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FEOQ MODEL WITH OCTAGONAL FUZZY DEMAND RATE AND OPTIMIZE WITH SIGNED DISTANCE METHOD

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Abstract: Uncertainties in the stock control system development is one of the main concerns of researchers, several studies has been done in this research field with different fuzzy numbers by using different defuzzification methods. In this present article, at first, we have studied the scenario of increasing demand rate with the time and shortage are fully backlogged. Due to some reasons, there is flexible(uncertain) demand rate for the production, so the demand rate has to be taken as octagonal fuzzy number and the model converted into fuzzy model. The purpose of presented model is to optimize the total inventory cost in both the crisp and the fuzzy situations. The designated model included with Signed distance method for an appropriate numerical example, sensitivity analysis and graphical illustration for better explanation of the article.

Keywords: Fuzzy inventory model, octagonal fuzzy demand, deteriorating items, signed distance method.

MSC: 03B52, 78M50, 90B05, 97M40, 90B25.

1. INTRODUCTION

In the industrial area, there is more complex situation for producers. In the large multinational companies, the factor influencing any decision and it is a complex situation. In this situation there is several uncertainties to make a decision in the favor of company. The factors like, customer demand, raw material, capacity of equipment etc. are the main cause for complex situation. It is possible to make suitable decision to remove these uncertainties with the help of mathematical model and optimize the objectives and results. The uncertainty is the main factor to affects the manufacturing industries; operation research is the essential tool to deal with this complex problem.

The operational research techniques are useful for the production management to selecting the locality where the product is in high demand and how much produce, calculate the optimal production, availability of raw material etc. OR is useful to the deal with financial problems and control the different investment proposals for an appropriate profit for the production company.

In the field of inventory control, many researchers have been their research on the topic of inventory management. But in the recent research, there are several type of uncertainties like demand fluctuation, high deteriorating rate, inflation rate and other cost objects. For these critical situation, the fuzzy theory helps us to get optimal results. Fuzzy set theory has been applied to several fields like optimization, project network, transportation air pollution, project scheduling psychology, field of health. In the field of research, L. A. Zadeh [1] developed the fuzzy set theory which is used in several research in different fields like mathematics, statistics, used in the programming of vehicle and video games, robotics and driverless vehicles etc.in this article the fuzzy theory is used in inventory models. With the help of fuzzy numbers, we can control the vagueness of highly fluctuated demand factor. Chauhan and Sharma [2] proposed their work for preservation technology with fuzzy cost and introduced a model with reliable and optimistic result. They presented the model for deteriorating items which can preserve for the long time period. They illustrated an example for the inventory model which is suitable for the article. Wu [3] proposed an EOQ model of inventory system with backlog and shortage, where the rate of backlogging is variable. With the continuity in the research work with fuzzy theory, Soni and joshi [4] presented an generalized EOQ model under trade credit policy. They discussed the fuzzy concept on all the cost factors and for defuzzification, they used Graded Mean Integration Representation Method. They proposed an inventory model with a profit maximization under fuzzy techniques. Some customers would not like to wait for backlogging during the shortage period, so Wang et al. [5] studied on inventory modeling for deteriorating items with shortages and partial backlogging. This article helps us to understand the concept of partial backlogging with an inventory model. In the field of application of fuzzy theory, Banu and Mondal [6] presented an inventory model for two-level trade credit policy under fuzzy environment. They use q-fuzzy number for the uncertain demand of goods and defuzzified the model with the centroid method. They also included the effect of trade credit on customers. Garai et al. [7] developed a fuzzy inventory model in which the demand is price dependent and the fuzzy holding cost. They developed a fully fuzzy model and developed an algorithm for defuzzification of that fully fuzzy inventory model. They illustrated an example for this fully fuzzy inventory model. In this manner, Saha et al. [8] proposed a model for inventory problem which consider the retailers and distributors to control the stock of several seasonal products. They presented the problem with partial backlog and ramp type demand and also discussed the price discount policy for retailers. In the end, they provided an verified example for their model. A two-level supply chain model is developed by Pakhira et al. [9]. In this model they discussed the problem of retailer's warehouse, which has a limited capacity in the market. Then the retailer rent another warehouse for the stock of goods. They also included the channel profit, promotional cost etc. This model is also solved in fuzzy environment and a Particle Swarm Optimization technique is used to find the optimal decisions. S. Fathalizadeh et al. [10] presented two models along with a deteriorating rate and inflation. They have used two different optimization methods, the discount cost method, and the average annual cost method. In this article, in the fuzzy environment they used the fuzzy nonlinear programming method to solve fuzzy average cost problem. Rajput et al. [11] proposed an inventory model with a different type of demand function, discussed the importance of fuzzy parameters in healthcare industries. They used triangular fuzzy number for the demand and defuzzified the model with signed distance method and get the maximum profit for all three models. In the recent research study Triupathi et al. [12] proposed their article about fuzzy theory applications in the field of operations research as in manufacturing industries for automotive components. They used fuzzy axiomatic design (FAD) technique to manufacture the product. They also used triangular fuzzy number as input for the sustainable manufacturing. An inventory model proposed by Tai et al. [13], they discussed inventory model with two replenishment policies-(1) quantity-based (2) time-based policy and two inspection positions- (1) one inspection (2) continuous monitoring. In recent studies shows that Mahato S.K. et al. [14] discussed the optimization of a complex network system. They use trapezoidal fuzzy number in the place of parameters and then use signed distance method for defuzzification. They also use genetic algorithm for interger variables. Rajput et al. [15] developed the inventory model with fuzzy demand rate. They used signed distance method for defuzzification and get the optimal inventory cost. They used triangular fuzzy number as well as defuzzification by signed distance method. In the recent research, Rajput et al. [16] proposed an inventory model for both the crisp and the fuzzy environments. They included reliability induced demand and fuzzy parameters. Graded Mean Integration method for defuzzification for the model in fuzzy sense. They provided an example to satisfy the result in both the environments. Padiyar et al. [17] developed a production inventory model for perishable items under learning inspection. The study considers two different demands for two different warehouses which are exponential and selling price dependent demand with the help of shortage. They discussed own warehouse (OW) and rented warehouse (RW), and optimize the total inventory cost. Total cost of model is minimized for both crisp and fuzzy environment by using triangular fuzzy numbers and defuzzify the total cost by Singed distance, centroid, and GMI method. In 2021 Rajput et al. [18] presented an EOQ model for deteriorating items under inflation, the research article deals with effects of deterioration and inflation without any shortage in both the crisp and the fuzzy environments. They used a pentagonal fuzzy number and then optimize the total inventory cost with the GMI defuzzification method.

The present study introduced an inventory model FMCG products. In this situation, the uncertainty of demand rate has been solved with the application of octagonal fuzzy number and the Signed distance defuzzification method. The EOQ model has been considered and solved in the crisp, and the fuzzy environment then comparison in between them. The introduction is in section 1, some basic definitions related to fuzzy number and defuzzification method are in section 2. In section 3, we introduce some assumptions and notations related to the proposed model. Mathematical modeling is described in section 4 with subsections 4.1 and 4.2, section 5 represents the algorithm for optimality criteria, and numerical illustration example and sensitivity analysis shows in section 6. Then, we construct the conclusion for the application of fuzzy theory is in section 7. You can use this template as a sample.

2. MANUSCRIPT PREPARATION

2.1. Preliminaries

Definition 1. ***Fuzzy Set-*** *Let $\tilde{Z}$ be a set of fuzzy numbers on real numbers $R = (-\infty, \infty)$. Then $\tilde{Z}$ is said to be a fuzzy set, if its membership function is*

$$\mu_{\tilde{Z}(x)} = \begin{cases} 1, & if x = q \\ 0, & if x \neq q \end{cases}$$

Definition 2. *α-**Cut**- Let $\tilde{Z}$ be a fuzzy set on R. For $0 \leq \alpha \leq 1$ the $\alpha-$ cut of $\tilde{Z}$ is all the points x of $\tilde{Z}$ such that $\mu_{\tilde{Z}(x)} \geq \alpha$*

$$Z(\alpha) = \{x/\mu_{\tilde{Z}(x) \geq \alpha}\}$$

Definition 3. ***Octagonal fuzzy number-*** *A fuzzy number $\tilde{Z}$=$(z_1, z_2, z_3, z_4, z_5, z_6, z_7, z_8)$ is a octagonal fuzzy number(figure (1)), if its membership function $\mu_{\tilde{Z}(x)}$ is*

$$\mu_{\tilde{Z}(x)} = \begin{cases} 0, & \text{if } x < z_1 \ \& \ z_8 \leq x \\ k\left(\frac{x-z_1}{z_2-z_1}\right), & \text{if } z_1 \leq x \leq z_2 \\ k, & \text{if } z_2 \leq x \leq z_3 \\ k+(1-k)\left(\frac{x-z_3}{z_4-z_3}\right), & \text{if } z_3 \leq x \leq z_4 \\ 1, & \text{if } z_4 \leq x \leq z_5 \\ k+(1-k)\left(\frac{z_6-x}{z_6-z_5}\right), & \text{if } z_5 \leq x \leq z_6 \\ k, & \text{if } z_6 \leq x \leq z_7 \\ k\left(\frac{z_8-x}{z_8-z_7}\right), & \text{if } z_7 \leq x \leq z_8 \end{cases} \quad \text{for } \ 0 \leq k \leq 1$$

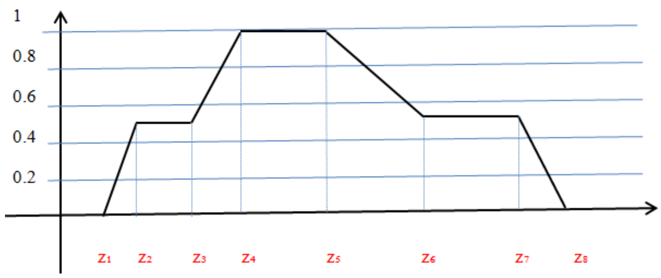


Figure 1: Graphical presentation of octagonal fuzzy number.

Remark-. If k = 0, then the octagonal fuzzy number reduces into the trapezoidal fuzzy number (z_3, z_4, z_5, z_6) and if k=1, then it reduces into the trapezoidal number (z_1, z_4, z_5, z_8).

Definition 4. ***Signed Distance method-*** *Since $\tilde{Z}$ is a fuzzy number, $Z_L(\alpha)$ and $Z_R(\alpha)$ are left α-cut and right α-cut of $\tilde{Z}$ and they are integrable and exist for $\alpha \in (0,1)$, then the distance between $\tilde{P}$ and 0, which is known as signed distance is given by;*

$$d(\tilde{Z}, 0) = \frac{1}{2}\int_0^1 [Z_L(\alpha) + Z_R(\alpha)]d\alpha$$

2.2. Assumptions and Notations

2.2.1. Assumptions

- In the present inventory model, the demand rate is $A(t) = o(1 + \beta(t))$ for $\alpha > 0$ and $0 < \beta < 1$, is an increasing function with respect to the time.
- The replenishment is instantaneous.
- The lead time for this inventory model is zero.
- There are some shortages and fully backlogged.

2.2.2. Notations

$A(t)$	Demand rate.
o & β	The demand parameters.
q	Order quantity.
θ	Deteriorating rate.
N	Length of the cycle in years.
OC	Ordering cost for the inventory.
H	Holding cost per unit.
S	Shortage cost per unit.
D	Deteriorating cost per unit.
TIC	Total inventory cost.

2.3. Formulation of Mathematical model

2.3.1. Crisp Model

For the development of inventory model in crisp environment, let us take the inventory level is q and the level of inventory decreases due to the demand and deteriorating nature of products and become zero at cycle time t. So, there is some shortage of goods in inventory. For the optimal solution we try to solve this mathematical model with some differential equations below; Let $L(t)$ be th level of inventory at time t then

$$\frac{dL(t)}{dt} + rL(t) = -A(t), \qquad 0 \le t \le t_1 \tag{1}$$

$$\frac{dL(t)}{dt} = -A(t), \qquad t_1 \le t \le N \tag{2}$$

Then the solution of eqn. 1 and 2 is

$$L(t) = o\left[(t_1 - t) + \frac{r}{2}(t_1 - t)^2\right] + o\beta\left[t_1(t_1 - t) - \frac{(t_1 - t)^2}{2} + \frac{rt_1(t_1 - t)^2}{2} - \frac{r(t_1 - t)^3}{6}\right] \tag{3}$$

Holding cost for the model is

$$HC = H\int_0^{t_1} L(t)\ dt = Ho\left[\frac{t_1^2}{2} + \frac{rt_1^3}{6}\right] + Ho\beta\left[\frac{t_1^3}{3} + \frac{rt_1^4}{8}\right] \tag{4}$$

Deteriorating cost is

$$DC = D\left[q - \int_0^{t_1} A(t)\ dt\right] = Do\left[\frac{rt_1^2}{2} + \frac{\beta rt_1^3}{3}\right] \tag{5}$$

Shortage cost is

$$SC = S\left[-\int_0^{t_1} L(t)\ dt\right] = S\left[\frac{o(t_1 - N)^2}{2} - \frac{o\beta}{2}(t_1^2 N - \frac{N^3}{3} - \frac{2t_1^3}{3})\right] \tag{6}$$

From the eqn. 3, 4, 5 and 6, the total inventory cost for the presented model is

$$TIC = [OC + HC + DC + SC]$$

$$TIC = \frac{1}{N}\left[D\left(\frac{1}{3}ort_1^3 + \frac{1}{2}ort_1^2\right) + o\beta H\left(\frac{rt_1^4}{8} + \frac{t_1^3}{3}\right) + oH\left(\frac{rt_1^3}{6} + \frac{t_1^2}{2}\right) + S\left(\frac{1}{2}o(t_1 - N)^2 - \frac{1}{2}(o\beta)\left(-\frac{N^3}{3} + Nt_1^2 - \frac{2t_1^3}{3}\right)\right) + \mathrm{OC}\right] \tag{7}$$

2.3.2. Fuzzy Model

In uncertain world, there are several uncertainties in consumption of goods. In present time period, there are uncertainties in the daily use FMCG products. The demand of these products increased, so the uncertain(fuzzy) behavior of demand of these products is discussed here. Therefore, the demand parameter o is in the form of octagonal fuzzy number $(o_1, o_2, o_3, o_4, o_5, o_6, o_7, o_8)$ (from the definition (3)) and the associate fuzzy model presented below,
The total fuzzy inventory cost is

$$\widetilde{TIC} = [\widetilde{OC} + \widetilde{HC} + \widetilde{DC} + \widetilde{SC}]$$

$$\widetilde{TIC} = \frac{1}{N}\left[D\tilde{o}\left(\frac{1}{3}rt_1^3 + \frac{1}{2}rt_1^2\right) + \tilde{o}\beta H\left(\frac{rt_1^4}{8} + \frac{t_1^3}{3}\right) + \tilde{o}H\left(\frac{rt_1^3}{6} + \frac{t_1^2}{2}\right) + S\left(\frac{1}{2}\tilde{o}(t_1 - N)^2 - \frac{1}{2}(\tilde{o}\beta)\left(-\frac{N^3}{3} + Nt_1^2 - \frac{2t_1^3}{3}\right)\right) + \mathrm{OC}\right] \tag{8}$$

To solve the above eqn 8 with the help of fuzzy arithmetic operations and apply the Signed Distance defuzzification method discussed in definition 4, we get the total inventory cost in fuzzy environment is

$$\begin{aligned}\widetilde{TIC} = &\frac{1}{N}\Bigg[\mathrm{OC} + \frac{1}{4}D\left((o_3+o_4+o_5+o_6)(1-k) + (o_1+o_2+o_7+o_8)k\right)\left(\frac{rt_1^3}{3} + \frac{rt_1^2}{2}\right) + \\ &\frac{1}{4}\beta H\left((o_3+o_4+o_5+o_6)(1-k) + (o_1+o_2+o_7+o_8)k\right)\left(\frac{rt_1^4}{8} + \frac{t_1^3}{3}\right) + \\ &\frac{1}{4}H\left((o_3+o_4+o_5+o_6)(1-k) + (o_1+o_2+o_7+o_8)k\right)\left(\frac{rt_1^3}{6} + \frac{t_1^2}{2}\right) + \\ &\frac{1}{4}S\left((o_3+o_4+o_5+o_6)(1-k) + (o_1+o_2+o_7+o_8)k\right) \\ &\left(\frac{1}{2}(t_1 - N)^2 - \frac{1}{2}\beta\left(-\frac{N^3}{3} + Nt_1^2 - \frac{2t_1^3}{3}\right)\right)\Bigg]\end{aligned} \tag{9}$$

2.4. Optimality Criteria

To find the minimum value of the TIC with respect to t_1, we have to follow these steps;

Step 1- Start with $\dfrac{TIC}{dt} = 0$ and get t_1^*.

Step 2- Using t_1^*, which found in step 1, evaluate the value of $TIC(t_1^*)$ from the cost function equation in crisp and fuzzy environment.

Step 3- If the second derivative of $\dfrac{TIC^2}{dt_1^2} > 0$ for the critical value t_1^*. Then, we get the minimum value of cost function which is a convex function.

Step 4- Repeat step 2 and step 3 until we get the optimal solution for the cost function.

2.5. Numerical Example and Sensitivity Analysis

2.5.1. Numerical Example

A FMCG manufacture company, which faced the increasing demand of products in FMCG. The retailer spends $OC = Rs.$ 5000 per unit on each order and the demand parameters are $o = 1200$ and $\beta = 0.2$ units per year. The deteriorating cost for one unit is $d = Rs.\ 25$. There is a huge demand of the product so there are some shortage in the inventory and the shortage cost is $S = Rs.\ 20$. He finds that the cost of holding items in stock is $H = Rs.\ 15$ per unit per year. The rate of deterioration is $r = Rs.\ 0.01$ per unit. The length of cycle time is 10 years for the manufacturing the items.In the fuzzy environment, retailer face the fuzzy demand parameters-

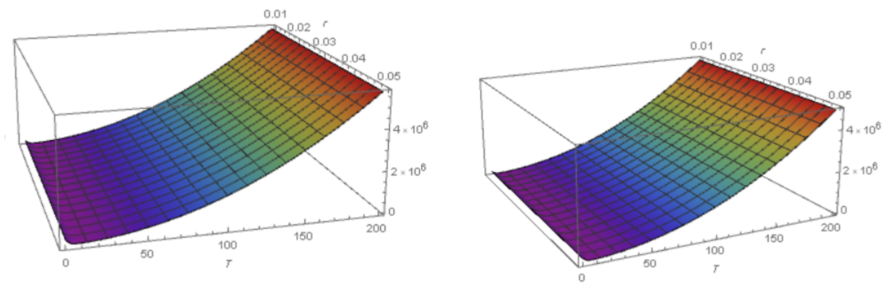
$\tilde{o} = (o_1, o_2, o_3, o_4, o_5, o_6, o_7, o_8) = (50, 70, 90, 130, 150, 250, 300, 350)$ and the value of k is 0.5.

Then from the eqn. (7) and (9), the optimal solution in crisp model and fuzzy model are presented in table (1) below.

Table 1: Optimal values in different environments

Model	t_1^*	TIC^*
Crisp	5.5265	18589.2
Fuzzy	5.5265	16215.0

From the table (1), in the crisp environment the optimal total inventory cost is Rs. 18589.2 with 505265 years approximately. While the optimal total inventory cost Rs. 16215.0 is minimum in fuzzy environment with the same time as crisp. So we admire that the application with the Signed distance defuuzfication method, the inventory cost is minimize with the same cycle time. So that the retailer can provide the products in limited time.



(a) Convex curve of total inventory cost for crisp model.

(b) Convex curve of total inventory cost for fuzzy model.

Figure 2: Convexity of the total inventory cost function for different environments.

Figure (2a) and (2b) shows that the convex characters of inventory cost function with respect to the time in both the crisp and th fuzzy environments. It is clear from both the figures, the cost has a minimum point at 5.5265 years, but the inventory cost is optimize for the fuzzy model. With the same optimal time optimal total inventory cost is Rs. 18589.2 in the crisp environment, while the optimal total inventory cost Rs. 16215.0 is minimum in fuzzy environment. With the help of fuzzy techniques, the inventory cost decreases almost 12.77% in fuzzy model compare with crisp model. Hence in the same cycle time, the company produces more products with low inventory cost with fuzzy operators.

2.5.2. Sensitivity Analysis

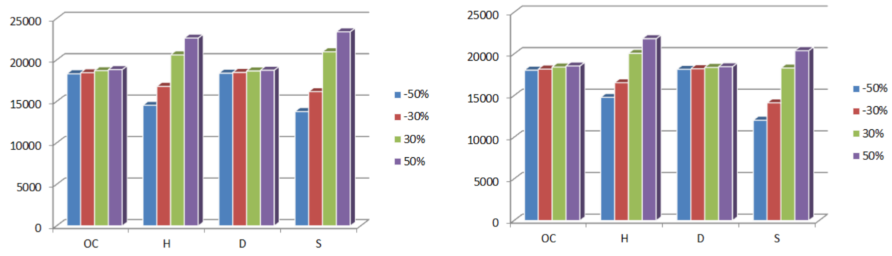
The consideration of a sensitivity analysis for the parametric changes from -50% to +50% of the parameters OC, H, D, and S respectively in the Table(2).

Table 2: Sensitivity analysis for total cost in general fuzzy environment.

Parameter	% change	Crisp total cost	Fuzzy total cost
	-50	18339.2	18054.4
OC	-25	18466.2	18179.4
	+25	18714.2	18429.4
	+50	18839.2	18554.4
	-50	14533.5	14781.0
H	-25	16818.2	16542.7
	+25	20617.0	20066.0
	+50	22644.9	21827.7
	-50	18410.4	18149.0
D	-25	18499.8	18226.7
	+25	18678.6	18382.0
	+50	18768.0	18459.7
	-50	13779.1	12036.2
S	-25	16184.2	14125.6
	+25	20994.2	18304.4
	+50	23399.2	20393.7

From the Table(2), it is seen that the cost parameters ordering cost (OC), holding cost (H), deteriorating cost (D) and shortage cost (S) are highly sensitive.

From the figure (3a) and (3b), it is clear that the change in inventory cost due to cost parameters holding cost (H) and shortage cost (S) are highly sensitive compare to other costs in both the models. While the other parameters cost ordering cost (OC) and deteriorating cost (D) has minor effects on inventory cost for both models.



(a) Sensitivity analysis for crisp model with cost parameters.

(b) Sensitivity analysis for fuzzy model with cost parameters.

Figure 3: Sensitive analysis for both the environments with respect to change in cost parameters.

3. CONCLUSION

The presented inventory model comprise that the concept of optimal management of the inventory system with respect to the time and also the demand function is uncertain. The proposed system of inventory is also dependent on the deterioration rate and the partial back-order of the items. For this model, we conclude that the total inventory cost for the inventory system under fuzzy environment has been reduced compared to the crisp environment. In the present situation, the fuzziness occurs in the different parameters which affects to the whole inventory management. Thus for the future scope, there is wide applicability of this model of the inventory in various domains as it helps in the reduction of the total cost of the inventory system. This paper provides a compulsive topic for the further study of such kind of important inventory models and it can be extended in numerous ways for future research with ramp type demand, exponential demand, Verhulst's model type demand rate, trapezoidal demand rate, inflation, etc.

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