}^R(*)\tilde{B}^R=[({\xi_l}^{LL}\cdot{\eta_l}^{LL},\underline{\xi}^M\cdot\underline{\eta}^M,\overline{\xi}^M\cdot\overline{\eta}^M,{\xi_r}^{UL}\cdot{\eta_r}^{UL}):({\xi_l}^{LU}\cdot{\eta_l}^{LU},\underline{\xi}^M\cdot\underline{\eta}^M,\overline{\xi}^M\cdot\overline{\eta}^M,{\xi_r}^{UU}\cdot{\eta_r}^{UU})]$

(v) Division:

$\frac{\tilde{A}^R}{\tilde{B}^R}=[(\frac{{\xi_l}^{LL}}{{\eta_r}^{UL}},\frac{\underline{\xi}^M}{\overline{\eta}^M},\frac{\overline{\xi}^M}{\underline{\eta}^M},\frac{{\xi_r}^{UL}}{{\eta_l}^{LL}}):(\frac{{\xi_l}^{LU}}{{\eta_r}^{UU}},\frac{\underline{\xi}^M}{\overline{\eta}^M},\frac{\overline{\xi}^M}{\underline{\eta}^M},\frac{{\xi_r}^{UU}}{{\eta_l}^{LU}})]$

3. Fuzzy Rough Linear Programming Models for Matrix Games

Let a fuzzy rough payoff matrix be given as $[\tilde{a}^R_{ij}]_{m\times n}$ having the mixed strategies $\eta=(\tilde{\beta}^R_1,\tilde{\beta}^R_2,\ldots\tilde{\beta}^R_m)^T$ and $\gamma=(\tilde{\gamma}^R_1,\tilde{\gamma}^R_2,\ldots\tilde{\gamma}^R_n)^T$ respectively. Then according to maximin and minimax principle player I choose such a strategy that maximizes his minimum expected gain i.e.

$$\begin{cases} \max_{\tilde{\beta}_i^R} \left[\min\{\sum_{i=1}^{m} \tilde{a}_{i1}^R \tilde{\beta}_i^R, \sum_{i=1}^{m} \tilde{a}_{i2}^R \tilde{\beta}_i^R, \ldots \sum_{i=1}^{m} \tilde{a}_{in}^R \tilde{\beta}_i^R\} \right] \\ \text{such that} \quad \tilde{\beta}_1^R + \tilde{\beta}_2^R + \ldots + \tilde{\beta}_m^R \approx \tilde{1}^R \\ \text{and} \quad \tilde{\beta}_i^R \geq 0; \quad \forall \quad i = 1, 2, \ldots m \end{cases} \tag{1}$$

Now if $\min\{\sum_{i=1}^{m} \tilde{a}_{i1}^R \tilde{\beta}_i^R, \sum_{i=1}^{m} \tilde{a}_{i2}^R \tilde{\beta}_i^R, \ldots \sum_{i=1}^{m} \tilde{a}_{in}^R \tilde{\beta}_i^R\} \approx \tilde{u}^R$ is the expected minimum gain for player I, then the problem (1) can be written as

$$\begin{cases} \max \quad \tilde{u}^R \\ \text{s.t.} \quad \sum_{i=1}^{m} \tilde{a}_{i1}^R \tilde{\beta}_i^R \succeq \tilde{u}^R \\ \sum_{i=1}^{m} \tilde{a}_{i2}^R \tilde{\beta}_i^R \succeq \tilde{u}^R \\ \cdots \\ \sum_{i=1}^{m} \tilde{a}_{in}^R \tilde{\beta}_i^R \succeq \tilde{u}^R \\ \tilde{\beta}_1^R + \tilde{\beta}_2^R + \ldots + \tilde{\beta}_m^R \approx \tilde{1}^R \\ \text{and} \quad \tilde{\beta}_i^R, \tilde{u}^R \geq 0; \quad \forall \quad i = 1, 2, \ldots m \end{cases} \tag{2}$$

Similarly the player II chooses that strategy which minimizes his maximum expected loss i.e.

$$\begin{cases} \min_{\tilde{\gamma}_j^R} \left[\max\{\sum_{j=1}^{n} \tilde{a}_{1j}^R \tilde{\gamma}_j^R, \sum_{j=1}^{n} \tilde{a}_{2j}^R \tilde{\gamma}_j^R, \ldots \sum_{j=1}^{n} \tilde{a}_{mj}^R \tilde{\gamma}_j^R\} \right] \\ \text{such that} \quad \tilde{\gamma}_1^R + \tilde{\gamma}_2^R + \ldots + \tilde{\gamma}_n^R \approx \tilde{1}^R \\ \text{and} \quad \tilde{\gamma}_j^R \geq 0; \quad \forall \quad j = 1, 2, \ldots n \end{cases} \tag{3}$$

Also if $\max\{\sum_{j=1}^{n}\tilde{a}_{1j}^R\tilde{\gamma}_j^R, \sum_{j=1}^{n}\tilde{a}_{2j}^R\tilde{\gamma}_j^R, \ldots \sum_{j=1}^{n}\tilde{a}_{mj}^R\tilde{\gamma}_j^R\} \approx \tilde{v}^R$ is the expected maximum loss for player II, then the problem (3) can be written as

$$\begin{cases} \min \quad \tilde{v}^R \\ \text{s.t.} \quad \displaystyle\sum_{j=1}^{n}\tilde{a}_{1j}^R\tilde{\gamma}_j^R \preceq \tilde{v}^R \\ \displaystyle\sum_{j=1}^{n}\tilde{a}_{2j}^R\tilde{\gamma}_j^R \preceq \tilde{v}^R \\ \cdots \\ \displaystyle\sum_{j=1}^{n}\tilde{a}_{mj}^R\tilde{\gamma}_j^R \preceq \tilde{v}^R \\ \tilde{\gamma}_1^R + \tilde{\gamma}_2^R + \ldots + \tilde{\gamma}_n^R \approx \tilde{1}^R \\ \text{and} \quad \tilde{\gamma}_j^R, \tilde{v}^R \geq 0; \quad \forall \quad j = 1, 2, \ldots n \end{cases} \tag{4}$$

4. Proposed Procedure

To solve the FRLP models of matrix game as explained above by the problems (2),(4) we apply the following algorithm as.

Step 1 Using the principle of indifference when both the players adopt their optimal strategies of expected extreme payoffs as game value, irrespective of what the other player chooses from any row or column. The fuzzy rough linear programming problems (2) and (4) become

$$\begin{cases} \max \quad \tilde{u}^R \\ \text{subject to the double fuzzy constraints} \\ (\tilde{\beta}^R)^T\tilde{A}^R\tilde{\gamma}^R \succeq_{\tilde{p}^R} \tilde{u}^R \quad \forall \quad \tilde{\gamma}^R \in \gamma \\ e^T\tilde{\beta}^R \approx \tilde{1}^R \\ \text{and} \quad \tilde{\beta}^R, \tilde{u}^R \geq 0 \\ \text{where} \quad e^T = (1, 1, \ldots 1)\text{m times.} \end{cases} \tag{5}$$

And

$$\begin{cases} \min \quad \tilde{v}^R \\ \text{subject to the double fuzzy constraints} \\ (\tilde{\beta}^R)^T\tilde{A}^R\tilde{\gamma}^R \preceq_{\tilde{q}^R} \tilde{v}^R \quad \forall \quad \tilde{\beta}^R \in \beta \\ e^T\tilde{\gamma}^R \approx \tilde{1}^R \\ \text{and} \quad \tilde{\gamma}^R, \tilde{v}^R \geq 0 \\ \text{where} \quad e^T = (1, 1, \ldots 1)\text{n times.} \end{cases} \tag{6}$$

Here FR values $\tilde{u}^R, \tilde{v}^R \in N(R)$ (set of all fuzzy rough numbers) and $\tilde{p}^R, \tilde{q}^R$ are FR adequacies corresponding to players.

Step 2 Consider the extreme row or column strategy from the fuzzy rough strategy sets β and γ to extract the double fuzzy constraints of problems (5) and (6) for the player I and II respectively.

$$\begin{cases} \max \quad \tilde{u}^R \\ \text{subject to the double fuzzy constraints} \\ (\tilde{\beta}^R)^T \tilde{A}_j^R \succeq_{\tilde{p}^R} \tilde{u}^R \quad \forall \;\; j = 1, 2, \ldots n \\ e^T \tilde{\beta}^R \approx \tilde{1}^R \\ \text{and} \quad \tilde{\beta}^R, \tilde{u}^R \geq 0 \\ \text{where} \quad e^T = (1, 1, \ldots 1)\text{m times.} \end{cases} \tag{7}$$

And

$$\begin{cases} \min \quad \tilde{v}^R \\ \text{subject to the double fuzzy constraints} \\ \tilde{A}_i^R \tilde{\gamma}^R \preceq_{\tilde{q}^R} \tilde{v}^R \quad \forall \;\; i = 1, 2, \ldots m \\ e^T \tilde{\gamma}^R \approx \tilde{1}^R \\ \text{and} \quad \tilde{\gamma}^R, \tilde{v}^R \geq 0 \\ \text{where} \quad e^T = (1, 1, \ldots 1)\text{n times.} \end{cases} \tag{8}$$

Where the symbols $\tilde{A}_i^R$ and $\tilde{A}_j^R$ stands for the i^{th} row and j^{th} column of FR payoff matrix $[\tilde{a}_{ij}^R]_{m \times n}$ for $i = 1, 2, \ldots m$ and $j = 1, 2, \ldots n$.

Step 3 Express the double fuzzy constraints as fuzzy inequalities of the problems (7) and (8) to the linear combination of expected gain or loss of the players I and II respectively and the fuzzy rough adequacies by using the Yager′s resolution method for the parameters $\lambda, \mu \in [0, 1]$. Then the improved fuzzy rough linear programming problem takes the form as

$$\begin{cases} \max \quad \tilde{u}^R \\ \text{subject to the constraints} \\ \displaystyle\sum_{i=1}^{m} \tilde{a}_{ij}^R \tilde{\beta}_i^R \geq \tilde{u}^R - \tilde{p}^R(1 - \lambda) \quad \forall \;\; j = 1, 2, \ldots, n \\ \displaystyle\sum_{i=1}^{m} \tilde{\beta}_i^R = \tilde{1}^R \\ \lambda \leq 1 \\ \text{and} \quad \tilde{\beta}_i^R, \tilde{u}^R, \lambda \geq 0 \;\; \forall \;\; i = 1, 2, \ldots m. \end{cases} \tag{9}$$

And

$$\begin{cases} \min \quad \tilde{v}^R \\ \text{subject to the constraints} \\ \sum_{j=1}^{n} \tilde{a}_{ij}^R \tilde{\gamma}_j^R \leq \tilde{v}^R + \tilde{q}^R(1-\mu) \quad \forall \ \ i = 1,2,\ldots,m \\ \sum_{j=1}^{n} \tilde{\gamma}_j^R = \tilde{1}^R \\ \mu \leq 1 \\ \text{and} \quad \tilde{\gamma}_j^R, \tilde{v}^R, \mu \geq 0 \ \ \forall \ \ j = 1,2,\ldots n. \end{cases} \tag{10}$$

Step 4 Determine upper and lower problems $(FLPP-I)^{UAI}$ and $(FLPP-I)^{LAI}$ from the problem (9) for the player I using upper approximation interval (UAI) and lower approximation interval (LAI) of the TFRNs, which provides solutions to $(FLPP-I)^{UAI}$ and $(FLPP-I)^{LAI}$ as
(FLPP-I)^{UAI}

$$\begin{cases} \max \quad \left({u_l}^{LU}, \underline{u}^M, \overline{u}^M, {u_r}^{UU}\right) \\ \text{subject to the constraints} \\ \sum_{i=1}^{m} \left({(a_{ij})_l}^{LU}, \underline{a_{ij}}^M, \overline{a_{ij}}^M, {(a_{ij})_r}^{UU}\right) * \left({(\beta_i)_l}^{LU}, \underline{\beta_i}^M, \overline{\beta_i}^M, {(\beta_i)_r}^{UU}\right) \geq \\ \left({u_l}^{LU}, \underline{u}^M, \overline{u}^M, {u_r}^{UU}\right) - \left({p_l}^{LU}, \underline{p}^M, \overline{p}^M, {p_r}^{UU}\right)(1-\lambda) \ \forall \ j = 1,2,\ldots,n \\ \sum_{i=1}^{m} \left({(\beta_i)_l}^{LU}, \underline{\beta_i}^M, \overline{\beta_i}^M, {(\beta_i)_r}^{UU}\right) = \tilde{1}^U \\ \lambda \leq 1 \\ \text{and } {(\beta_i)_l}^{LU}, \underline{\beta_i}^M, \overline{\beta_i}^M, {(\beta_i)_r}^{UU}, \lambda, {u_l}^{LU}, \underline{u}^M, \overline{u}^M, {u_r}^{UU} \geq 0 \ \ \forall \ i = 1,2,\ldots,m \\ \text{where } \tilde{1}^U = \left({1_l}^{LU}, \underline{1}^M, \overline{1}^M, {1_r}^{UU}\right) \end{cases} \tag{11}$$

And (FLPP-I)^{LAI}

$$\begin{cases} \max \quad \left(u_l{}^{LL}, \underline{u}^M, \overline{u}^M, u_r{}^{UL}\right) \\ \text{subject to the constraints} \\ \sum_{i=1}^{m} \left((a_{ij})_l{}^{LL}, \underline{a_{ij}}^M, \overline{a_{ij}}^M, (a_{ij})_r{}^{UL}\right) * \left((\beta_i)_l{}^{LL}, \underline{\beta_i}^M, \overline{\beta_i}^M, (\beta_i)_r{}^{UL}\right) \geq \\ \left(u_l{}^{LL}, \underline{u}^M, \overline{u}^M, u_r{}^{UL}\right) - \left(p_l{}^{LL}, \underline{p}^M, \overline{p}^M, p_r{}^{UL}\right)(1-\lambda) \ \forall\ j = 1, 2, \ldots, n \\ \sum_{i=1}^{m} \left((\beta_i)_l{}^{LL}, \underline{\beta_i}^M, \overline{\beta_i}^M, (\beta_i)_r{}^{UL}\right) = \tilde{1}^L \\ \lambda \leq 1 \\ \text{and } (\beta_i)_l{}^{LL}, \underline{\beta_i}^M, \overline{\beta_i}^M, (\beta_i)_r{}^{UL}, \lambda, u_l{}^{LL}, \underline{u}^M, \overline{u}^M, u_r{}^{UL} \geq 0 \quad \forall\ i = 1, 2, \ldots, m \\ \text{where } \tilde{1}^L = \left(1_l{}^{LL}, \underline{1}^M, \overline{1}^M, 1_r{}^{UL}\right) \end{cases} \tag{12}$$

Step 5 Also determine upper and lower fuzzy linear programming problems $(FLPP-II)^{UAI}$ and $(FLPP-II)^{LAI}$ from the problem (10) for the player II as explained in step 4 to provide rather and complete satisfactory solutions respectively.
(FLPP-II)^{UAI}

$$\begin{cases} \min \quad \left(v_l{}^{LU}, \underline{v}^M, \overline{v}^M, v_r{}^{UU}\right) \\ \text{subject to the constraints} \\ \sum_{j=1}^{n} \left((a_{ij})_l{}^{LU}, \underline{a_{ij}}^M, \overline{a_{ij}}^M, (a_{ij})_r{}^{UU}\right) * \left((\gamma_j)_l{}^{LU}, \underline{\gamma_j}^M, \overline{\gamma_j}^M, (\gamma_j)_r{}^{UU}\right) \leq \\ \left(v_l{}^{LU}, \underline{v}^M, \overline{v}^M, v_r{}^{UU}\right) + \left(q_l{}^{LU}, \underline{q}^M, \overline{q}^M, q_r{}^{UU}\right)(1-\mu) \ \forall\ i = 1, 2, \ldots, m \\ \sum_{j=1}^{n} \left((\gamma_j)_l{}^{LU}, \underline{\gamma_j}^M, \overline{\gamma_j}^M, (\gamma_j)_r{}^{UU}\right) = \tilde{1}^U \\ \mu \leq 1 \\ \text{and } (\gamma_j)_l{}^{LU}, \underline{\gamma_j}^M, \overline{\gamma_j}^M, (\gamma_j)_r{}^{UU}, \mu, v_l{}^{LU}, \underline{v}^M, \overline{v}^M, v_r{}^{UU} \geq 0 \quad \forall\ j = 1, 2, \ldots, n \\ \text{where } \tilde{1}^U = \left(1_l{}^{LU}, \underline{1}^M, \overline{1}^M, 1_r{}^{UU}\right) \end{cases} \tag{13}$$

And $(\text{FLPP-II})^{LAI}$

$$\begin{cases} \min \quad \left(v_l{}^{LL}, \underline{v}^M, \overline{v}^M, v_r{}^{UL}\right) \\ \text{subject to the constraints} \\ \sum_{j=1}^{n} \left((a_{ij})_l{}^{LL}, \underline{a_{ij}}^M, \overline{a_{ij}}^M, (a_{ij})_r{}^{UL}\right) * \left((\gamma_j)_l{}^{LL}, \underline{\gamma_j}^M, \overline{\gamma_j}^M, (\gamma_j)_r{}^{UL}\right) \leq \\ \left(v_l{}^{LL}, \underline{v}^M, \overline{v}^M, v_r{}^{UL}\right) + \left(q_l{}^{LL}, \underline{q}^M, \overline{q}^M, q_r{}^{UL}\right)(1-\mu) \ \forall\ i = 1, 2, \ldots, m \\ \sum_{j=1}^{n} \left((\gamma_j)_l{}^{LL}, \underline{\gamma_j}^M, \overline{\gamma_j}^M, (\gamma_j)_r{}^{UL}\right) = \tilde{1}^L \\ \mu \leq 1 \\ \text{and } (\gamma_j)_l{}^{LL}, \underline{\gamma_j}^M, \overline{\gamma_j}^M, (\gamma_j)_r{}^{UL}, \mu, v_l{}^{LL}, \underline{v}^M, \overline{v}^M, v_r{}^{UL} \geq 0 \ \ \forall\ j = 1, 2, \ldots, n \\ \text{where } \tilde{1}^L = \left(1_l{}^{LL}, \underline{1}^M, \overline{1}^M, 1_r{}^{UL}\right) \end{cases} \tag{14}$$

Step 6 Disunite the fuzzy linear programming problems (11) and (12) into six sub CLPPs namely lower-upper (LU), lower-lower (LL), lower-medium (LM), upper-medium (UM), upper-lower (UL) and upper-upper (UU) respectively as $(\text{CLPP-I})^{LU}$

$$\begin{cases} \max \quad u_l{}^{LU} \\ \text{subject to the constraints} \\ \sum_{i=1}^{m} (a_{ij})_l{}^{LU} (\beta_i)_l{}^{LU} \geq u_l{}^{LU} - p_r{}^{UU}(1-\lambda) \ \forall\ j = 1, 2, \ldots, n \\ \sum_{i=1}^{m} (\beta_i)_l{}^{LU} = 1_l{}^{LU} \\ \lambda \leq 1 \\ \text{and } (\beta_i)_l{}^{LU}, \lambda, u_l{}^{LU} \geq 0 \ \ \forall\ i = 1, 2, \ldots, m \end{cases} \tag{15}$$

$(\text{CLPP-I})^{LL}$

$$\begin{cases} \max \quad {u_l}^{LL} \\ \text{subject to the constraints} \\ \sum_{i=1}^{m} {(a_{ij})_l}^{LL} {(\beta_i)_l}^{LL} \geq {u_l}^{LL} - {p_r}^{UL}(1-\lambda) \ \forall \ j = 1, 2, \ldots, n \\ \sum_{i=1}^{m} {(\beta_i)_l}^{LL} = {1_l}^{LL} \\ \lambda \leq 1 \\ \text{and } {(\beta_i)_l}^{LL}, \lambda, {u_l}^{LL} \geq 0 \ \ \forall \ i = 1, 2, \ldots, m \end{cases} \tag{16}$$

$(\text{CLPP-I})^{LM}$

$$\begin{cases} \max \quad \underline{u}^{M} \\ \text{subject to the constraints} \\ \sum_{i=1}^{m} \underline{a_{ij}}^{M} \underline{\beta_i}^{M} \geq \underline{u}^{M} - \overline{p}^{M}(1-\lambda) \ \forall \ j = 1, 2, \ldots, n \\ \sum_{i=1}^{m} \underline{\beta_i}^{M} = \underline{1}^{M} \\ \lambda \leq 1 \\ \text{and } \underline{\beta_i}^{M}, \lambda, \underline{u}^{M} \geq 0 \ \ \forall \ i = 1, 2, \ldots, m \end{cases} \tag{17}$$

$(\text{CLPP-I})^{UM}$

$$\begin{cases} \max \quad \overline{u}^{M} \\ \text{subject to the constraints} \\ \sum_{i=1}^{m} \overline{a_{ij}}^{M} \overline{\beta_i}^{M} \geq \overline{u}^{M} - \underline{p}^{M}(1-\lambda) \ \forall \ j = 1, 2, \ldots, n \\ \sum_{i=1}^{m} \overline{\beta_i}^{M} = \overline{1}^{M} \\ \lambda \leq 1 \\ \text{and } \overline{\beta_i}^{M}, \lambda, \overline{u}^{M} \geq 0 \ \ \forall \ i = 1, 2, \ldots, m \end{cases} \tag{18}$$

$(\text{CLPP-I})^{UL}$

$$\begin{cases} \max \quad {u_r}^{UL} \\ \text{subject to the constraints} \\ \sum_{i=1}^{m} {(a_{ij})_r}^{UL} {(\beta_i)_r}^{UL} \geq {u_r}^{UL} - {p_l}^{LL}(1-\lambda) \ \forall \ j = 1, 2, \ldots, n \\ \sum_{i=1}^{m} {(\beta_i)_r}^{UL} = {1_r}^{UL} \\ \lambda \leq 1 \\ \text{and } {(\beta_i)_r}^{UL}, \lambda, {u_r}^{UL} \geq 0 \ \ \forall \ i = 1, 2, \ldots, m \end{cases} \tag{19}$$

$(\text{CLPP-I})^{UU}$

$$\begin{cases} \max \quad {u_r}^{UU} \\ \text{subject to the constraints} \\ \sum_{i=1}^{m} {(a_{ij})_r}^{UU} {(\beta_i)_r}^{UU} \geq {u_r}^{UU} - {p_l}^{LU}(1-\lambda) \ \forall \ j = 1, 2, \ldots, n \\ \sum_{i=1}^{m} {(\beta_i)_r}^{UU} = {1_r}^{UU} \\ \lambda \leq 1 \\ \text{and } {(\beta_i)_r}^{UU}, \lambda, {u_r}^{UU} \geq 0 \ \ \forall \ i = 1, 2, \ldots, m \end{cases} \tag{20}$$

Step 7 Also disunite the fuzzy linear programming problems (13) and (14) into six sub CLPPs namely according as explained in step 6.

$(\text{CLPP-II})^{LU}$

$$\begin{cases} \min \quad {v_l}^{LU} \\ \text{subject to the constraints} \\ \sum_{j=1}^{n} {(a_{ij})_l}^{LU} {(\gamma_j)_l}^{LU} \leq {v_l}^{LU} + {q_l}^{LU}(1-\mu) \ \forall \ i = 1, 2, \ldots, m \\ \sum_{j=1}^{n} {(\gamma_j)_l}^{LU} = {1_l}^{LU} \\ \mu \leq 1 \\ \text{and } {(\gamma_j)_l}^{LU}, \mu, {v_l}^{LU} \geq 0 \ \ \forall \ j = 1, 2, \ldots, n \end{cases} \tag{21}$$

(CLPP-II)^{LL}

$$\begin{cases} \min \quad {v_l}^{LL} \\ \text{subject to the constraints} \\ \sum_{j=1}^{n} (a_{ij})_l{}^{LL}(\gamma_j)_l{}^{LL} \leq {v_l}^{LL} + {q_l}^{LL}(1-\mu) \ \forall \ i = 1,2,\ldots,m \\ \sum_{j=1}^{n} (\gamma_j)_l{}^{LL} = {1_l}^{LL} \\ \mu \leq 1 \\ \text{and } (\gamma_j)_l{}^{LL}, \mu, {v_l}^{LL} \geq 0 \ \ \forall \ j = 1,2,\ldots,n \end{cases} \tag{22}$$

(CLPP-II)^{LM}

$$\begin{cases} \min \quad \underline{v}^{M} \\ \text{subject to the constraints} \\ \sum_{j=1}^{n} \underline{a_{ij}}^{M}\underline{\gamma_j}^{M} \leq \underline{v}^{M} + \underline{q}^{M}(1-\mu) \ \forall \ i = 1,2,\ldots,m \\ \sum_{j=1}^{n} \underline{\gamma_j}^{M} = \underline{1}^{M} \\ \mu \leq 1 \\ \text{and } \underline{\gamma_j}^{M}, \mu, \underline{v}^{M} \geq 0 \ \ \forall \ j = 1,2,\ldots,n \end{cases} \tag{23}$$

(CLPP-II)^{UM}

$$\begin{cases} \min \quad \overline{v}^{M} \\ \text{subject to the constraints} \\ \sum_{j=1}^{n} \overline{a_{ij}}^{M}\overline{\gamma_j}^{M} \leq \overline{v}^{M} + \overline{q}^{M}(1-\mu) \ \forall \ i = 1,2,\ldots,m \\ \sum_{j=1}^{n} \overline{\gamma_j}^{M} = \overline{1}^{M} \\ \mu \leq 1 \\ \text{and } \overline{\gamma_j}^{M}, \mu, \overline{v}^{M} \geq 0 \ \ \forall \ j = 1,2,\ldots,n \end{cases} \tag{24}$$

$(\text{CLPP-II})^{UL}$

$$\begin{cases} \min \quad {v_r}^{UL} \\ \text{subject to the constraints} \\ \sum_{j=1}^{n} (a_{ij})_r{}^{UL} (\gamma_j)_r{}^{UL} \leq {v_r}^{UL} + {q_r}^{UL}(1-\mu) \; \forall \; i = 1, 2, \ldots, m \\ \sum_{j=1}^{n} (\gamma_j)_r{}^{UL} = {1_r}^{UL} \\ \mu \leq 1 \\ \text{and } (\gamma_j)_r{}^{UL}, \mu, {v_r}^{UL} \geq 0 \;\; \forall \; j = 1, 2, \ldots, n \end{cases} \tag{25}$$

$(\text{CLPP-II})^{UU}$

$$\begin{cases} \min \quad {v_r}^{UU} \\ \text{subject to the constraints} \\ \sum_{j=1}^{n} (a_{ij})_r{}^{UU} (\gamma_j)_r{}^{UU} \leq {v_r}^{UU} + {q_r}^{UU}(1-\mu) \; \forall \; i = 1, 2, \ldots, m \\ \sum_{j=1}^{n} (\gamma_j)_r{}^{UU} = {1_r}^{UU} \\ \mu \leq 1 \\ \text{and } (\gamma_j)_r{}^{UU}, \mu, {v_r}^{UU} \geq 0 \;\; \forall \; j = 1, 2, \ldots, n \end{cases} \tag{26}$$

Step 8 Solve the obtained CLPPs as explained in steps 6 and 7 by the standard simplex method for the players.
Step 9 The obtained optimal solution of CLPPs in step 8 provides the complete and satisfactory solutions for players. We also get the best strategies and game value for both players.

5. Numerical Examples

5.1. Example

Taking a zero sum FMG $\tilde{A}^R = \left[\tilde{a}_{ij}^R\right]_{m\times n}$ with two players whose payoffs are TFRNs is given as follows

$$\tilde{A}^R = \begin{bmatrix} \tilde{a}_{11}^R & \tilde{a}_{12}^R \\ \tilde{a}_{21}^R & \tilde{a}_{22}^R \end{bmatrix}$$

Where $\tilde{a}_{11}^R = [(175, 180, 185, 190) : (170, 180, 185, 195)]$;
$\tilde{a}_{12}^R = [(150, 155, 157, 158) : (145, 155, 157, 160)]$;

$\tilde{a}_{21}^R = [(80, 87, 92, 100) : (75, 87, 92, 105)]$;
$\tilde{a}_{22}^R = [(175, 180, 185, 190) : (170, 180, 185, 195)]$; Assuming that the margins for both the players are $\tilde{p}^R$ and $\tilde{q}^R$, which are given as
$\tilde{p}^R = [(0.06, 0.08, 0.10, 0.12) : (0.04, 0.08, 0.10, 0.14)]$;
$\tilde{q}^R = [(0.10, 0.12, 0.16, 0.18) : (0.07, 0.12, 0.16, 0.19)]$;
Having $\tilde{1}^R = [(0.5, 0.8, 1.0, 1.2) : (0.3, 0.8, 1.0, 1.5)]$, then using equations (11)-(14) as explained in steps 4 and 5 and equations (15)-(26) as explained in steps 6 and 7, the given TPZSMG with TFRNs as payoffs can be split into CLPPs for both the players as follows

$(\text{CLPP-I})^{LU}$

$$\begin{cases} \max \quad {u_l}^{LU} \\ \text{subject to the constraints} \\ 170{(\beta_1)_l}^{LU} + 75{(\beta_2)_l}^{LU} \geq {u_l}^{LU} - 0.14(1-\lambda) \\ 145{(\beta_1)_l}^{LU} + 170{(\beta_2)_l}^{LU} \geq {u_l}^{LU} - 0.14(1-\lambda) \\ {(\beta_1)_l}^{LU} + {(\beta_2)_l}^{LU} = 0.3 \\ \lambda \leq 1 \\ \text{and } {(\beta_1)_l}^{LU}, {(\beta_2)_l}^{LU}, \lambda, {u_l}^{LU} \geq 0 \end{cases} \tag{27}$$

$(\text{CLPP-I})^{LL}$

$$\begin{cases} \max \quad {u_l}^{LL} \\ \text{subject to the constraints} \\ 175{(\beta_1)_l}^{LL} + 80{(\beta_2)_l}^{LL} \geq {u_l}^{LL} - 0.12(1-\lambda) \\ 150{(\beta_1)_l}^{LL} + 175{(\beta_2)_l}^{LL} \geq {u_l}^{LL} - 0.12(1-\lambda) \\ {(\beta_1)_l}^{LL} + {(\beta_2)_l}^{LL} = 0.5 \\ \lambda \leq 1 \\ \text{and } {(\beta_1)_l}^{LL}, {(\beta_2)_l}^{LL}, \lambda, {u_l}^{LL} \geq 0 \end{cases} \tag{28}$$

$(\text{CLPP-I})^{LM}$

$$\begin{cases} \max \quad \underline{u}^M \\ \text{subject to the constraints} \\ 180\underline{\beta_1}^M + 87\underline{\beta_2}^M \geq \underline{u}^M - 0.10(1-\lambda) \\ 155\underline{\beta_1}^M + 180\underline{\beta_2}^M \geq \underline{u}^M - 0.10(1-\lambda) \\ \underline{\beta_1}^M + \underline{\beta_2}^M = 0.8 \\ \lambda \leq 1 \\ \text{and } \underline{\beta_1}^M, \underline{\beta_2}^M \lambda, \underline{u}^M \geq 0 \end{cases} \tag{29}$$

$(\text{CLPP-I})^{UM}$

$$\begin{cases} \max \quad \overline{u}^M \\ \text{subject to the constraints} \\ 185\overline{\beta_1}^M + 92\overline{\beta_2}^M \geq \overline{u}^M - 0.08(1-\lambda) \\ 157\overline{\beta_1}^M + 185\overline{\beta_2}^M \geq \overline{u}^M - 0.08(1-\lambda) \\ \overline{\beta_1}^M + \overline{\beta_2}^M = 1 \\ \lambda \leq 1 \\ \text{and } \overline{\beta_1}^M, \overline{\beta_2}^M \lambda, \overline{u}^M \geq 0 \end{cases} \tag{30}$$

$(\text{CLPP-I})^{UL}$

$$\begin{cases} \max \quad {u_r}^{UL} \\ \text{subject to the constraints} \\ 190{(\beta_1)_r}^{UL} + 100{(\beta_2)_r}^{UL} \geq {u_r}^{UL} - 0.06(1-\lambda) \\ 158{(\beta_1)_r}^{UL} + 190{(\beta_2)_r}^{UL} \geq {u_r}^{UL} - 0.06(1-\lambda) \\ {(\beta_1)_r}^{UL} + {(\beta_2)_r}^{UL} = 1.2 \\ \lambda \leq 1 \\ \text{and } {(\beta_1)_r}^{UL}, {(\beta_2)_r}^{UL}, \lambda, {u_r}^{UL} \geq 0 \end{cases} \tag{31}$$

$(\text{CLPP-I})^{UU}$

$$\begin{cases} \max \quad {u_r}^{UU} \\ \text{subject to the constraints} \\ 195{(\beta_1)_r}^{UU} + 105{(\beta_2)_r}^{UU} \geq {u_r}^{UU} - 0.04(1-\lambda) \\ 160{(\beta_1)_r}^{UU} + 195{(\beta_2)_r}^{UU} \geq {u_r}^{UU} - 0.04(1-\lambda) \\ {(\beta_1)_r}^{UU} + {(\beta_2)_r}^{UU} = 1.5 \\ \lambda \leq 1 \\ \text{and } {(\beta_1)_r}^{UU}, {(\beta_2)_r}^{UU}, \lambda, {u_r}^{UU} \geq 0 \end{cases} \tag{32}$$

And $(\text{CLPP-II})^{LU}$

$$\begin{cases} \min \quad {v_l}^{LU} \\ \text{subject to the constraints} \\ 170{(\gamma_1)_l}^{LU} + 145{(\gamma_2)_l}^{LU} \leq {v_l}^{LU} + 0.07(1-\mu) \\ 75{(\gamma_1)_l}^{LU} + 170{(\gamma_2)_l}^{LU} \leq {v_l}^{LU} + 0.07(1-\mu) \\ {(\gamma_1)_l}^{LU} + {(\gamma_2)_l}^{LU} = 0.3 \\ \mu \leq 1 \\ \text{and } {(\gamma_1)_l}^{LU}, {(\gamma_2)_l}^{LU}, \mu, {v_l}^{LU} \geq 0 \end{cases} \tag{33}$$

$(\text{CLPP-II})^{LL}$

$$\begin{cases} \min \quad {v_l}^{LL} \\ \text{subject to the constraints} \\ 175{(\gamma_1)_l}^{LL} + 150{(\gamma_2)_l}^{LL} \leq {v_l}^{LL} + 0.10(1-\mu) \\ 80{(\gamma_1)_l}^{LL} + 175{(\gamma_2)_l}^{LL} \leq {v_l}^{LL} + 0.10(1-\mu) \\ {(\gamma_1)_l}^{LL} + {(\gamma_2)_l}^{LL} = 0.5 \\ \mu \leq 1 \\ \text{and } {(\gamma_1)_l}^{LL}, {(\gamma_2)_l}^{LL}, \mu, {v_l}^{LL} \geq 0 \end{cases} \tag{34}$$

$(\text{CLPP-I})^{LM}$

$$\begin{cases} \min \quad \underline{v}^{M} \\ \text{subject to the constraints} \\ 180\underline{\gamma_1}^{M} + 155\underline{\gamma_2}^{M} \leq \underline{v}^{M} + 0.12(1-\mu) \\ 87\underline{\gamma_1}^{M} + 180\underline{\gamma_2}^{M} \leq \underline{v}^{M} + 0.12(1-\mu) \\ \underline{\gamma_1}^{M} + \underline{\gamma_2}^{M} = 0.8 \\ \mu \leq 1 \\ \text{and } \underline{\gamma_1}^{M}, \underline{\gamma_2}^{M}\mu, \underline{v}^{M} \geq 0 \end{cases} \tag{35}$$

$(\text{CLPP-II})^{UM}$

$$\begin{cases} \min \quad \overline{v}^M \\ \text{subject to the constraints} \\ 185\overline{\gamma_1}^M + 157\overline{\gamma_2}^M \leq \overline{v}^M + 0.16(1-\mu) \\ 92\overline{\gamma_1}^M + 185\overline{\gamma_2}^M \leq \overline{v}^M + 0.16(1-\mu) \\ \overline{\gamma_1}^M + \overline{\gamma_2}^M = 1 \\ \mu \leq 1 \\ \text{and } \overline{\gamma_1}^M, \overline{\gamma_2}^M \mu, \overline{v}^M \geq 0 \end{cases} \tag{36}$$

$(\text{CLPP-II})^{UL}$

$$\begin{cases} \min \quad {v_r}^{UL} \\ \text{subject to the constraints} \\ 190{(\gamma_1)_r}^{UL} + 158{(\gamma_2)_r}^{UL} \leq {v_r}^{UL} + 0.18(1-\mu) \\ 100{(\gamma_1)_r}^{UL} + 190{(\gamma_2)_r}^{UL} \leq {v_r}^{UL} + 0.18(1-\mu) \\ {(\gamma_1)_r}^{UL} + {(\gamma_2)_r}^{UL} = 1.2 \\ \mu \leq 1 \\ \text{and } {(\gamma_1)_r}^{UL}, {(\gamma_2)_r}^{UL}, \mu, {v_r}^{UL} \geq 0 \end{cases} \tag{37}$$

$(\text{CLPP-II})^{UU}$

$$\begin{cases} \min \quad {v_r}^{UU} \\ \text{subject to the constraints} \\ 195{(\gamma_1)_r}^{UU} + 160{(\gamma_2)_r}^{UU} \leq {v_r}^{UU} + 0.19(1-\mu) \\ 105{(\gamma_1)_r}^{UU} + 195{(\gamma_2)_r}^{UU} \leq {v_r}^{UU} + 0.19(1-\mu) \\ {(\gamma_1)_r}^{UU} + {(\gamma_2)_r}^{UU} = 1.5 \\ \mu \leq 1 \\ \text{and } {(\gamma_1)_r}^{UU}, {(\gamma_2)_r}^{UU}, \mu, {v_r}^{UU} \geq 0 \end{cases} \tag{38}$$

Table 1: Value of game $(\tilde{u}^R)$ for player I

Problem	$\lambda = 0$	$\lambda = 0.25$	$\lambda = 0.5$	$\lambda = 0.75$	$\lambda = 1$
$(\text{CLPP-I})^{LU}$	45.2025	45.1675	45.1325	45.0975	45.0625
$(\text{CLPP-I})^{LL}$	77.7242	77.6942	77.6642	77.6342	77.6042
$(\text{CLPP-I})^{LM}$	128.3373	128.3123	128.2873	128.2623	128.2373
$(\text{CLPP-I})^{UM}$	163.5593	163.5393	163.5193	163.4993	163.4793
$(\text{CLPP-I})^{UL}$	199.7321	199.7171	199.7021	199.6871	199.6721
$(\text{CLPP-I})^{UU}$	254.7400	254.7300	254.7200	254.7100	254.7000

Table 2: Value of game $(\tilde{v}^R)$ for player II

Problem	$\mu = 0$	$\mu = 0.25$	$\mu = 0.5$	$\mu = 0.75$	$\mu = 1$
$(\text{CLPP-II})^{LU}$	44.9925	45.0100	45.0275	45.0450	45.0625
$(\text{CLPP-II})^{LL}$	77.5042	77.5292	77.5542	77.5792	77.6042
$(\text{CLPP-II})^{LM}$	128.1173	128.1473	128.1773	128.2073	128.2373
$(\text{CLPP-II})^{UM}$	163.3193	163.3593	163.3993	163.4393	163.4793
$(\text{CLPP-II})^{UL}$	199.4921	199.5371	199.5821	199.6271	199.6721
$(\text{CLPP-II})^{UU}$	254.5100	254. 5575	254. 6050	254.6525	254.7000

Tables 1 and 2 illustrate the sensitivity analysis of game value for players I and II with different values of parameter λ. We have used Wolfram Cloud to solve the problem.

5.2. Example

Taking payoff in FMG with TFRNs as follows

$\tilde{A}^R = \begin{bmatrix} \tilde{a}_{11}^R & \tilde{a}_{12}^R \\ \tilde{a}_{21}^R & \tilde{a}_{22}^R \end{bmatrix}$

Where $\tilde{a}_{11}^R = [(1, 2, 3, 4) : (0.5, 2, 3, 4.5)]$;
$\tilde{a}_{12}^R = [(2.5, 5, 7.5, 10) : (1, 5, 7.5, 11.5)]$;
$\tilde{a}_{21}^R = [(3.5, 7, 10.5, 14) : (2, 7, 10.5, 15)]$;
$\tilde{a}_{22}^R = [(1.5, 3, 4.5, 6) : (1, 3, 4.5, 8)]$; Assuming that the margins for both the players are $\tilde{p}^R$ and $\tilde{q}^R$, which are given by
$\tilde{p}^R = [(0.06, 0.08, 0.10, 0.12) : (0.04, 0.08, 0.10, 0.14)]$;
$\tilde{q}^R = [(0.10, 0.12, 0.16, 0.18) : (0.07, 0.12, 0.16, 0.19)]$;
Having $\tilde{1}^R = [(0.5, 0.8, 1.0, 1.2) : (0.3, 0.8, 1.0, 1.5)]$, then using equations (11)-(14) as explained in steps 4 and 5 and equations (15)-(26) as explained in steps 6 and 7, the given TPZSMG with TFRNs as payoffs can be split into CLPPs for both the players as follows

$(\text{CLPP-I})^{LU}$

$$\begin{cases} \max \quad {u_l}^{LU} \\ \text{subject to the constraints} \\ 0.5{(\beta_1)_l}^{LU} + 2{(\beta_2)_l}^{LU} \geq {u_l}^{LU} - 0.14(1-\lambda) \\ 1{(\beta_1)_l}^{LU} + 1{(\beta_2)_l}^{LU} \geq {u_l}^{LU} - 0.14(1-\lambda) \\ {(\beta_1)_l}^{LU} + {(\beta_2)_l}^{LU} = 0.3 \\ \lambda \leq 1 \\ \text{and } {(\beta_1)_l}^{LU}, {(\beta_2)_l}^{LU}, \lambda, {u_l}^{LU} \geq 0 \end{cases} \tag{39}$$

$(\text{CLPP-I})^{LL}$

$$\begin{cases} \max \quad {u_l}^{LL} \\ \text{subject to the constraints} \\ 1{(\beta_1)_l}^{LL} + 3.5{(\beta_2)_l}^{LL} \geq {u_l}^{LL} - 0.12(1-\lambda) \\ 2.5{(\beta_1)_l}^{LL} + 1.5{(\beta_2)_l}^{LL} \geq {u_l}^{LL} - 0.12(1-\lambda) \\ {(\beta_1)_l}^{LL} + {(\beta_2)_l}^{LL} = 0.5 \\ \lambda \leq 1 \\ \text{and } {(\beta_1)_l}^{LL}, {(\beta_2)_l}^{LL}, \lambda, {u_l}^{LL} \geq 0 \end{cases} \tag{40}$$

$(\text{CLPP-I})^{LM}$

$$\begin{cases} \max \quad \underline{u}^{M} \\ \text{subject to the constraints} \\ 2\underline{\beta_1}^{M} + 7\underline{\beta_2}^{M} \geq \underline{u}^{M} - 0.10(1-\lambda) \\ 5\underline{\beta_1}^{M} + 3\underline{\beta_2}^{M} \geq \underline{u}^{M} - 0.10(1-\lambda) \\ \underline{\beta_1}^{M} + \underline{\beta_2}^{M} = 0.8 \\ \lambda \leq 1 \\ \text{and } \underline{\beta_1}^{M}, \underline{\beta_2}^{M} \lambda, \underline{u}^{M} \geq 0 \end{cases} \tag{41}$$

$(\text{CLPP-I})^{UM}$

$$\begin{cases} \max \quad \overline{u}^{M} \\ \text{subject to the constraints} \\ 3\overline{\beta_1}^{M} + 10.5\overline{\beta_2}^{M} \geq \overline{u}^{M} - 0.08(1-\lambda) \\ 7.5\overline{\beta_1}^{M} + 4.5\overline{\beta_2}^{M} \geq \overline{u}^{M} - 0.08(1-\lambda) \\ \overline{\beta_1}^{M} + \overline{\beta_2}^{M} = 1 \\ \lambda \leq 1 \\ \text{and } \overline{\beta_1}^{M}, \overline{\beta_2}^{M} \lambda, \overline{u}^{M} \geq 0 \end{cases} \tag{42}$$

$(\text{CLPP-I})^{UL}$

$$\begin{cases} \max \quad {u_r}^{UL} \\ \text{subject to the constraints} \\ 4{(\beta_1)_r}^{UL} + 14{(\beta_2)_r}^{UL} \geq {u_r}^{UL} - 0.06(1-\lambda) \\ 10{(\beta_1)_r}^{UL} + 6{(\beta_2)_r}^{UL} \geq {u_r}^{UL} - 0.06(1-\lambda) \\ {(\beta_1)_r}^{UL} + {(\beta_2)_r}^{UL} = 1.2 \\ \lambda \leq 1 \\ \text{and } {(\beta_1)_r}^{UL}, {(\beta_2)_r}^{UL}, \lambda, {u_r}^{UL} \geq 0 \end{cases} \tag{43}$$

$(\text{CLPP-I})^{UU}$

$$\begin{cases} \max \quad {u_r}^{UU} \\ \text{subject to the constraints} \\ 4.5{(\beta_1)_r}^{UU} + 15{(\beta_2)_r}^{UU} \geq {u_r}^{UU} - 0.04(1-\lambda) \\ 11.5{(\beta_1)_r}^{UU} + 8{(\beta_2)_r}^{UU} \geq {u_r}^{UU} - 0.04(1-\lambda) \\ {(\beta_1)_r}^{UU} + {(\beta_2)_r}^{UU} = 1.5 \\ \lambda \leq 1 \\ \text{and } {(\beta_1)_r}^{UU}, {(\beta_2)_r}^{UU}, \lambda, {u_r}^{UU} \geq 0 \end{cases} \tag{44}$$

And $(\text{CLPP-II})^{LU}$

$$\begin{cases} \min \quad {v_l}^{LU} \\ \text{subject to the constraints} \\ 0.5{(\gamma_1)_l}^{LU} + 1{(\gamma_2)_l}^{LU} \leq {v_l}^{LU} + 0.07(1-\mu) \\ 2{(\gamma_1)_l}^{LU} + 1{(\gamma_2)_l}^{LU} \leq {v_l}^{LU} + 0.07(1-\mu) \\ {(\gamma_1)_l}^{LU} + {(\gamma_2)_l}^{LU} = 0.3 \\ \mu \leq 1 \\ \text{and } {(\gamma_1)_l}^{LU}, {(\gamma_2)_l}^{LU}, \mu, {v_l}^{LU} \geq 0 \end{cases} \tag{45}$$

$(\text{CLPP-II})^{LL}$

$$\begin{cases} \min \quad {v_l}^{LL} \\ \text{subject to the constraints} \\ 1{(\gamma_1)_l}^{LL} + 2.5{(\gamma_2)_l}^{LL} \leq {v_l}^{LL} + 0.10(1-\mu) \\ 3.5{(\gamma_1)_l}^{LL} + 1.5{(\gamma_2)_l}^{LL} \leq {v_l}^{LL} + 0.10(1-\mu) \\ {(\gamma_1)_l}^{LL} + {(\gamma_2)_l}^{LL} = 0.5 \\ \mu \leq 1 \\ \text{and } {(\gamma_1)_l}^{LL}, {(\gamma_2)_l}^{LL}, \mu, {v_l}^{LL} \geq 0 \end{cases} \tag{46}$$

$(\text{CLPP-I})^{LM}$

$$\begin{cases} \min \quad \underline{v}^{M} \\ \text{subject to the constraints} \\ 2\underline{\gamma_1}^{M} + 5\underline{\gamma_2}^{M} \leq \underline{v}^{M} + 0.12(1-\mu) \\ 7\underline{\gamma_1}^{M} + 3\underline{\gamma_2}^{M} \leq \underline{v}^{M} + 0.12(1-\mu) \\ \underline{\gamma_1}^{M} + \underline{\gamma_2}^{M} = 0.8 \\ \mu \leq 1 \\ \text{and } \underline{\gamma_1}^{M}, \underline{\gamma_2}^{M}\mu, \underline{v}^{M} \geq 0 \end{cases} \tag{47}$$

$(\text{CLPP-II})^{UM}$

$$\begin{cases} \min \quad \overline{v}^M \\ \text{subject to the constraints} \\ 3\overline{\gamma_1}^M + 7.5\overline{\gamma_2}^M \leq \overline{v}^M + 0.16(1-\mu) \\ 10.5\overline{\gamma_1}^M + 4.5\overline{\gamma_2}^M \leq \overline{v}^M + 0.16(1-\mu) \\ \overline{\gamma_1}^M + \overline{\gamma_2}^M = 1 \\ \mu \leq 1 \\ \text{and } \overline{\gamma_1}^M, \overline{\gamma_2}^M \mu, \overline{v}^M \geq 0 \end{cases} \tag{48}$$

$(\text{CLPP-II})^{UL}$

$$\begin{cases} \min \quad {v_r}^{UL} \\ \text{subject to the constraints} \\ 4{(\gamma_1)_r}^{UL} + 10{(\gamma_2)_r}^{UL} \leq {v_r}^{UL} + 0.18(1-\mu) \\ 14{(\gamma_1)_r}^{UL} + 6{(\gamma_2)_r}^{UL} \leq {v_r}^{UL} + 0.18(1-\mu) \\ {(\gamma_1)_r}^{UL} + {(\gamma_2)_r}^{UL} = 1.2 \\ \mu \leq 1 \\ \text{and } {(\gamma_1)_r}^{UL}, {(\gamma_2)_r}^{UL}, \mu, {v_r}^{UL} \geq 0 \end{cases} \tag{49}$$

$(\text{CLPP-II})^{UU}$

$$\begin{cases} \min \quad {v_r}^{UU} \\ \text{subject to the constraints} \\ 4.5{(\gamma_1)_r}^{UU} + 11.5{(\gamma_2)_r}^{UU} \leq {v_r}^{UU} + 0.19(1-\mu) \\ 15{(\gamma_1)_r}^{UU} + 8{(\gamma_2)_r}^{UU} \leq {v_r}^{UU} + 0.19(1-\mu) \\ {(\gamma_1)_r}^{UU} + {(\gamma_2)_r}^{UU} = 1.5 \\ \mu \leq 1 \\ \text{and } {(\gamma_1)_r}^{UU}, {(\gamma_2)_r}^{UU}, \mu, {v_r}^{UU} \geq 0 \end{cases} \tag{50}$$

Table 3: Value of game $(\tilde{u}^R)$ for player I

Problem	$\lambda = 0$	$\lambda = 0.25$	$\lambda = 0.5$	$\lambda = 0.75$	$\lambda = 1$
$(\text{CLPP-I})^{LU}$	0.4400	0.4050	0.3700	0.3350	0.3000
$(\text{CLPP-I})^{LL}$	1.1557	1.1257	1.0957	1.0657	1.0357
$(\text{CLPP-I})^{LM}$	3.4143	3.3893	3.3643	3.3393	3.3143
$(\text{CLPP-I})^{UM}$	6.2943	6.2743	6.2543	6.2343	6.2143
$(\text{CLPP-I})^{UL}$	10.0029	9.9879	9.9729	9.9579	9.9429
$(\text{CLPP-I})^{UU}$	14.6650	14.6550	14.6450	14.6350	14.6250

Table 4: Value of game $(\tilde{v}^R)$ for player II

Problem	$\mu = 0$	$\mu = 0.25$	$\mu = 0.5$	$\mu = 0.75$	$\mu = 1$
$(\text{CLPP-II})^{LU}$	0.2300	0.2475	0.2650	0.2825	0.3000
$(\text{CLPP-II})^{LL}$	0.9357	0.9607	0.9857	1.0107	1.0357
$(\text{CLPP-II})^{LM}$	3.1943	3.2243	3.2543	3.2843	3.3143
$(\text{CLPP-II})^{UM}$	6.0543	6.0943	6.1343	6.1743	6.2143
$(\text{CLPP-II})^{UL}$	9.7629	9.8079	9.8529	9.8979	9.9429
$(\text{CLPP-II})^{UU}$	14.4350	14.4825	14.5300	14.5775	14.6250

Tables 3 and 4 illustrate the sensitivity analysis of game value for players I and II with different values of parameter λ. We have used Wolfram Cloud to solve the problem.

6. Conclusion

This study is about framing models to solve TPZSFMG with TFRNs as payoffs. To discover an optimal solution, a pair of FRLPPs is explored. The TPZSMG's max-min optimality criteria are transformed into upper and lower approximation intervals FLPPs and CLPPs for each player in the game. Using the proposed methodology, it is solved for various values of the parameters λ and μ in [0,1]. The proposed model is found to be suitable for solving TPZSFMG. Finally, the model is established using numerical examples. These numerical examples show that increasing the parameters λ and μ diminishes the value of the game for player I and increases it for player II. Future research can be expanded to neutrosophic and picture fuzzy sets.

Acknowledgements: We acknowledge the suggestions made by the journal's editor and anonymous reviewers, which helped us to improve the overall quality of the work.

REFERENCES

[1] Abraham, S., and Punniyamoorthy, M., "A fuzzy approach using asymmetrical triangular distribution in a two-person zero-sum game for a multi-criteria decision-making problem", *Quantum Machine Intelligence*, 3 (1) (2021) 1-24.

[2] Ammar, E., and Emsimir, A., "A mathematical model for solving fuzzy integer linear programming problems with fully rough intervals", *Granular Computing*, 6 (2021) 567–578.

[3] Aubin, J. P., "Mathematical Methods of Game and Economic Theory", *North-Holland, Amsterdam*, 1979.

[4] Aubin, J. P., "Cooperative fuzzy game", *Mathematics of Operations Research*, 6 (1981) 1-13.

[5] Bector, C. R., Chandra, S., and Vijay, V., "Duality in linear programming with fuzzy parameters and matrix games with fuzzy payoffs", *Fuzzy Sets and Systems*, 146 (2004) 253-269.

[6] Behera, D., Peters, K., and Edalatpanah, S. A., "Alternative methods for linear programming problem under triangular fuzzy uncertainty", *Journal of Statistics and Management Systems*, (2021) 1-19.

[7] Bigdeli, H., and Hassanpour, H., "An approach to solve multi-objective linear production planning games with fuzzy parameters", *Yugoslav Journal of Operations Research*, 28 (2) (2018) 237-248.

[8] Brikaa, M. G., Zheng, Z., and Ammar, E., "Fuzzy multi-objective programming approach for constrained matrix games with payoffs of fuzzy rough numbers", *Symmetry*, 11 (5) (2019) 7021-7026.

[9] Butnariu, D., "Fuzzy games: A description of the concept", *Fuzzy Sets and Systems*, 1 (1978) 181-192.

[10] Campos, L., and Verdegay, J. L., "Linear programming problems and ranking of fuzzy numbers", *Fuzzy Sets and Systems*, 32 (1) (1989) 1-11.

[11] Campos, L., "Fuzzy linear programming models to solve fuzzy matrix games", *Fuzzy Sets and Systems*, 32 (3) (1989) 275-289.

[12] Chen, Y. W., and Larbani, M., "Two person zero sum game approach for fuzzy multiple attribute decision making problems", *Fuzzy Sets and Systems*, 157 (2006) 34-51.

[13] Collins, W. D., and Hu, C., "Studying interval valued matrix games with fuzzy logic", *Soft Computing*, 12 (2008) 147-155.

[14] Das, S. K., Edalatpanah, S. A., "Application of Linear Programming in Diet Problem Under Pythagorean Fuzzy Environment", *Pythagorean Fuzzy Sets, Springer, Singapore*, (2021) 315-327.

[15] Das, S. K., Edalatpanah, S. A., and Dash, J. K., "A novel lexicographical-based method for trapezoidal neutrosophic linear programming problem", *Neutrosophic Sets and Systems*, 46 (1) (2021) 151-179.

[16] Ding, J., Li, C. L., and Zhu, G. S., "Two-person zero-sum matrix games on credibility space", *Eighth International Conference on Fuzzy Systems and Knowledge Discovery (FSKD), IEEE*, 2 (2011) 912-916.

[17] Dubey, R., Deepmala, and Mishra, V. N., "Higher-order symmetric duality in nondifferentiable multiobjective fractional programming problem over cone constraints", *Statistics, Optimization and Information Computing*, 8 (1) (2020) 187-205.

[18] Dubey, R., Kumar, R., Alam, K., Mishra, L. N., and Mishra, V. N., "A class of new type unified non-differentiable higher order symmetric duality theorems over arbitrary cones under generalized assumptions", *Yugoslav Journal of Operations Research.*

[19] Dubey, R., Vandana and Mishra, V. N., "Second order multiobjective symmetric programming problem and duality relations under (F, G_f)-convexity", *Global Journal of Engineering Science and Researches*, 5 (8) (2018) 187-199.

[20] Dubey, R., Vandana, Mishra, V. N., and Karateke, S., "A class of second order nondifferentiable symmetric duality relations under generalized assumptions", *Journal of Mathematics and Computer Science*, 21 (2) (2020) 120–126.

[21] Dubois, D., and Prade, H., "Rough fuzzy sets and fuzzy rough sets", *International Journal of General Systems*, 17 (2) (1990) 191-209.

[22] Ebrahimnejad, A., and Nasseri, S. H., "Using complementary slackness property to solve linear programming with fuzzy parameters", *Fuzzy Information and Engineering*, 3 (2009) 233-245.

[23] Farnam, M., and Darehmiraki, M., "Solution procedure for multi-objective fractional programming problem under hesitant fuzzy decision environment", *Journal of Fuzzy Extension & Applications*, 2 (4) (2021) 364-376.

[24] Khalifa, H. A., and Masoud, M., "Solving two-person zero-sum games with interval valued fuzzy payoffs", *International Journal of Applied Optimization Studies*, 2 (4) (2019) 01-08.

[25] Kumar, S., "Max-min solution approach for multi-objective matrix game with fuzzy goals", *Yugoslav Journal of Operations Research*, 26 (1) (2016) 2334-6043.

[26] Kumar, G., and Jangid, V., "Linear programming models to solve fully fuzzy two person zero sum matrix game", *Malaya Journal of Matematik*, 8(3) (2020) 775-781.

[27] Kumar, R. S., and Keerthana, D., "A solution of fuzzy game matrix using defuzzification method", *Malaya Journal of Matematik*, 5(1) (2015) 68-72.

[28] Maeda, T., "On characterization of equilibrium strategy of two person zero sum games with fuzzy payoffs", *Fuzzy Sets and Systems*, 139 (2003) 283-296.

[29] Maleki, H. R., "Ranking functions and their applications to fuzzy linear programming", *Far East Journal of Mathematical Sciences*, 4 (2002) 283-301.

[30] Mi, X., Liao, H., Zeng, X. J., and Xu, Z., "The two-person and zero-sum matrix game with probabilistic linguistic information", *Information Sciences*, 570 (2021) 487–499.

[31] Nanda, S., and Majumda, S., "Fuzzy rough sets", *Fuzzy Sets and Systems*, 45 (1992) 157-160.

[32] Naqvi, D., Aggarwal, A., Sachdev, G., and Khan, I., "Solving I-fuzzy two person zero-sum matrix games: Tanaka and Asai approach", *Granular Computing*, 6 (2) (2021) 399-409.

[33] Nash, J. F., "Equilibrium points in n-person games", *Proceedings of The National Academy of Sciences*, 36 (1950) 48-49.

[34] Nehi, H. M., and Hajmohamadi, H., "A ranking function method solving fuzzy multi-objective linear programming", *Annals of Fuzzy Mathematics and Informatics*, 10 (10) (2011) 1-20.

[35] Neumann, J. V., and Morgenstern, O., "Theory of Games and Economic Behavior", *Princeton University press*, 1944.

[36] Nishizaki, I., and Sakawa, M., "Fuzzy and Multi-objective Games for Conflict Resolution", *Physica-Verlag, Heidelberg*, 2001.

[37] Nurmi, H., "A fuzzy solution to a majority voting game", *Fuzzy Sets and Systems*, 5 (1981) 187-198.

[38] Pandian, P., Natarajan, G., and Akilbasha, A., "Fuzzy interval integer transportation problems", *International Journal of Pure and Applied Mathematics*, 119 (9) (2018) 133-142.

[39] Pawlak, Z., "Rough sets", *International Journal of Computer and Information Sciences*, 11 (5) (1982) 341-356.

[40] Pawlak, Z., "Rough Sets: Theoretical aspects of reasoning about data", *Kluwer Academic Publishers, Dordrecht, The Netherlands*, 1991.

[41] Peykani, P., Nouri, M., Eshghi, F., Khamechian, M. and Farrokhi-Asl, M., "A novel mathematical approach for fuzzy multi-period multi-objective portfolio optimization problem under uncertain environment and practical constraints", *Journal of Fuzzy Extension & Applications*, 2 (3) (2021) 191-203.

[42] Ramik, J., "Duality in fuzzy linear programming with possibility and necessity relations", *Fuzzy Sets and Systems*, 157 (2006) 1283-1302.

[43] Roy, S. K., and Mondal, S. N., "An approach to solve fuzzy interval valued matrix games", *International Journal of Operational Research*, 26 (3) (2016) 253-267.

[44] Roy, S. K., and Mula, P., "Solving matrix game with rough payoffs using genetic algorithm", *International Journal of Operational Research*, 16 (1) (2016) 117-130.

[45] Sahoo, L., "An approach for solving fuzzy matrix games using signed distance method", *Journal of Information and Computing Science*, 12 (1) (2017) 73-80.

[46] Sakawa, M., and Nishizaki, I., "Max-min solution for fuzzy multi-objective matrix games", *Fuzzy Sets and Systems*, 67 (1) (1994) 53-69.

[47] Seikh, M. R., Nayak, P. K., and Pal, M., "An alternative approach for solving fuzzy matrix

games", *International Journal of Mathematics and Soft Computing*, 5 (1) (2015) 79-92.

[48] Subha, V. S., and Dhanalakshmi, P., "Some similarity measures of rough interval Pythagorean fuzzy sets", *Journal of Fuzzy Extension & Applications*, 1 (4) (2020) 304-313.

[49] Sun, J., "Two-person zero-sum stochastic linear-quadratic differential games", *SIAM Journal on Control and Optimization*, 59 (3) (2021) 1804-1829.

[50] Vandana, Dubey, R., Deepmala, Mishra, L. N., and Mishra, V. N., "Duality relations for a class of a multiobjective fractional programming problem involving support functions", *American Journal of Operations Research*, 8 (4) (2018) 294-311.

[51] Vijay, V., Chandra, S., and Bector, C. R., "Matrix games with fuzzy goals and fuzzy payoffs", *Omega*, 33 (2005) 425-429.

[52] Wang, Q., Huang, Y., Kong, S., Ma, X., Liu, Y., Das, S. K., and Edalatpanah, S. A., "A Novel Method for Solving Multiobjective Linear Programming Problems with Triangular Neutrosophic Numbers", *Hindawi Journal of Mathematics*, 2021 (2021) 1-8.

[53] Wu, W. Z., Mi, J. S., and Zhang, W. X., "Generalized fuzzy rough sets", *Information Sciences*, 151 (2003) 263-282.

[54] Xu, J., and Yao, L., "A class of two person zero sum matrix games with rough payoffs", *International Journal of Mathematics and Mathematical Sciences*, 2010 (3) (2010) 1-22.

[55] Yager, R. R., "A procedure for ordering fuzzy numbers of the unit interval", *Information Sciences*, 24 (1981) 143-161.

[56] Zadeh, L. A., "Fuzzy sets", *Information and Control*, 8 (3) (1965) 338-353.

[57] Zadeh, L. A., "Fuzzy sets as a basis for a theory of possibility", *Fuzzy Sets and Systems*, 1 (1978) 3-28.

[58] Zimmermann, H. J., "Fuzzy programming and linear programming with several objective functions", *Fuzzy Sets and Systems*, 1 (1978) 45-55.