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ACHIEVING OPTIMAL DESIGN OF THE PRODUCTION LINE WITH OBTAINABLE RESOURCE CAPACITY

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Abstract: The Maximal Profit Model for reaching an optimal design of the production line undergoing the limitations of obtainable resources is presented in this paper. This model is treated as an integer programming problem, and an efficient step-by-step algorithm to solve this problem is also constructed. In addition, it is discussed that the operation cost of a machine does not include idle and breakdown situations while the maintenance cost for a broken machine should be considered. This study offers a better tool for achieving the optimal design of a flexible production line and reveals the special applicability of the shortest path in production line design.

Keywords: Maximal profit model, integer programming, obtainable resources.

1. INTRODUCTION

A good production line is designed to be an efficient and profit-enhancement way for manufacturing products. It can be applied to both manual and automated manufacturing workstations. The advanced development of flexible machines provides more options for the layout of the production line. In practice, a firm already has a series of production stages for a given product before manufacturing. A flexible machine [2,3] is designed to combine two or more production stages into one workstation. Therefore, a flexible machine can perform a sequence of different operations, but a typical machine is merely for a single operation. A production line is generally configured by a sequence of workstations, and each workstation has one machine or more (the same type of machines in parallel). This is shown in Figure 1.

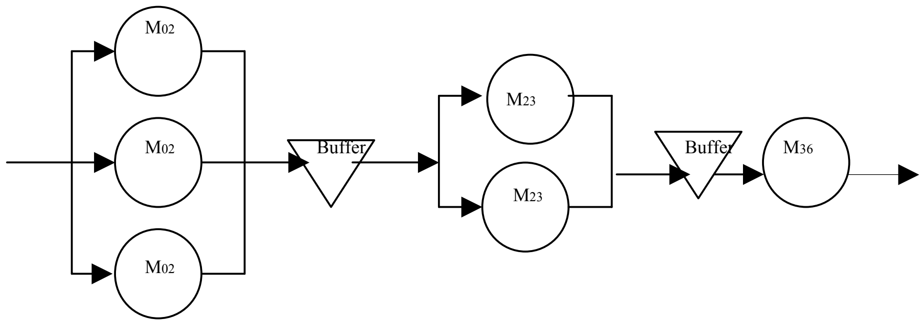

Figure 1: Schematic diagram showing a flexible production line.

The storage space between workstations serves as a buffer. When a buffer on the output side of a workstation is filled, this workstation then temporarily stops. This workstation will not operate until products are finished by the next downstream workstation and the buffer has space again for output. This is the so-called blocking [1, 4, 8]. In addition, a workstation may stop awhile when there is no input available (i.e. the buffer is empty). This is called starvation [1, 4, 8].

Different workstations may need different time for processing a unit product. The maximum processing time of all workstations determines the entire production rate of the whole line. Here, the workstation that has the maximum processing time is called a bottleneck workstation [1]. Thus, the thorough production rate of the line is equal to the output rate of the bottleneck workstation [1, 2, 3, 7]. If a station spends less time than the bottleneck workstation in completing the process, it will be idle for the remainder of its production cycle.

Adar Kalir and Yohanan Arzi (1998) presented the way to search for a profit-maximizing configuration of workstations (both machine types and number have to be determined) along a production line with typical or flexible machines. Buffer space is considered to be infinite in their study. The main work of their study focuses on the unreliable machines and multiple parallel machines (with the same type) which can be used in every workstation. The unreliable machine [2, 3] is defined as the one where machine failure can occur randomly.

Due to different production rates of workstations and unreliable machines, workstations may sometimes be blocked, starved, or broken. Whenever blocking or starvation happens, the related machines should be idle to wait. The objective function proposed by Kalir *et al.* (1998) shows that regardless of whether the machines are idle or broken, the operation cost still needs to be paid. However, from our point of view, while the machines are in idle or break-down situations, the operation cost is negligible. This is because the consumption of input materials does not exist, and electricity fees of idle machines are relatively small when compared to those of the whole system. In addition, the maintenance cost for a break-down machine has to be taken into consideration.

The design of a production line can be composed of both typical and flexible machines. To reach the profit-maximizing layout, the reliability of each machine and the obtainable resource capacity are considered simultaneously. In fact, a production line consisting of flexible machines is very common in industry, and there are many studies dealing with this topic. However, the limitations of the obtainable resources are rarely discussed. The available machine number and types refer to the machines that are not actually operated in a firm. In this study, the maximum available machine number of each type in a firm is regarded as the obtainable resource capacity.

2. ASSUMPTIONS AND NOTATIONS

Before formulating this study, there are several assumptions for the optimal design of a production line described. They are:

1. The production line only makes the same type of product during the manufacturing process and a series of production stages of this product are given.
2. It's an automated production line; every workstation has a specific sequence of production stages and consists of the same type of machines. The configurations of workstations (*i.e.* the corresponding sequence of production stages for each workstation) are not known in advance.
3. In every machine, there is only one part that can be processed at one time.
4. No scrapping parts are considered during the manufacturing process.
5. All products are sold at its given price at once after production.
6. The idle and break-down machines do not charge for operation.
7. While blocking or starvation occurs, the related machine will be idle quickly and automatically; whenever a buffer has more space or input again, the related machine will soon function automatically.
8. The cost of buffer space is ignored because it is far less than the operation and maintenance costs of machines; i.e. the buffer space can be large.

The problem of this study is to determine such a configuration of workstations which maximizes the profit. The following are the notations in this study.

n : the number of production stages.

$N : N = (1,2,...,n)$ sequence consisting of production stages $k,\ k = 1,2,...,n$.

MS : the set of all available machine types.

$ij : 0 \le i < j \le n$, indicating a workstation functioning from production stage $i+1$ to j in sequence. ij is said to be feasible if there exists a machine of type $M_{ij} \in MS$; where M_{ij} is a machine type which performs production stages from $i+1$ to j in sequence.

$F : F = \{ij \mid 0 \le i < j \le n$ and ij is a feasible workstation$\}$.

$t_{ij},\ ij \in F$: the processing time per unit product performed by a single machine of type M_{ij} .

$l_{ij},\ ij \in F$ $l_{ij}, ij \in F$: the maximum available number of machines for machine type M_{ij} offered by the firm.

r_{ij} : the reliability of machine type M_{ij} which is defined by $r_{ij} = \varepsilon_{ij} / (\varepsilon_{ij} + \delta_{ij})$; where ε_{ij} and δ_{ij} are the mean time between failures and the mean time to fix a single machine of type M_{ij} respectively.

$PR_{\min}$: minimum thorough production rate of the production line imposed by a firm.

$PR_{\max}$: maximum thorough production rate of the production line imposed by a firm.

c_{ij} : operation cost (dollar(s) per unit time) of a single machine type M_{ij} .

$$C\text{: } C = \left\{ C_{ij} \,\middle|\, ij \in F \text{ and } C_{ij} = \left[c_{ij} + c_{ij}^{f} \left(\frac{1}{r_{ij}} - 1 \right) \right] t_{ij} \right\}.$$

$$L\text{: } L = \left\{ L_{ij} \,\middle|\, ij \in F \text{ and } L_{ij} = \frac{l_{ij} r_{ij}}{t_{ij}} \right\}.$$

c_{ij}^{f} : maintenance cost (dollar(s) per unit time) of a single machine type M_{ij} . *i.e.* $c_{ij}^{f} \delta_{ij}$ is the mean maintenance fee of a single machine type M_{ij} .

p : contribution per unit product (including operation cost and maintenance cost related to the production line).

Decision Variables:

$y_{ij},\ ij \in F : y_{ij} = 1$ if workstation ij is selected to be in a configuration; otherwise 0.

$x_{ij},\ ij \in F$: number of parallel machines of type M_{ij} .

Z : profit of the production line.

3. MODEL DEVELOPMENT

After introducing the notations and decision variables, the mathematical model can be presented. In order to reach the maximal profit of the production line, the mathematical model, Maximal Profit (MP) Model, is formulated. It is described as follows.

MP Model:

$$\begin{cases} \max\limits_{x_{ij}, y_{ij}} \quad Z = \left[p - \sum\limits_{ij \in F} \left(c_{ij} + c_{ij}^{f} \left(\dfrac{1}{r_{ij}} - 1 \right) \right) t_{ij} y_{ij} \right] \left[\left(\min\limits_{ij \in F} \dfrac{x_{ij} r_{ij}}{t_{ij}} \right) y_{ij} \right] & (1) \\ \text{s.t.} \quad PR_{\min} \le \left(\min\limits_{ij \in F} \dfrac{x_{ij} r_{ij}}{t_{ij}} \right) y_{ij} \le PR_{\max}, \ \forall y_{ij} = 1 & (2) \\ \quad y_{ij} \in \{0,1\},\ ij \in F, \text{ and for each production stage } k \in N, \ \sum\limits_{\substack{ij \\ i+1 \le k \le j}} y_{ij} = 1 & (3) \\ \quad x_{ij},\ ij \in F, \text{are integers and satisfy } y_{ij} \le x_{ij} \le l_{ij} y_{ij}. & (4) \end{cases}$$

The objective function (1) is to search the maximum profit per unit time for a flexible production line; where $\left(\min\limits_{ij \in F} \dfrac{x_{ij} r_{ij}}{t_{ij}} \right) y_{ij}, \forall y_{ij} = 1$ presents the production rate of bottleneck workstation in the production line. Constraint (2) shows the limitations of the thorough production rate of the line imposed by a firm to prevent the change of p by the amount produced. The maximum limit (the predicted upper limit of the demand) is the quantity of products that can be sold. In this study, it is assumed that all production can be sold for the given price within the limitations of the production rate. Constraints (3) ensure that in a feasible configuration each production stage k is associated with only one of the feasible workstations ij for which $i+1 \le k \le j$, i.e. selected workstations include all production stages such that there are no two workstations containing the same production stage. Constraints (4) guarantee that for a selected workstation ij, the number of machines x_{ij} is nonzero integer and less than or equal to its upper bound l_{ij}, while for an unselected workstation, the number of machines is zero.

4. STEP-BY-STEP ALGORITHM

After introducing the MP Model, the step-by-step algorithm for reaching an optimal solution is proposed in this study. In this study, the proposed algorithm starts from Network (N,F,C) which is associated with MP Model in the following way: Each production stage k, $k \in N$, is represented as a node of the network and then a "source" node 0 is added. Nodes i and j are connected by arc (i, j) with weight $C_{ij} \in C$ if the corresponding workstation ij is feasible. Let us mention that, obviously, Network (N,F,C) is connected as for each production stage k there exists a machine type which performs it and, consequently, the network contains arc $(k-1,k)$ for each $k \in N$.

Now each feasible configuration of workstations, defined by constraints (2)-(4), can be treated as a path from source node 0 to sink node n [5, 6] which, together with numbers of machines corresponding to its arcs, represents a feasible path of the problem. Because each feasible path (configuration) has its own bottleneck value,

$\left(\min_{ij \in F} \frac{x_{ij} r_{ij}}{t_{ij}}\right) y_{ij}, \forall y_{ij} = 1$, all feasible paths with passing through the same bottleneck workstation are sorted into the same group. The main idea of the algorithm is to identify a shortest path from a group of paths passing through arc ij (arc ij presents the bottleneck workstation with minimum production rate $\frac{x_{ij} r_{ij}}{t_{ij}}$) in the present network to be a candidate solution; and arc ij is then removed from the present network to form a new one. Then, the next candidate solution is established from that new network by repeating the above procedures. The same process revolves over and over again until there is no feasible path to be identified. In our algorithm, each candidate solution is the best solution from each group, so the optimal solution is obtained from among all these candidate solutions.

Therefore, the step-by-step algorithm for solving MP Model is developed and listed below: First, let $Y = \{y_{ij} \mid ij \in F \text{ and } y_{ij} \text{ meets constraint (3)}\}$, then $\hat{Y} = \{ij \mid y_{ij} \in Y \text{ and } y_{ij} = 1\}$ forms a path (a feasible configuration of workstations for a production line) between nodes 0 and n. Then initialization:

Calculate values C_{ij} and L_{ij} for all $ij \in F$ (see Section 2),

Order all L_{ij} according to non-decreasing order, such that

$$L_{i_1 j_1} \le L_{i_2 j_2} \le \cdots \le L_{i_m j_m}, \text{ where } m = |F|.$$

$\alpha = 1,\ F_1 = F$, Go to Step 0.

Step 0: In Network$(N, F\alpha, C)$, if there exists a path between nodes 0 and n, go to Step 1; otherwise, go to Step 3.

Step 1: If $\begin{cases} L_{i_\alpha j_\alpha} < PR_{\min}, \text{ let } \bar{L}_{i_\alpha j_\alpha} = 0, \text{ go to next step.} \\ PR_{\min} \le L_{i_\alpha j_\alpha} \le PR_{\max}, \text{ let } \bar{L}_{i_\alpha j_\alpha} = L_{i_\alpha j_\alpha}, \text{ go to next step.} \\ L_{i_\alpha j_\alpha} > PR_{\max}, \text{ let } \bar{L}_{i_\alpha j_\alpha} = PR_{\max}, \text{ go to next step} \end{cases}$

where $\bar{L}_{i_\alpha j_\alpha}$ is the thorough production rate (determined by workstation $i_\alpha j_\alpha$) of the production line which meets the constraint (2) of MP Model.

If $\bar{L}_{i_\alpha j_\alpha} = 0$, then $\underset{i_\alpha j_\alpha}{path} = \phi,\ O.V.\left[\underset{i_\alpha j_\alpha}{path}\right] = 0$ and go to Step 2a. ; where $\underset{i_\alpha j_\alpha}{path}$ is a path which passes through arc $i_\alpha j_\alpha$ with the shortest distance between node 0 and node n in Network $(N, F\alpha, C)$ and $O.V.$ means the objective value.

Step 2: Let PATH$(N, F\alpha, C)$ be a set of all paths in Network $(N, F\alpha, C)$ from node 0 to node n which contains arc $(i_\alpha j_\alpha)$.

If PATH$(N, F\alpha, C) = \phi$, then $\underset{i_\alpha j_\alpha}{path} = \phi,\ O.V.\left[\underset{i_\alpha j_\alpha}{path}\right] = 0$ and $\bar{L}_{i_\alpha j_\alpha} = 0$;

Otherwise, it is valid that:

$$\max_{\hat{Y}\in \text{PATH}(N,F_\alpha,C)} (p - \sum_{(i,j)\in\hat{Y}} C_{ij})\bar{L}_{i_\alpha j_\alpha}$$

$$= p\bar{L}_{i_\alpha j_\alpha} - \left[\min_{\hat{Y}\in \text{PATH}(N,F_\alpha,C)} (\sum_{(i,j)\in\hat{Y}} C_{ij})\right]\bar{L}_{i_\alpha j_\alpha}$$

$$= p\bar{L}_{i_\alpha j_\alpha} - \left[\min_{j\le i_\alpha} \sum_{(i,j)\in\hat{Y}} C_{ij} + C_{i_\alpha j_\alpha} + \min_{i\ge j_\alpha} \sum_{(i,j)\in\hat{Y}} C_{ij}\right]\bar{L}_{i_\alpha j_\alpha}$$

$$\equiv O.V.\left[\underset{i_\alpha j_\alpha}{path}\right]$$

Save $O.V.\left[\underset{i_\alpha j_\alpha}{path}\right]$, $\underset{i_\alpha j_\alpha}{path}$, and $\bar{L}_{i_\alpha j_\alpha}$.

Step 2a: Set $\alpha = \alpha + 1$ and remove arc $i_\alpha j_\alpha$ from F_α; return to Step 0.

Step 3: Find $O.V.\left[\underset{i_{\alpha^*} j_{\alpha^*}}{path}\right] = \max_{1\le\beta\le\alpha-1}\left\{O.V.\left[\underset{i_\beta j_\beta}{path}\right]\right\}$, $path^* = \underset{i_{\alpha^*} j_{\alpha^*}}{path}$ and associated production rate $\bar{L}^*$ of the line. $\bar{L}^* = L_{i_{\alpha^*} j_{\alpha^*}}$.

If $O.V.\left[path^*\right] = 0$, it means that there are no feasible solutions of MP Model.

Otherwise, the optimal solutions y_{ij}^*, x_{ij}^* of MP model are equal to:

$y_{ij}^* = 1$ for $(i,j) \in path^*$, otherwise 0

$x_{ij}^* = \left[\dfrac{\bar{L}^* t_{ij}}{r_{ij}}\right]^+$, for $(i,j) \in path^*$, otherwise 0.

$Z^* = O.V.\left[path^*\right]$;

In addition, a numerical example is illustrated in Appendix.

Let us mention that optimal values x_{ij}^* for $(i,j) \in path^*$ and $(i_{\alpha^*}, j_{\alpha^*})$ are not unique. Obviously, x_{ij}^* can be equal to any value to satisfy

$$\left[\frac{\bar{L}^* t_{ij}}{r_{ij}}\right]^+ \le x_{ij}^* \le l_{ij} \ .$$

In Step 3 of our algorithm, x_{ij}^* are defined to be equal to their lower bounds. Actually, the larger the number of production stages is, the more complex the algorithm is.

5. CONCLUSIONS

Designing a production line requires the considerations of operation cost, maintenance cost, and the reliability of each machine type. The obtainable machine number, type, and the production rate of each workstation are also taken into consideration simultaneously. Definitely, it is a complicated and hard-solving issue. However, by means of MP Model, the above issue becomes concrete and solvable.

Compared to the algorithm of Kalir et al. proposed in 1998, all feasible configurations of workstations for a production line need to be determined before using their algorithm, but ours do not. In addition, the special application of the shortest-path problem is proposed to expand its applicability field in this paper. Moreover, two viewpoints, i.e. the idle or breakdown machine wastes no operation cost and the maintenance cost for a breakdown machine needs to be considered, are expressed in this study. In fact, after reaching the optimal solution, the total operation cost and total maintenance cost of the machines in the production line can be computed. The proportion of them functions as an important indicator in the cost analysis. Consequently, the applicability of the MP Model is certainly expanded. In sum, this paper introduces a better and a more efficient way to design a flexible production line.

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APPENDIX: NUMERICAL EXAMPLE

This example considers a product consisting of seven production stages. The obtainable resource capacity l_{ij} and associated information of each machine type are described in Table 1.

Table 1: The data of numerical example

Indices ij	Machine types M_{ij}	Processing time t_{ij} (hrs)	Reliability r_{ij}	Maximum available number of machines l_{ij}	Operation Cost c_{ij} ($/hr)	Maintenance Cost c_{ij}^{f} ($/hr)
01	M_{01}	0.44	0.90	8	20.0	8.5
12	M_{12}	1.10	0.85	8	14.3	8.0
23	M_{23}	0.30	0.90	8	15.0	8.0
34	M_{34}	0.90	0.90	8	21.7	8.6
45	M_{45}	0.50	0.90	8	20.0	8.5
56	M_{56}	0.18	0.95	8	18.3	8.0
67	M_{67}	0.15	0.90	8	18.3	8.0
25	M_{25}	1.00	0.95	6	26.0	10.0
36	M_{36}	1.20	0.90	6	21.0	9.4
46	M_{46}	0.60	0.95	6	23.3	9.6

Let $p = \$80$, $PR_{\min} = 5$, $PR_{\max} = 7$, and all $L_{i_\alpha j_\alpha}$ are calculated and listed below.

$$L_{36} = 4.500 \le L_{25} = 5.700 \le L_{12} = 6.182 \le L_{34} = 8.000 \le L_{46} = 9.500$$
$$\le L_{45} = 14.400 \le L_{01} = 16.364 \le L_{23} = 24.000 \le L_{56} = 42.222 \le L_{67} = 48.000.$$

Network (N, F, C) is shown in Figure 2. Then, initialize $\alpha = 1$, $F = F_1$, go to step 0.

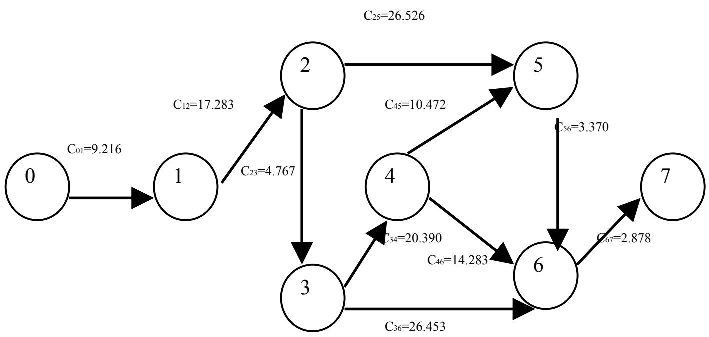

Figure 2: Network (N, F, C); $F = F_1$

I. Compute $O.V.\left[\underset{36}{path}\right]$

Step 0: There are paths between nodes 0 and n in Network(N, F_1, C), go to Step 1.

Step 1: $L_{36} = 4.5 < PR_{\min} = 5$, let $\bar{L}_{36} = 0, \underset{36}{path} = \phi, O.V.\left[\underset{36}{path}\right] = 0$, go to Step 2a.

Step 2a: Remove arc 36 from Network(N, F_1, C),
then, apply $\alpha = 1 + 1 = 2$, return to Step 0.

II. Compute $O.V.\left[\underset{25}{path}\right]$

Step 0: There are paths between nodes 0 and n in Network(N, F_2, C)(shown in Figure 3), go to Step 1.

Step 1: $PR_{\min} = 5 < L_{25} = 5.7 < PR_{\max} = 7$, let $\bar{L}_{25} = 5.7$ and go to Step 2.

Step 2: $\text{PATH}(N, F_2, C) = \{(01, 12, 25, 56, 67)\}$

$$\max_{\hat{Y} \in \text{PATH}(N, F_2, C)} \left(p - \sum_{(i,j) \in \hat{Y}} C_{ij} \right) \bar{L}_{25}$$

$$= 20.727 \times 5.7 = 118.144$$

$$\equiv O.V.\left[\underset{25}{path}\right]$$

Save $O.V.\left[\underset{25}{path}\right] = 118.144, \underset{25}{path} = \{(01, 12, 25, 56, 67)\}$, and $\bar{L}_{25} = 5.7$.

Step 2a: Remove arc 25 from Network(N, F_2, C),
then apply $\alpha = 2 + 1 = 3$, return to Step 0.

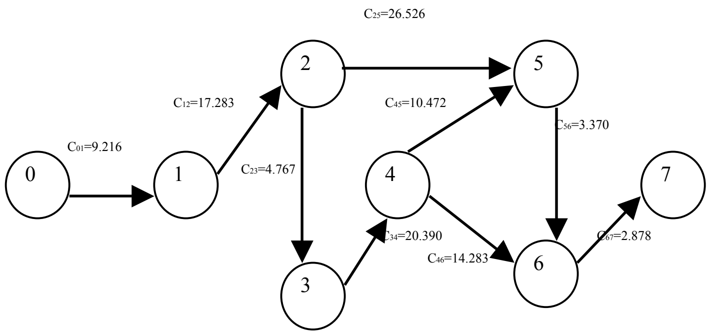

Figure 3: Network (N, F_2, C)

III. Compute $O.V.\left[\underset{12}{path}\right]$

Step0: There are paths between nodes 0 and n in Network(N, F_3, C)(shown in Fig. 4), go to Step 1.

Step 1: $PR_{\min} = 5 < L_{12} = 6.182 < PR_{\max} = 7$, let $\bar{L}_{12} = 6.182$ and go to Step 2.

Step 2: $\text{PATH}(N, F3, C) = \{(01,12,23,34,45,56,67),(01,12,23,34,46,67)\}$

$$\max_{\hat{Y} \in \text{PATH}(N,F3,C)} \left(p - \sum_{(i,j)\in\hat{Y}} Cij \right) \bar{L}_{12}$$

$$= 11.624 \times 6.182 = 71.860$$

$$\equiv O.V.\left[\underset{12}{path}\right]$$

Save $O.V.\left[\underset{12}{path}\right] = 71.860, \underset{12}{path} = \{(01,12,23,34,45,56,67)\}$, and $\bar{L}_{12} = 6.182$.

Step 2a: Remove arc 12 from Network$(N, F3, C)$,

then apply $\alpha = 3+1 = 4$, return to Step 0.

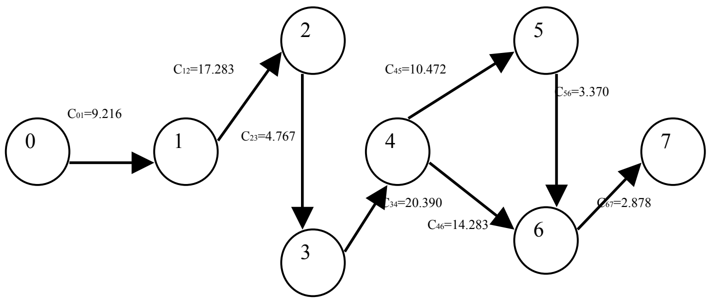

Figure 4: Network (N, F_3, C)

IV. Compute $O.V.\left[\underset{34}{path}\right]$

Step 0: There are no paths between nodes 0 and n in Network$(N, F4, C)$(shown in Fig. 5), stop and go to Step 3.

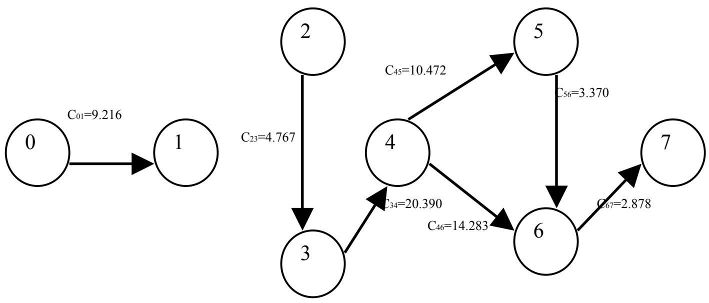

Figure 5: Network (N, F_4, C)

Step 3: Find $O.V.\left[\underset{25}{path}\right] = 118.144, path^* = \{(01,12,25,56,67)\}$, and $\bar{L}^* = 5.7$

$$y_{01}^* = y_{12}^* = y_{25}^* = y_{56}^* = y_{67}^* = 1$$

$$x_{01}^* = 3,\ x_{12}^* = 8,\ x_{25}^* = 6,\ x_{56}^* = 2,\ x_{67}^* = 1.$$

$$Z^* = 118.144$$

From the illustration above, there are five workstations in the optimal production line. The first workstation (with three parallel M_{01} machines) only performs the production stage 1 and the second workstation (with eight M_{12} machines in parallel) performs the production stage 2 only. Production stages 3, 4, and 5 are combined as the third workstation (having six parallel M_{25} machines). Then, production stage 6 is functioning as the fourth workstation (arranging two parallel M_{56} machines). Finally, the last workstation (with one M_{67} machine) only performs the production stage 7. The thorough production rate of the line is 5.70 unit per hour and it is determined by the third workstation. Under such a design, the profit accomplished by the line is 118.144 dollars per hour.