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ECONOMIC ORDER QUANTITY MODEL FOR IMPERFECT QUALITY ITEMS WITH ENVIRONMENTAL REGULATIONS

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Abstract: In real life situation, advance technologies are used to produce items but due to some technical error imperfect items are produced. Therefore it is necessary to separate these imperfect items by complete screening process and such items are sold at discounted price at the end of screening process. This paper study about the EOQ model for imperfect items with shortage and zero lead time. Additionally, it provides a model to reduce emissions by including carbon cap, carbon tax, carbon cap and offset and carbon price. The main purpose of the developed model is to reduce the carbon emission when there are imperfect items using KKT(Karush Kuhn Tucker) conditions and the impact of imperfect quality items can be seen in the sensitive analysis part. The retailer should be more vigilant while ordering.Numerical examples are provided to illustrate the procedure.

Keywords: Inventory management, economic order, carbon emission, environmental regulations, screening cost, imperfect quality.

MSC: 90B05(90B90), 90C15(90C27).

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1. INTRODUCTION

Economic order quantity is the quantity which is used to minimize total costs. Ford W. Haris and R.H. Wilson developed this model in 1913. Bouchery and Dallery [1] consider sustainability into classical inventory model. Arslan and Turkay [2] have contributed to Economic order quantity model by including sustainability considerations which embrace environmental and social criteria with standard economic consideration. Wang et al. [3] developed a EOQ model with renewal reward theory to derive the Expected total profit per unit time. Lee et al. [4] developed a model for sustainable economic order quantity with stochastic lead time and multi-model transportation options. Agarwal et al. [5] provided a classification policy for inventory and loss profit of recurring items is determined at each level. Babar et. al. [6] study about offline inspection system where inspection is performed by the human labor of varying skill levels. It has been proved that inspection performance of inspectors improves significantly with learning and revision of allocation of inspectors with the proposed model ensure better utilization of available manpower, maintain good quality and reduce cost as well.

Sheikh et al. [7] developed two EOQ models with and without shortages and considering purchasing and holding cost constant. Mittal et al.[8] proposed an inventory model for deteriorating imperfect items with learning effect and carbon emissions.

Carbon emission is increasing day by day and many firms are working to reduce carbon emissions. Government has also taken many steps to reduce emission such as carbon tax, cap and offset. Therefore, Wang and Hua [9] investigates management of carbon footprints in firms under carbon emission trading mechanism. Benjaafar et al. [10] developed a model to investigate how far carbon reduction requirements can be addressed by operational adjustments as a supplement to costly investments in carbon-reducing. Chen, Benjaafar and Elomri [11] provide a model a condition in which emission can be reducing by modifying order quantity. Toptal et al. [12] explained the reduction of carbon emission using the government regulations i.e, carbon cap, carbon tax and carbon cap-and-trade and compare the results in terms of cost and emission between different policies. Mittal et al. [13] provide an economic production model to elaborate human errors effect on emission cost, transportation cost and Expected total profit of the retailer. Kanna et al. [14] the impact of preservation technology for deteriorating items and also noticed that pattern of demand for many products is relying on its usage and availability. Daryanto et al. [15] introduced an Economic order quantity model which includes effect of defective rates, different sources of carbon emission, different demand rates, selling price and holding cost for defective products, and shortages backorder.

Since there are perfect quality items as well as defective items, therefore, In 2000, Salameh and Jaber [16] proposed EPQ/EOQ model in which a production/inventory situation where items received / produced are of imperfect quality and extends the standard EOQ/EPQ model for imperfect items. Chang [17] introduced a model with complete screening process and imperfect quality items are sold as single batch with discount before recieving the next shipment. Jaggi and Mittal [18] developed a model for spoilable items in which there are constant deterioration and demand rate is time dependent under inflation and money value. Wee et al. [19] developed an inventory model for imperfect items and shortages. Jaggi et al. [20] introduced a model for retailer working with imperfect items with deteriorating nature under inflation and permissible delay in payments. Jaggi and Mittal [21] developed a model for deteriorating items with imperfect quality and also an assumption has been made that screening rate is more than demand. Jaber et al. [22] reviewed the model of Salameh and Jaber(2000) and elongate by making an assumption that shipment is coming from a distant supplier and thus it is not feasible to imperfect items with an additional order to the same supplier. Mittal et al. [23] discussed about method for redesigning the ordering policy by incorporating the cross - selling effect and also compares ordering policy for imperfect items developed by applying rules derived from apriori algorithm. Yadav et al. [24] provided a supply chain model which is being used to provide the interaction and democracy of the participants in the supply chain, the buyer and seller, is pitched by non-cooperative and cooperative game theoretical approaches. Mittal et al. [25] discussed about the impact of emissions on ordering policy of retailer for items which are deteriorated under permissible delay in payment where price and demand varies with passage of time. Kumar et al. [26] provided that when a machine shifts from in-control to out of control state, defective items are produced then reworking is performed to make these items perfect. Jayaswal et al. [27] discussed a fiscal construction feature model for imperfect quality items with trade credit policy is analyzed under the effects of learning.

Many researchers have worked on reducing carbon emission including imperfect items. Nobil et al. [28] proposed a model to calculate optimal reorder point for inventory model in Salameh and Jaber(2000) by which appropriate timing of an order can be determined. Sarkar et al. [29] developed a three - echelon sustainable supply chain model with a single - supplier, single manufacture and multiple retailer is considered. Also, control the carbon emission and reduce the imperfect items to maintain the sustainability. Daryanto et al. [30] considered EOQ model with carbon emissions from transportation and warehouse operations. Furthermore, include imperfect items and complete backordering is assumed.

2. MOTIVATION AND CONTRIBUTION

Carbon emission reduction is requisite all over the world. In this paper, KKT(Karush-Kuhn-Tucker) has been implemented to solve the non-linear constraint equation and try to reduce the total cost and carbon emission using government regulations i.e, carbon cap and carbon tax. Also, there will be an increment in order quantity and shortages whenever the imperfect items increases which notably impact the total expected profit.

3. NOTATIONS AND ASSUMPTIONS

Assumptions -

1. Demand rate is considered constant throughout the model.
2. Shortages are allowed and completely backlogged.
3. Lead time is constant and known.
4. Instantaneous replenishment is considered.
5. Emission through backordering is not considered in this model.
6. Each inventory contains defective items with percentage i with probability density function $P(i)$ is known.
7. Imperfect items has been sold as a single batch with discount at price.

Notations -

Q - order quantity (per cycle)

k - unit variable cost ($ per unit)

F - fixed cost ($ per unit)

i - percentage of defective items in Q

$P(i)$ - probability density function of i

x - screening rate, $x > D$

d - unit screening cost($ per unit)

T - cycle length

E - Expected value

h - holding cost ($ per unit)

B - shortage Quantity(per cycle)

b - shortage cost($ per unit per year)

$\hat{F}$ - emission associated with ordering (per unit)

$\hat{h}$ - emission associated with inventory holding (per unit)

$\hat{k}$ - emission associated with production/purchasing (per unit)

p_e - penalty for per unit carbon emitted

D - demand per year.

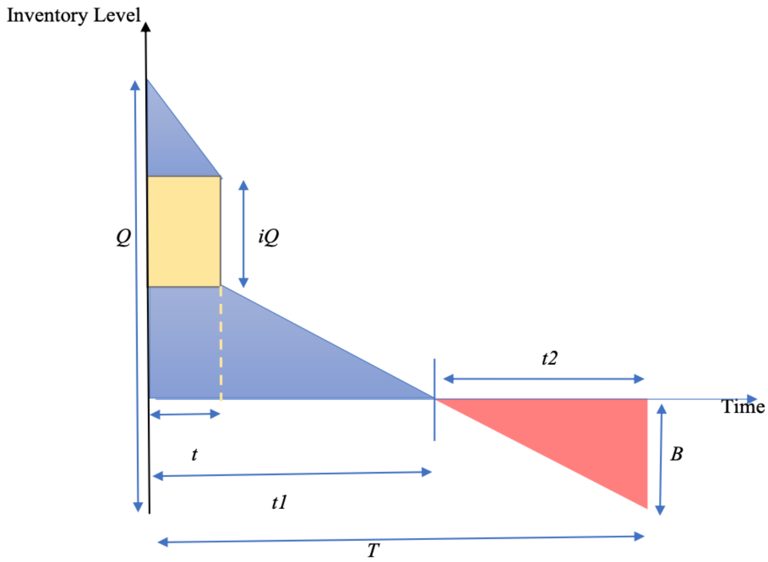

Figure 1: Inventory system with complete backordering.

4. PROBLEM DEFINITION AND MATHEMATICAL MODELLING

A model with imperfect items has been studied with carbon emission impact. There are many different ways for pricing of carbon. One of them is to impose a financial penalty(tax) per unit of carbon emitted when there is constrained carbon cap C on emission. Therefore, in this model, it is assumed that there is 'i' percentage of defective items in inventory delivered with uniform probability density function $f(i)$ and inventory screening process is done with the fixed rate 'x'. Figure 1 depicts that backordering is permissible and Q is the order quantity which includes perfect, imperfect items and shortages, where t is the screening time of Q unit ordered per cycle. Additionally, carbon tax is imposed with a penalty '$p_e > 0$' per unit carbon emitted.

Thus, the total cost during a cycle is

$$TC(Q,B) = F + kQ + dQ + h\left[\frac{((1-i)Q-B)^2}{2D} + \frac{iQ^2}{x}\right] + \frac{1}{2}\frac{bB^2}{D} \tag{1}$$

Total emission per cycle

$$TE(Q,B) = \hat{F} + \hat{k}Q + \hat{h}\left[\frac{((1-i)Q-B)^2}{2D} + \frac{iQ^2}{x}\right] \tag{2}$$

Our intention is to minimize the Expected total cost per unit time where Expected total emission per unit time is restricted to carbon cap C. Also, Expected total cost will be minimized when there is penalty p_e per unit carbon emitted.

4.1. Carbon Cap

In cap policy, there is a certain cap provided by the government to reduce the carbon emission. Here, the Expected total cost is defined as,

$$E[TC] = F + kQ + dQ + h\left[\frac{((E[1-i])Q-B)^2}{2D} + \frac{E[i]Q^2}{x}\right] + \frac{1}{2}\frac{bB^2}{D} \tag{3}$$

Therefore, Expected total cost per unit time is

$$E[TCU] = \frac{E[TC]}{E[T]} = \frac{\frac{FD}{Q} + (k+d)D + h\left[\frac{((E[1-i])Q-B)^2}{2Q} + \frac{E[i]QD}{x}\right] + \frac{1}{2}\frac{bB^2}{Q}}{1-E[i]} \tag{4}$$

Where,

$$E[T] = \frac{Q}{D}(1-E[i])$$

The Expected total emission is,

$$E[TEU] = \frac{E[TE]}{E[T]} = \frac{\frac{\hat{F}D}{Q} + \hat{k}D + \hat{h}\left[\frac{((E[1-i])Q-B)^2)}{2Q} + \frac{E[i]QD}{x}\right]}{1-E[i]} \tag{5}$$

If there is no shortages that is, $B = 0$ then Eq.(4) will provide the total cost for imperfect quality items and value of Expected total cost will be minimized when $Q_c^* = \sqrt{\frac{FD}{h(E[1-i])^2/2+E[i]D/x)}}$ and minimized Expected total emission is

$$\frac{1}{1-E[i]}\left[\sqrt{2\hat{F}D\hat{h}\left[(E[1-i])^2 + \frac{E[i]D}{x}\right]} + \hat{k}D\right]$$

when $Q_e^* = \sqrt{\frac{\hat{F}D}{\hat{h}(E[1-i])^2/2+E[i]D/x)}}$. Therefore, formally the problem can be stated as -

Minimize Total Expected cost per unit time when Expected total emission is less than or equal to C and the solution can be attained when

$$C \geq \frac{1}{1-E[i]}\left[\sqrt{2\hat{F}D\hat{h}\left[(E[1-i])^2 + \frac{E[i]D}{x}\right]} + \hat{k}D\right]$$

and the feasible region consists of all pairs $(Q, 0)$ such that $Q_1 \leq Q \leq Q_2$, where

$$Q_2, Q_1 = \frac{\hat{C} \pm \sqrt{\hat{C}^2 - 4\hat{F}\hat{h}\left[\frac{(E[1-i])^2}{2} + \frac{E[i]D}{x}\right]}}{\hat{h}\left[\frac{(E[1-i])^2}{2} + \frac{E[i]D}{x}\right]}$$

Q_1 and Q_2 can be find out when Expected total emission is equal to C.

In Theorem 1, optimal solution will be find out using five cases.

Theorem 1. *In cap policy, optimal solution can be find out using the following cases-*

$$(Q^*, B^*) = \begin{cases} (Q_c^*, 0) & \text{if } Q_1 \leq Q_c^* \leq Q_2 \\ (Q_1, 0) & \text{if } Q_c^* < Q_1 < \sqrt{((Q_c^*)^2 + (Q_e^*)^2)/2} \\ (Q_2, 0) & \text{if } \sqrt{((Q_c^*)^2 + (Q_e^*)^2)/2} < Q_2 < Q_c^* \\ (Q_3, B_3) & \text{if } Q_{B_c}^* < Q_3 < Q_{B_e}^* \\ (Q_4, B_4) & \text{if } Q_{B_e}^* < Q_4 < Q_{B_c}^* \\ (Q_5, B_5) & \text{Otherwise} \end{cases} \tag{6}$$

where $Q_c^* = \sqrt{\frac{FD}{hM}}$, $Q_e^* = \sqrt{\frac{\hat{F}D}{\hat{h}M}}$ *and* Q_3

$$= \frac{(C(1-E[i]) - \hat{k}D + B\hat{h}(E[1-i])) - \sqrt{(C(1-E[i]) - \hat{k}D + B\hat{h}(E[1-i]))^2 - 4\hat{h}M\left(\hat{F}D + \frac{\hat{h}B^2}{2}\right)}}{2\hat{h}M},$$

$$Q_4 = \frac{(C(1-E[i]) - \hat{k}D + B\hat{h}(E[1-i])) + \sqrt{(C(1-E[i]) - \hat{k}D + B\hat{h}(E[1-i]))^2 - 4\hat{h}M\left(\hat{F}D + \frac{\hat{h}B^2}{2}\right)}}{2\hat{h}M}.$$

Proof. (See Appendix) □

Here, Minimum value of expected total emission is at Q_e^* and maximum at Q_5 and B_5.

4.2. Carbon tax

In tax policy, there is a process to foist a penalty(tax) for per unit carbon emitted. Let p_e be the penalty with $p_e > 0$. Therefore, total cost by applying penalty per unit carbon emitted

$$TC_{p_e} = TC + p_e TE$$

Now, the Expected total cost is defined as,

$$E[TC_{p_e}] = F + kQ + dQ + h\left[\frac{((E[1-i])Q - B)T}{2} + \frac{E[i]Q^2}{x}\right] + \frac{1}{2}\frac{bB^2}{D} + p_e\Bigg[F + \hat{k}Q + \hat{h}\left(\frac{(E[1-i])Q - B))T}{2} + \frac{E[i]Q^2}{x}\right)\Bigg]$$

(7)

Therefore Expected total cost per unit time is

$$ETC_{p_e}U = \frac{E[TC_{p_e}]}{E[T]} = \frac{F + kQ + dQ + h\left[\frac{(E[1-i]))Q-B)^2)T}{2} + \frac{E[i]Q^2}{x}\right] + \frac{1}{2}\frac{bB^2}{D}}{E[T]} +$$

$$p_e\left[\frac{F + \hat{k}Q + \hat{h}\left(\frac{(E[1-i]))Q-B)^2)T}{2} + \frac{E[i]Q^2}{x}\right)}{E[T]}\right] \tag{8}$$

Where,

$$E[T] = (1 - E[i])\frac{Q}{D}$$

Thus,

$$ETC_{p_e}U = \frac{E[TC_{p_e}]}{E[T]} = \frac{\frac{FD}{Q} + (k+d)D + h\left[\frac{(E[1-i]))Q-B)^2)}{2} + \frac{E[i]QD}{x}\right] + \frac{1}{2}\frac{bB^2}{Q}}{1 - E[i]} +$$

$$\frac{\frac{\hat{F}D}{Q} + \hat{k}D + \hat{h}\left[\frac{(E[1-i]))Q-B)^2)}{2} + \frac{E[i]QD}{x}\right]}{1 - E[i]} \tag{9}$$

Thus, by Theorem 2 optimal solution of the problem can be find out.

Theorem 2. *An optimal solution for the above stated problem is :*

$$(Q^{**}, G^{**}) = \left(\sqrt{\frac{F + p_e\hat{F}(1+p_e)(B^2/2)D}{(h + p_e\hat{h})M}}, \frac{E[1-i](1+p_e)Q}{(1+p_e+b)}\right)$$

with $Q \geq 0$, $B \geq 0$

5. NUMERICAL SOLUTIONS

Here, numerical study to further check the impact of cap policy on annual expected cost and emission will be presented. Parameters in this numerical are taken from Chen et al. [9]. Considering two set of cases where, $D = 600$units/year, $i = 0.02$, $x = 175200$unit/year, $d = \$0.5$/unit, $p_e = \$8$/unit of carbon emitted, $b = \$2$/unit/year will remain same, therefore the cases are : i) $\frac{F}{h} > \frac{\hat{F}}{\hat{h}}$ ii) $\frac{F}{h} < \frac{\hat{F}}{\hat{h}}$

Case (i). $F = \$120$ /cycle, $h = \$4$/unit/year, $\hat{F} = 10$g/unit, $\hat{h} = 2$ /unit/year, $k = \$5$/unit,$\hat{k} = 1$g/unit, Since, it is known that percentage defective random variable i is uniformly distributed and can have any value within the range $[\gamma, \delta]$ where $\gamma = 0$ and $\delta = 0.04$.
Probability density function for i is

$$P(i) = \begin{cases} 25, & 0 \leq i \leq 0.04 \\ 0, & \text{otherwise} \end{cases}$$

Therefore, the minimized expected total cost $ETCU = \$3805.62$ /year when optimal order quantity for cost is 335.269 unit/year and backorder quantity is 219.042 unit/year and the emission $ETEU = 667.06$ g/year.

Case (ii). $F = \$10$ /cycle, $h = \$2$/unit/year, $\hat{F} = 120$g/unit, $\hat{h} = 4$ /unit/year, $k = \$1$/unit, $\hat{k} = 5$g/unit In this case, the minimized expected total cost $ETCU = \$1027.93$ /year when optimal order quantity for cost is 111.764 unit/year and backorder quantity is 54.7644 unit/year and thes emission $ETEU = 3773.38$g/year From both the cases, increasing of order quantity will decrease the emission and there will be increase of emission if the order quantity decreases.

6. SENSITIVE ANALYSIS

To analyze the impact of parameters h, k, d and b on Expected total cost, the sensitive analysis has been performed using values $D = 600$ units/year, $F = \$120$ /cycle, $h = \$2$/unit/year, $\hat{F} = 2$g/unit, $\hat{h} = 3$ /unit/year, $k = \$5$/unit, $\hat{k} = 1$g/unit. Impact of these parameters is shown in Figure 2.

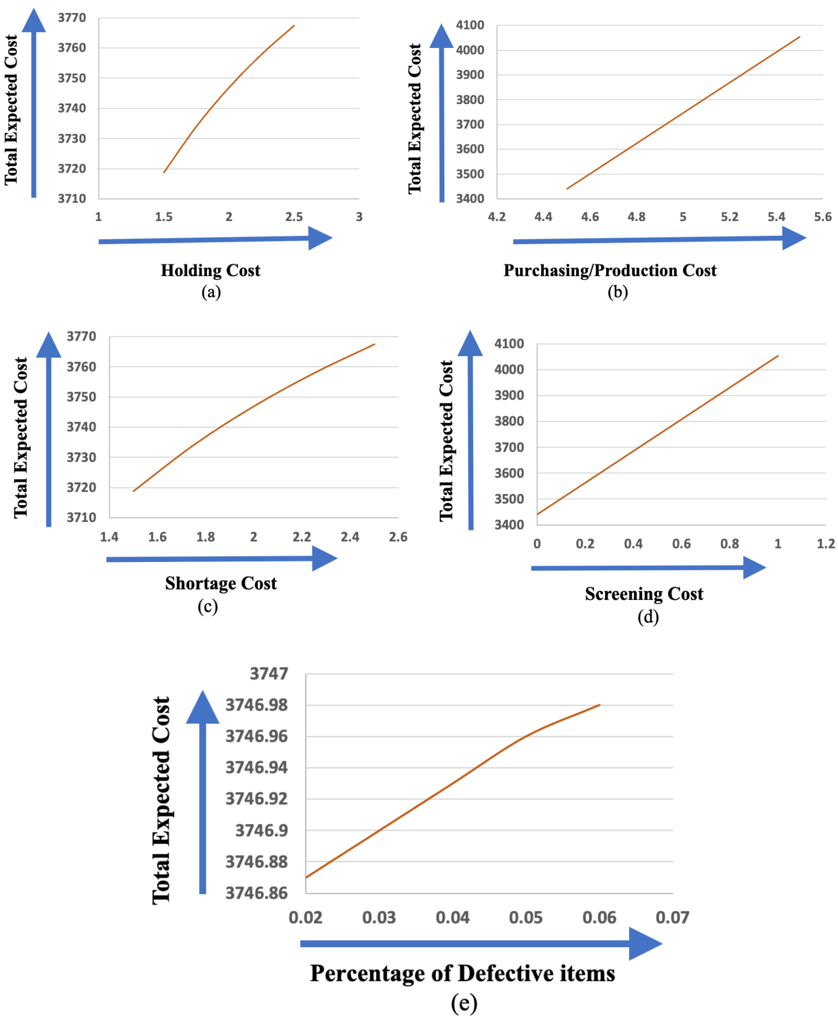

Figure 2: Effect of change of parameters on total expected cost.

Based on the effect of parameters, Figure 2 represents -

1. In Figure 2(a), we studied that when the holding cost increases it will increase the total expected cost.
2. From Figure 2(b), it is analyzed that screening cost effects the total expected cost as it increases whenever screening cost increase.
3. From Figure 2(c), it is seen that there will be an increment in total expected cost when the set up cost increases.
4. From Figure 2(d), it is studied that if backordering cost increase then the total expected cost will increase.

5. In Figure 2(e), it is seen that total expected cost will increase whenever there will be an increment in percentage of defective items.

7. CONCLUSION

In this paper, generalized production lot size model with backordering and government regulations has been proposed. Parameters of the emission regulation are assumed to be exogenic. The result reveals when the imperfect quality items increases then order quantity and shortages will also increase which impacted the total expected profit significantly. Also, KKT(Karush-Kuhn Tucker) conditions has been used to find out the result for imperfect items. This paper is limited to only constant demand, it can be further extended to probabilistic demand. For future research, some more realistic assumptions such as green technology, partial backordering and controllable lead time would be more accurate to extend the model.

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sustainable inventory model with controllable carbon emissions in green-warehouse farms," *Journal of Cleaner Production*, vol. 298, p. 126777, 2021.

A. Appendix

A1. Proof of Theorem 1

In carbon cap policy , KKT(Karush-Kuhn-Tucker) conditions has been used to find the optimal solution for emission constraint. Feasible solution exists when there are contraints such that

$$\frac{\frac{\hat{F}D}{Q} + \hat{k}D + \hat{h}\left[\frac{((E[1-i])Q-B)^2}{2Q} + \frac{E[i]QD}{x}\right]}{1-E[i]} < C$$

and $Q, B \geq 0$

By using KKT conditions, there is global optimality when optimality conditions has been used. Therefore,

$$\begin{aligned} \frac{1}{1-E[i]}&\left[\frac{-FD}{Q^2} + h\left(\frac{(E[1-i])^2}{2} - \frac{B^2}{2Q^2} + \frac{E[i]D}{x}\right) - \frac{bB^2}{2Q^2}\right] \\ &+\frac{\lambda_1}{1-E[i]}\left[\frac{(E[1-i])^2}{2} - \frac{B^2}{2Q^2} + \frac{E[i]D}{x}\right] - \mu_1 = 0 \end{aligned} \tag{10}$$

$$\frac{1}{1-E[i]}\left[\frac{-h(Q(E[1-i])-B)}{Q} + \frac{bB}{Q} + \lambda_1\left\{\frac{-\hat{h}(Q(E[1-i])-B)}{Q}\right\}\right] - \mu_2 = 0 \tag{11}$$

$$\lambda_1\left[C - \frac{1}{1-E[i]}\left(\frac{\hat{F}D}{Q} + \hat{k}D + \hat{h}\left\{\frac{((E[1-i])Q-B)^2}{2Q} + \frac{E[i]QD}{x}\right\}\right)\right] \leq 0 \tag{12}$$

$$\mu_1 Q = 0$$

$$\mu_2 B = 0$$

where multipliers λ_1, μ_1 and μ_2 may be greater than or equal to zero. There could be eight possible cases but the feasible solution can be attain using the following three.

Case 1. $\lambda_1 = 0$, $\mu_1 = 0$, $\mu_2 = 0$

If $\lambda_1 = 0$, $\mu_1 = 0$ then Eq.(9) becomes,

$$\frac{-FD}{Q^2} + h\left(\frac{(E[1-i])^2}{2} - \frac{B^2}{2Q^2} + \frac{E[i]D}{x}\right) - \frac{bB^2}{2Q^2} = 0 \tag{13}$$

and since $\mu_2 B = 0$ and $\mu_2 > 0$ then B = 0.

Eq.(12) now becomes

$$\frac{-FD}{Q^2} + h\left(\frac{(E[1-i])^2}{2} + \frac{E[i]D}{x}\right) = 0 \tag{14}$$

Let us consider $M = \dfrac{(E[1-i])^2}{2} + \dfrac{E[i]D}{x}$. Therefore, the order quantity will be

$$Q = \sqrt{\frac{FD}{hM}} = Q_c^* \tag{15}$$

where Q_c^* is the optimal solution for imperfect items.
To get the optimal solution order quantity must satisfy the

$$\frac{1}{1-E[i]}\left[\frac{\hat{F}D}{Q} + \hat{k}D + \hat{h}MQ\right] \leq C$$

Using this equation there will be a global interval $[Q_1, Q_2]$, where

$$Q_1 = \frac{(C(1-E[i]) - \hat{k}D) - \sqrt{(C(1-E[i]) - \hat{k}D)^2 - 4\hat{F}D\hat{h}M}}{2\hat{h}M}$$

$$Q_2 = \frac{(C(1-E[i]) - \hat{k}D) + \sqrt{(C(1-E[i]) - \hat{k}D)^2 - 4\hat{F}D\hat{h}M}}{2\hat{h}M}$$

For a solution to be feasible $(C(1-E[i]) - \hat{k}D)^2 - 4\hat{F}D\hat{h}M \geq 0$ and hence $C \geq \hat{k}D + \sqrt{4\hat{F}D\hat{h}M}$ Thus, if $C \geq \hat{k}D + \sqrt{4\hat{F}D\hat{h}M}$ and $Q_1 \leq Q_c^* \leq Q_2$ then $Q^* = Q_c^*$ and $B = 0$

Case 2. $\lambda_1 > 0$, $\mu_1 = 0$, $\mu_2 > 0$

From Eq.(9) and (10), we have

$$\frac{1}{1-E[i]}\left[\frac{-FD}{Q^2} + h\left(M - \frac{B^2}{2Q^2}\right) - \frac{bB^2}{2Q^2}\right] + \frac{\lambda_1}{1-E[i]}\left[\frac{-\hat{F}D}{Q^2} + \hat{h}\left(M - \frac{B^2}{2Q^2}\right)\right] = 0 \tag{16}$$

and

$$\frac{1}{1-E[i]}\left[\frac{-h(Q(E[1-i]) - B)}{Q} + \frac{bB}{Q} + \lambda_1\left\{\frac{-\hat{h}(Q(E[1-i]) - B)}{Q}\right\}\right] - \mu_2 = 0 \tag{17}$$

Since $\mu_2 > 0$ then B = 0. Therefore, the above equalities becomes

$$\frac{1}{1-E[i]}\left[\frac{-FD}{Q^2} + hM\right] + \frac{\lambda_1}{1-E[i]}\left[\frac{-\hat{F}D}{Q^2} + \hat{h}M\right] = 0 \tag{18}$$

Also, $\lambda_1 > 0$ then from Eq.(11) we have

$$C - \frac{1}{1-E[i]}\left(\frac{\hat{F}D}{Q} + \hat{k}D + \hat{h}MQ\right) = 0 \tag{19}$$

Q_1 and Q_2 satisfy the above equality. Thus, there must have $C \geq \hat{k}D + \sqrt{4\hat{F}D\hat{h}M}$ to get the feasible solution. Further, let us consider two cases as follows :

Case 2.1. $C = \hat{k}D + \sqrt{4\hat{F}D\hat{h}M}$

In this case, $Q_1 = Q_2 = \sqrt{\dfrac{\hat{F}D}{\hat{h}M}} = Q_e^*$ and also Eq.(17) can be rewritten as

$$\lambda_1 = \frac{\dfrac{FD}{Q^2} - hM}{-\dfrac{\hat{F}D}{Q^2} + \hat{h}M} \tag{20}$$

This equation exists for any positive value of λ_1 and $\dfrac{F}{h} = \dfrac{\hat{F}}{\hat{h}}$ and since $\lambda_1 > 0$ and $\mu_2 > 0$ then by using Eq.(16), $\lambda_1 \leq \frac{h}{\hat{h}}$. Thus if $\frac{F}{h} = \frac{\hat{F}}{\hat{h}}$ then $Q^* = Q_c^*$ and $B^* = 0$.

Case 2.2. $C > \hat{k}D + \sqrt{4\hat{F}D\hat{h}M}$

In this case, $Q_1 \neq Q_2$. Then either $Q = Q_1$ or $Q = Q_2$ to get the feasible solution. Since $\lambda_1 > 0$, $B = 0$ then from Eq.(17), it obtained

$$\lambda_1 = \frac{FD - hMQ^2}{-\hat{F}D + \hat{h}MQ^2}$$

then to get optimality, we must have

$$0 < \frac{FD - hMQ^2}{-\hat{F}D + \hat{h}MQ^2} < \frac{h}{\hat{h}} \tag{21}$$

From the above inequality there are two possibilities that is either $FD - hMQ^2 > 0$ and $-\hat{F}D + \hat{h}MQ^2 > 0$ or $FD - hMQ^2 < 0$ and $-\hat{F}D + \hat{h}MQ^2 < 0$.
Thus, let us prove first that both the numerator and denomerator are less than zero.
Since, we already know that for optimality $C > \hat{k}D + \sqrt{4\hat{F}D\hat{h}M}$. It can be rewritten as

$$2(C - \hat{k}D)^2 - 8\hat{F}D\hat{h}M > 0$$

$$2(C - \hat{k}D)^2 - 2(C - \hat{k}D)\sqrt{C - \hat{k}D)^2 - 4\hat{F}D\hat{h}M} - 8\hat{F}D\hat{h}M < 0$$

$$\frac{((C - \hat{k}D) - \sqrt{(C - \hat{k}D)^2 - 4\hat{F}D\hat{h}M})^2}{2\hat{h}M} - 2\hat{F}D < 0$$

$$\left(\frac{(C-\hat{k}D)-\sqrt{(C-\hat{k}D)^2-4\hat{F}D\hat{h}M}}{2\hat{h}M}\right)^2 2\hat{h}M-2\hat{F}D<0$$

$$-\hat{F}D+\hat{h}MQ_1^2<0$$

Therefore, according to Eq.(20), we must have $FD-hMQ_1^2<0$ and $0<\dfrac{FD-hMQ_1^2}{-\hat{F}D+\hat{h}MQ_1^2}<\dfrac{h}{\hat{h}}$. By solving these two equations together, the result can be formulated as $Q_1>\sqrt{\dfrac{FD}{hM}}=Q_c^*$ and $Q_1<\sqrt{\frac{(Q_c^*)^2+(Q_e^*)^2}{2}}$, then $Q^*=Q_1$ and $B^*=0$.

In a similar manner, we can show that $FD-hMQ_2^2>0$, $-\hat{F}D+\hat{h}MQ_2^2>0$ and $\dfrac{FD-hMQ^2}{-\hat{F}D+\hat{h}MQ^2}<\dfrac{h}{\hat{h}}$. After formulating the above results, the result can be shown as $Q_2<\sqrt{\dfrac{FD}{hM}}=Q_c^*$ and $Q_2>\sqrt{\frac{(Q_c^*)^2+(Q_e^*)^2}{2}}$, then $Q^*=Q_2$ and $B^*=0$.

Case 3. $\lambda_1>0,\mu_1=0,\mu_2=0$
Since $\mu_1=0$ and $\mu_2=0$ then Eq.(9) and (10) can be written as

$$\frac{1}{1-E[i]}\left[\frac{-FD}{Q^2}+h\left(M-\frac{B^2}{2Q^2}\right)-\frac{bB^2}{2Q^2}\right]+\frac{\lambda_1}{1-E[i]}\left[\frac{-\hat{F}D}{Q^2}+\hat{h}\left(M-\frac{B^2}{2Q^2}\right)\right]=0 \quad (22)$$

$$\frac{1}{1-E[i]}\left[\frac{-h(Q(E[1-i])-B)}{Q}+\frac{bB}{Q}+\lambda_1\left\{\frac{-\hat{h}(Q(E[1-i])-B)}{Q}\right\}\right]=0 \quad (23)$$

Now, for $\lambda_1>0$ we rewrite the Eq.(11) as

$$C-\frac{1}{1-E[i]}\left(\frac{\hat{F}D}{Q}+\hat{k}D+\hat{h}\left\{\frac{((E[1-i])Q-B)^2}{2Q}+\frac{E[i]QD}{x}\right\}\right)=0 \quad (24)$$

By evaluating the above equation, we obtain

$$Q_3=\frac{(C(1-E[i])-\hat{k}D+B\hat{h}(E[1-i]))-\sqrt{(C(1-E[i])-\hat{k}D+B\hat{h}(E[1-i]))^2-4\hat{h}M\left(\hat{F}D+\frac{\hat{h}B^2}{2}\right)}}{2\hat{h}M}$$

$$Q_4 = \frac{(C(1 - E[i]) - \hat{k}D + B\hat{h}(E[1 - i])) + \sqrt{(C(1 - E[i]) - \hat{k}D + B\hat{h}(E[1 - i]))^2 - 4\hat{h}M\left(\hat{F}D + \frac{\hat{h}B^2}{2}\right)}}{2\hat{h}M}$$

Here, Q_3 , Q_4 exist only if $(C(1-E[i])-\hat{k}D+B\hat{h}(E[1-i]))^2 \geq 4\hat{h}M\left(\hat{F}D + \frac{\hat{h}B^2}{2}\right)$.
Let us consider two cases as follows:

Case 3.1. $(C(1 - E[i]) - \hat{k}D + B\hat{h}(E[1 - i]))^2 = 4\hat{h}M\left(\hat{F}D + \frac{\hat{h}B^2}{2}\right)$

From this equality, $Q_3(B) = Q_4(B) = \sqrt{\frac{(\hat{F}D + \hat{h}B^2/2)}{\hat{h}M}} = Q^*_{B_e} = Q_5$, where $Q^*_{B_e}$ is an optimal solution for emission when shortages are there. We should have from Eq.(22)

$$B < \frac{hQE[1 - i]}{h + b}$$

When $Q = Q^*$ then Eq.(21) holds for any positive value of λ_1 as long as $\frac{\hat{F}}{\hat{h}} + \frac{bB^2}{2Dh}$

Now, from $C(E[1 - i]) - \hat{k}D + B\hat{h}E[1 - i] = \sqrt{4\hat{h}M\left(\hat{F}D + \frac{\hat{h}B^2}{2}\right)}$

$$(C(E[1 - i]) - \hat{k}D + B\hat{h}E[1 - i])^2 = 4\hat{h}M\left(\hat{F}D + \frac{\hat{h}B^2}{2}\right)$$

$$C(E[1 - i]) - \hat{k}D + B\hat{h}(E[1 - i]) = 2\hat{h}M\sqrt{\frac{\left(\hat{F}D + \frac{\hat{h}B^2}{2}\right)}{\hat{h}M}}$$

$$C(E[1 - i]) - \hat{k}D + B\hat{h}(E[1 - i]) = 2\hat{h}MQ^*_{B_e}$$

$$B_5 = \frac{2\hat{h}MQ^*_{B_e} - C(E[1 - i]) + \hat{k}D}{\hat{h}(E[1 - i])}$$

Case 3.2. $(C(1 - E[i]) - \hat{k}D + B\hat{h}(E[1 - i]))^2 > 4\hat{h}M\left(\hat{F}D + \frac{\hat{h}B^2}{2}\right)$

In this case, $Q_3(B) \neq Q_4(B)$. From Eq.(22), $\lambda_1 = \frac{2FD - h(2MQ^2 - B^2) + bB^2}{-2\hat{F}D + \hat{h}(2MQ^2 - B^2)}$ and since, $\lambda_1 > 0$.
We can show that $-2\hat{F}D + \hat{h}(2MQ^2 - B^2) < 0$ and for optimal solution it must have,

$$2FD - h(2MQ^2 - B^2) + bB^2 < 0.$$

From $2FD - h(2MQ^2 - B^2) + bB^2 < 0$, $Q^*_{B_c} < Q_3 < Q^*_{B_e}$, and from Eq.(23) ,

$$B = \frac{b}{3a1} - \frac{(2^{1/3}(-a2^2 + 3a1a3))}{3a1(2a2^3 - 9a1a2a3 + 27a1^2a4 + \sqrt{4(-a2^2 + 3a1a3)^3 + (2a2^3 - 9a1a2a3 + 27a1^2a4)^2})^{1/3}} +$$

$$\frac{(2a2^3 - 9a1a2a3 + 27a1^2a4 + \sqrt{4(-a2^2 + 3a1a3)^3 + (2a2^3 - 9a1a2a3 + 27a1^2a4)^2})^{1/3}}{3(2^{1/3})a1}$$

where $a1 = b\hat{h}$, $a2 = b\hat{h}Q_3E[1-i]$, $a3 = -\hat{F}Dh + 2FD\hat{h}b$ and $a4 = (-\hat{F}Dh + 2FD\hat{h})Q_3E[1-i]$.

Similarly, it can be shown that $-2\hat{F}D + \hat{h}(2MQ^2 - B^2) > 0$ and therefore, $2FD - h(2MQ^2 - B^2) + bB^2 > 0$. From these two conditions we have,$Q^*_{B_e} < Q_4 < Q^*_{B_c}$ and

$$B = \frac{b}{3a1} - \frac{(2^{1/3}(-a2^2 + 3a1a3))}{3a1(2a2^3 - 9a1a2a3 + 27a1^2a4 + \sqrt{4(-a2^2 + 3a1a3)^3 + (2a2^3 - 9a1a2a3 + 27a1^2a4)^2})^{1/3}} +$$

$$\frac{(2a2^3 - 9a1a2a3 + 27a1^2a4 + \sqrt{4(-a2^2 + 3a1a3)^3 + (2a2^3 - 9a1a2a3 + 27a1^2a4)^2})^{1/3}}{3(2^{1/3})a1}$$

where, $a1 = b\hat{h}$, $a2 = b\hat{h}Q_4E[1-i]$, $a3 = -\hat{F}Dh + 2FD\hat{h}b$ and $a4 = (-\hat{F}Dh + 2FD\hat{h})Q_4E[1-i]$
and $Q^*_{B_c} = \sqrt{\dfrac{2FD + (h+b)B^2}{2Mh}}$ and $Q^*_{B_e} = \sqrt{\dfrac{2\hat{F}D + \hat{h}B^2}{2M\hat{h}}}$.

A2. Proof of Theorem 2

$$ETCU = \frac{E[TC_{p_e}(Q,B)]}{E[T]} = \frac{1}{1-E[i]}\Bigg[(F + p_e\hat{F})\left(\frac{D}{Q}\right) + (k + p_e\hat{k})D + dD$$
$$+(h + p_e\hat{h})\left(\frac{(E[1-i]Q - B)^2}{2Q} + \frac{E[i]QD}{x}\right) + \frac{1bB^2}{2Q}\Bigg] \tag{25}$$

Consider, $G1 = F + p_e\hat{F}$, $G2 = k + p_e\hat{k}$, $G3 = h + p_e\hat{h}$

Therefore,

$$ETCU = \frac{E[TC_{p_e}(Q,B)]}{E[T]} = \frac{1}{1-E[i]}\left[G1\left(\frac{D}{Q}\right) + G2D + dD + G3\left(\frac{(E[1-i]Q - B)^2}{2Q} + \frac{E[i]QD}{x}\right) + \frac{1bB^2}{2Q}\right]$$

$$\frac{\partial ETCU}{\partial Q} = \frac{1}{1-E[i]}\left[-G1\left(\frac{D}{Q^2}\right) + G3\left(\frac{(E[1-i])^2}{2} - \frac{B^2}{2Q^2} + \frac{E[i]D}{x}\right) - \frac{1bB^2}{2Q^2}\right]$$

$$\frac{\partial^2 ETCU}{\partial Q^2} = \frac{1}{Q^3(1-E[i])}\left[2G1D + G3B^2 + bB^2\right]$$

$$\frac{\partial ETCU}{\partial B} = \frac{1}{1-E[i]}\left[G3\left(\frac{(-2(E[1-i]) - B}{2Q}) + \frac{bB}{Q})\right]\right.$$

$$\frac{\partial^2 ETCU}{\partial B^2} = \frac{1}{Q(1-E[i])}(G3 + b)$$

$$\frac{\partial ETCU}{\partial Q \partial B} = \frac{-B}{(1-E[i])Q^2}[G3 + b]$$

$$\left(\frac{\partial ETCU}{\partial Q \partial B}\right)^2 = \frac{B^2}{(1-E[i])^2 Q^4}[G3 + b]^2$$

$$\left(\frac{\partial^2 ETCU}{\partial B^2}\right)\left(\frac{\partial^2 ETCU}{\partial Q^2}\right) = \left(\frac{1}{Q(1-E[i])}(G3{+}b))(\frac{1}{Q^3(1-E[i])}\left[2G1D + G3B^2 + bB^2\right]\right)$$

$$\left(\frac{\partial^2 ETCU}{\partial B^2}\right)\left(\frac{\partial^2 ETCU}{\partial Q^2}\right) - \left(\frac{\partial ETCU}{\partial Q \partial B}\right)^2 = \frac{1}{(1-E[i])^2 Q^4}[2G1D(G3 + b)]$$