Yugoslav Journal of Operations Research
33 (2023), Number 4, 549–575
DOI: https://doi.org/10.2298/YJOR230115007S

A PROMISING APPROACH FOR DECISION MODELING WITH SINGLE-VALUED NEUTROSOPHIC PROBABILISTIC HESITANT FUZZY DOMBI OPERATORS

Muhammad KAMRAN
Institute of Mathematics, Khwaja Fareed University of Engineering & Information Technology, Rahim Yar Khan 64200, Punjab, Pakistan
kamrankfueit@gmail.com

Shahzaib ASHRAF*
Institute of Mathematics, Khwaja Fareed University of Engineering & Information Technology, Rahim Yar Khan 64200, Punjab, Pakistan
shahzaib.ashraf@kfueit.edu.pk

Muhammad NAEEM
Department of Mathematics, Deanship of Applied Sciences, Umm Al-Qura University, Makkah, Saudi Arabia
mfaridoon@uqu.edu.sa

Received: January 2023 / Accepted: February 2023

Abstract: A combination of the single-valued neutrosophic set (SV-NS) and the probabilistic hesitant fuzzy set is the single-valued neutrosophic probabilistic hesitant fuzzy (SV-NPHF) environment (PHFS). It is intended for some unsatisfactory, ambiguous, and contradictory circumstances in which each element has a number of various values that are brought about by the situation's actuality. The decision-maker can quickly gather and analyze the facts by employing a strategic decision-making technique. On the other hand, uncertainty will be a big part of our daily lives when we are learning. We present a decision-making strategy for the SV-NPHF context to address this data ambiguity. The fundamental operational concepts for SV-NPHF information under Dombi aggregation operators were initially developed on the basis of this study. The SV-NPHF Dombi weighted arithmetic average (SV-NPHFDWAA) operator and SV-NPHF Dombi weighted arithmetic geometric (SV-NPHFDWAG) operators are two SV-NPHF Dombi aggregation Operators that are then examined. Following that, we look into further characterizations of the proposed operators, including idempotency, boundedness, and monotonicity. For the derived operators, we additionally developed the score and accuracy functions. When using SV-NPHF data in a multi attribute decision support system (MADSS), it is necessary to compare the effectiveness of various (AOs) in order to make the best decision. In addition, it is demonstrated how to use symmetry analysis to choose the optimal social media platform for earning and learning in a practical application of SV-NPHFDWAA and SV-NPHFDWAG.

Keywords: Dombi aggregation operators, neutrosophic sets, probabilistic hesitant fuzzy set, decision-making.

MSC: 03E72.

1. INTRODUCTION

There are a lot of uncertainties, imprecisions, and ambiguities in this universe. In actuality, the majority of concepts we come across on a daily basis are less precise and more ambiguous. Managing with trepidation or uncertainty is a significant problem in many fields, including economics, engineering, natural science, medical science, and sociology. Recently, many authors have developed a passion for making their points ambiguous. The complex relationship between clarity and ambiguity has been the focus of centuries of scholarly debate. When promising concept of a fuzzy set (FS) developed, Zadeh [1] was working to address the issue of handling ambiguous and imprecise information within the context of artificial intelligence. In this architecture, the membership function (MF) of each object is made up of a single number that falls between [0, 1]. The fact that it only supports MF and forbids the expression of non-membership function (NMF) is one of its drawbacks. Atanassov [2] designed an intuitionistic fuzzy set to overcome the restrictions imposed by FS (IFS). Using the MF and NMF of an IFS, both of which have values between 0 and 1, you may evaluate the structure of objects. In order to consolidate knowledge that is intuitively confusing, a broad variety of information aggregation have been devised. These methods of aggregation were developed to improve the information's plausibility and accuracy. IF weighted and other operators, as well as other aggregating operators (AOs), were created by Xu [3]. The types of averaging operators that are credited to Xu [4] include the IF weighted, ordered weighted, and hybrid averaging operators, among others. These operators are still very useful today [5]. An interval-valued intuitionistic fuzzy set (IVIFS), first introduced in [6], was further developed from an IFS, which is characterized by its interval MF and interval NMF in [0, 1].

Hesitancy is a natural occurrence in our environment [7]. It's challenging to find one of the viable choices with the same attributes in real life. Professionals are having trouble implementing choices due to the uncertainty and hesitation of the results. Torra and Narukawa [8] proposed the hesitant fuzzy set (HFS) approach to address reluctance, which permits elements to be set with numerous alternative values. HFS can be utilized to tackle a wide range of decision-making (DM) issues [9, 10]. By aggregating operators in group DM, many writers employed HFS to address difficulties [11, 12]. Modified versions of HF hybrid weighted averaging operator, HF hybrid weighted geometric operator, generalized form of quasi-HF hybrid weighted averaging operator, and generalized form of quasi-HF hybrid weighted geometric operator have been found by Liao and Xu [13]. Xu and Zhou introduced a novel idea of probabilistic HF sets (PHFSs) [14, 15]. Researchers investigated the notion of MADM significantly after being motivated by the effectiveness of PHFSs [16]. Both PHFSs and IFSs are incapable of dealing with the uncertain and inconsistent information found in belief systems. The neutrosophic set (NS), a philosophical field and mathematical tool for comprehending the origin, make-up, and range of neutralities, was initially proposed by Smarandache in [39]. The MF $\Theta_F(\xi)$, indeterminacy membership (IMF) $\delta_F(\xi)$ and NMF $\delta_F(\xi)$, where $\Theta_F(\xi)$, $\delta_F(\xi)$ and $\psi_F(\xi)$ are real standard or nonstandard subsets of $]0^-,1^+[$ respectively. Although FS, IFS, and other concepts are conceptually generalized in NS, applying it to actual scientific and practical circumstances will be difficult [19, 20]. A single-valued neutrosophic set (SV-NS), which can deal with inaccurate, ambiguous, and incompatible data issues, was proposed by Wang et al. [44] SV-NS has received a lot of attention from academics because it is a potent universal systematic approach. Ye described the information energy and correlation of SV-NSs in [17, 18]. The application of SV-NSs as a decision-making method was then explored by various authors [21, 22].

Motivation: Dombi [40] introduced the Dombi triangular-norm and Dombi triangular-conorm operations in 1982, defining them as having the preference of variability with parameter operation. For this benefit, Liu et al. [24] devised a multiple attribute group decision-making issue utilizing a Dombi Bonferroni mean operator under the intuitionistic fuzzy data and applied Dombi operations to intuitionistic fuzzy sets [25, 26]. In the single-valued neutrosophic information, Chen and Ye [27] suggested a multiple attribute decision-making issue using Dombi aggregations operations. Dombi operations are extended to neutrosophic cubic sets by Shi et al. [28] and used for travel decision-making issues. In order to build a multiple attribute decision-making (MADM) technique in a linguistic cubic scenario, Lu and Ye [30] first defined a Dombi aggregation operator for linguistic cubic variables. He [31] introduced the Dombi hesitant fuzzy information aggregation operators-based Typhoon disaster assessment method. A few aggregation methods are shown by Jana et al. [32] under picture fuzzy data for determining the various priority of the options throughout the decision-making process. On the basis of conventional arithmetic, geometric operations, and Dombi operations, Jana et al. [33] defined certain bipolar fuzzy Dombi aggregation operators. Wei and Wei [34] proposed some single-valued neutrosophic Dombi prioritised weighted aggregating operators for the aggregation of SVNNs and also investigated the properties of these operators. Many other researchers used Dombi operators for many decision-making problem [35, 36]. They also presented some operations of prioritised aggregation operators and Dombi operations of SVNNs that take into account the prioritized relationship between the SVNNs.

These examples and debates lead us to assume that the SV-NPHFS has a strong reliability to show the doubtful and likely data that appear in real-world problems. We heavily drew inspiration from the aforementioned decision-making issues in various fuzzy aggregation systems to create this study. The major goal of this article is to demonstrate a few SV-NPHF Dombi aggregations, or aggregation operators under neutrosophic data, for evaluating the various priorities of the options throughout the decision-making process. We suggest new aggregation operators for SV-NPHFSs using Dombi t-norm and Dombi t-conorm as a result of the discussion above. The aggregation operators are crucial in the decision-making process because they aggregate the ambiguous data. In light of this, we suggest a number of novel aggregation operators for SV-NPHF information, including Dombi weighted arithmetic average and Dombi weighted arithmetic geometric aggregation operators. The algorithm to handle the decision-making issues based on the suggested Dombi aggregation operators should then be launched. An example utilizing numbers shows the benefits and effectiveness of our suggested method for solving SV-NPHF-based decision-making issues. The decision-maker can choose the grades without being limited by the limitations of the current methods as a result. This structure additionally categorizes the problem by altering the physical significance of reference parameters. We endorse a number of new aggregation operators (AOs), such as the Dombi weighted arithmetic average and Dombi weighted arithmetic geometric aggregation operators under SV-NPHF information, for the following reasons:

- Since it is more versatile than other existing approaches because of the parameter present in it, the Dombi t-norm and t-conorms are utilized to construct aggregation operators.
- Decision-makers have more freedom because to the integrated SV-NS and PHFS concepts.
- In contrast to SV-NPHF Dombi weighted arithmetic average and Dombi weighted arithmetic geometric aggregation operators, SV-NWA and SV-NWG aggregation operators are unable to account for experts' levels of familiarity with the items under examination for first evaluation.
- This article focuses on more sophisticated and advanced data because the Dombi weighted arithmetic average and Dombi weighted arithmetic geometric aggregation operators are straightforward and cover the decision-making method.
- All shortcomings are addressed in the suggested work.

Some consequential endowment of the current study are as follows:

1 Firstly, introduce a concept of SV-neutrosophic probabilistic hesitant fuzzy sets.

2 Also explore the fundamental operational laws for SV-NPHFSs.

3 To design a DM approach for choosing the appropriate social media platform for earning and learning that employs the suggested aggregation operators to aggregate ambiguous information for decision-making difficulties.

4 We define new SV-NPHFDWAA and SV-NPHFDWAG operators.

5 We defined the score function for SV-NPHFSs with Dombi operators.

The structure of this article is as follows. In Part 3, the fundamental ideas behind FS, NS, PHFS, as well as a few fundamental operational laws, are reviewed. We introduce brand-new aggregation operators in Section 4, including SV-NPHFDWAA and SV-NPHFDWAG. In Section 5, a decision-making approach based on the proposed AOs is built, along with a solution to a numerical problem and numerical examples. We compare some of the current practises with the advised ones in Section 6. We arrive at a conclusion in Section 7.

2. PRELIMINARIES

In current section, Elementary concepts for Hesitant fuzzy sets (HFS), Neutrosophic sets (NS), Single valued Neutrosophic sets (SV-NS), SV-neutrosophic hesitant fuzzy set (SV-NHFS), Probabilistic hesitant fuzzy sets (PHFS), and SV-neutrosophic probabilistic hesitant fuzzy set (SV-NPHFS) are described.

Definition 1. *(See [37]) Suppose ζ be a fixed set. The mathematically representation of HFS D is defined as:*

$$D = \{\langle \xi, \Theta_{\sigma_D}(\xi)\rangle | \xi \in \zeta\}$$

where $\Theta_{\sigma_D}(\xi)$ is a set of some values in $[0,1]$, indicate the MF of the element $\xi \in \zeta$ to the set D.

Definition 2. *(See [38]) Suppose ζ be a fixed set. The mathematically representation Probabilistic HF set (PHFS) Γ is defined as:*

$$= \left\{\langle \xi, \Theta_{\sigma_\Gamma}(\xi)/\Omega_{\sigma(\xi)}\rangle | \xi \in \zeta\right\}$$

where $\Theta_{\sigma_\Gamma}(\xi)$ is a subset of $[0,1]$, and $\Theta_{\sigma_\Gamma}(\xi)/\Omega_\xi$ shows a MF of the element $\xi \in \zeta$ to the set Γ. And Ω_ξ shows the possibilities with the property that $\oplus_{i=1}^{s}\Omega_{\sigma_i} = 1$.

Definition 3. *(See [39, 41]) Suppose ζ be a fixed set and $\xi \in \zeta$. A neutrosophic set (NS) α in ζ is defined as MF $\Theta_\alpha(\xi)$, an IMF $\delta_\alpha(\xi)$ and a NMF $\psi_\alpha(\xi)$. $\Theta_\alpha(\xi), \delta_\alpha(\xi)$ and $\psi_\alpha(\xi)$ are real standard and non-standard subsets of $]0^-, 1^+[$ and*

$$\Theta_\alpha(\xi), \delta_\alpha(\xi), \psi_\alpha(\xi) : \zeta \longrightarrow]0^-, 1^+[$$

The mathematically representation neutrosophic set (NS) α is defined as:

$$\alpha = \{\langle \xi, \Theta_\alpha(\xi), \delta_\alpha(\xi), \psi_\alpha(\xi))\rangle \,|\, \xi \in \zeta\},$$

where

$$0^- < \Theta_\alpha(\xi) + \delta_\alpha(\xi) + \psi_\alpha(\xi) \leq 3^+.$$

Definition 4. *(See [44]) Suppose ζ be a fixed set and $\xi \in \zeta$. A single valued neutrosophic set (SV-NS) F in ζ is defined as MF $\Theta_F(\xi)$, an IMF $\delta_F(\xi)$ and a NMF $\psi_F(\xi)$. $\Theta_F(\xi), \delta_F(\xi)$ and $\psi_F(\xi)$ are real standard and non-standard subsets of $[0,1]$, and*

$$\Theta_F(\xi), \delta_F(\xi), \psi_F(\xi) : \zeta \longrightarrow [0,1].$$

The mathematically representation neutrosophic set (NS) F is defined as:

$$F = \{\langle \xi, \Theta_F(\xi), \delta_F(\xi), \psi_F(\xi)) \rangle \, | \xi \in \zeta\},$$

where

$$0 < \Theta_F(\xi) + \delta_F(\xi) + \psi_F(\xi) \leq 3.$$

Definition 5. *(See [23]) Suppose ζ be a fixed set. The mathematically representation of SV-NHFS $\Im$ is defined as:*

$$\Im = \{\langle \xi, \Theta_{\sigma_\Im}(\xi), \delta_{\sigma_\Im}(\xi), \psi_{\sigma_\Im}(\xi) \rangle | \xi \in \zeta\}$$

where $\Theta_{\sigma_\Im}(\xi), \delta_{\sigma_\Im}(\xi), \psi_{\sigma_\Im}(\xi)$ are set of some values in $[0,1]$, indicate the hesitant grade of MF, IMF and NMF of the element $\xi \in \zeta$ to the set $\Im$.

Definition 6. *(See [23]) For a fixed set Y, the $SV-NHFS$ ζ is mathematically represented as follows:*

$$\zeta = \{\left\langle \xi, \Theta_{\sigma_\zeta}(\xi), \delta_{\sigma_\zeta}(\xi), \psi_{\sigma_\zeta}(\xi) \right\rangle | \xi \in Y\},$$

where $\Theta_{\sigma_\zeta}(\xi), \delta_{\sigma_\zeta}(\xi)$ and $\psi_{\sigma_\zeta}(\xi)$ are sets of some values in $[0,1]$, called the MF, IMF and NMFs sequentially that must be satisfied the following properties:

$$\forall \xi \in Y, \forall \Theta_\zeta(\xi) \in \Theta_{\sigma_\zeta}(\xi), \forall \lambda_\zeta(\xi) \in \Theta_{\sigma_\zeta}(\xi),$$

and

$$\forall \delta_\zeta(\xi) \in \psi_{\sigma_\zeta}(\xi) \text{ with } \left(\max\left(\Theta_{\sigma_\zeta}(\xi)\right)\right) + \left(\min\left(\delta_{\sigma_\zeta}(\xi)\right)\right) + \left(\min\left(\delta_{\sigma_\zeta}(\xi)\right)\right) \leq 3,$$

and

$$\left(\min\left(\Theta_{\sigma_\zeta}(\xi)\right)\right) + \left(\min\left(\delta_{\sigma_\zeta}(\xi)\right)\right) + \left(\max\left(\psi_{\sigma_\zeta}(\xi)\right)\right) \leq 3.$$

For simplicity, we will use a pair $\zeta = (\Theta_{\sigma_\zeta}, \delta_{\sigma_\zeta}, \psi_{\sigma_\zeta})$ to mean $SV-NHFS$.

Definition 7. *Let ζ be a fixed set, the mathematical representation of SV-neutrosophic PHFS (SV-NPHFS) $\mathbb{N}$ is as,*

$$\mathbb{N} = \{\left\langle \xi, \Theta_{\sigma_\mathbb{N}}(\xi)/\eth_{\sigma(\xi)}, \delta_{\sigma_\mathbb{N}}(\xi)/\Omega_{\sigma(\xi)}, \psi_{\sigma_\mathbb{N}}(\xi)/\Lambda_{\sigma(\xi)} \right\rangle | \xi \in \zeta\},$$

where

$$\left\langle \Theta_{\sigma_\mathbb{N}}(\xi)/\eth_{\sigma(\xi)}, \delta_{\sigma_\mathbb{N}}(\xi)/\Omega_{\sigma(\xi)}, \psi_{\sigma_\mathbb{N}}(\xi)/\Lambda_{\sigma(\xi)} \right\rangle \longrightarrow [0,1],$$

and

$\Theta_{\sigma_\mathbb{N}}(\xi)/\eth_{\sigma(\xi)}$, $\delta_{\sigma_\mathbb{N}}(\xi)/\Omega_{\sigma(\xi)}$, $\psi_{\sigma_\mathbb{N}}(\xi)/\Lambda_{\sigma(\xi)}$ *shows a MF, IMF and NMF respectively of the element $\xi \in \zeta$ to the set $\mathbb{N}$. And $\eth_{\sigma(\xi)}, \Omega_{\sigma(\xi)}, \Lambda_{\sigma(\xi)}$ shows the possibilities with the property that*

$$\oplus_{i=1}^{s} \eth_{\sigma_i} = 1, \oplus_{i=1}^{s} \Omega_{\sigma_i} = 1 \text{ and } \oplus_{i=1}^{s} \Lambda_{\sigma_i} = 1.$$

Definition 8. *(See [39]) Let*

$$\Gamma_1 = (\Theta_{\sigma_{\Gamma_1}}, \delta_{\sigma_{\Gamma_1}}, \psi_{\sigma_{\Gamma_1}})$$

and

$$\Gamma_2 = (\Theta_{\sigma_{\Gamma_2}}, \delta_{\sigma_{\Gamma_2}}, \psi_{\sigma_{\Gamma_2}})$$

be two SV-NHFNs. The following are the basic set theoretic operations:

(1) $\Gamma_1 \cup \Gamma_2 = \left\{ \bigcup_{\substack{\Theta_1 \in \Theta_{\sigma_{\Gamma_1}} \\ \Theta_2 \in \Theta_{\sigma_{\Gamma_2}}}} \max(\Theta_1, \Theta_2), \bigcup_{\substack{\delta_1 \in \delta_{\sigma_{\Gamma_1}} \\ \delta_2 \in \delta_{\sigma_{\Gamma_2}}}} \min(\delta_1, \delta_2), \bigcup_{\substack{\lambda_1 \in \psi_{\sigma_{\Gamma_1}} \\ \lambda_2 \in \psi_{\sigma_{\Gamma_2}}}} \min(\lambda_1, \lambda_2) \right\};$

(2) $\Gamma_1 \cap \Gamma_2 = \left\{ \bigcup_{\substack{\Theta_1 \in \Theta_{\sigma_{\Gamma_1}} \\ \Theta_2 \in \Theta_{\sigma_{\Gamma_2}}}} \min(\Theta_1, \Theta_2), \bigcup_{\substack{\delta_1 \in \delta_{\sigma_{\Gamma_1}} \\ \delta_2 \in \delta_{\sigma_{\Gamma_2}}}} \max(\delta_1, \delta_2), \bigcup_{\substack{\lambda_1 \in \psi_{\sigma_{\Gamma_1}} \\ \lambda_2 \in \psi_{\sigma_{\Gamma_2}}}} \max(\lambda_1, \lambda_2) \right\};$

(3) $\Gamma_1^c = \left\{ \psi_{\sigma_{\Gamma_1}}, \delta_{\sigma_{\Gamma_1}}, \Theta_{\sigma_{\Gamma_1}} \right\}.$

Definition 9. *Let*

$$\Gamma_1 = (\Theta_{\sigma_{\Gamma_1}}(\xi)/\eth_{\xi_1}, \delta_{\sigma_{\Gamma_1}}(\xi)/\Omega_{\xi_1}, \psi_{\sigma_{\Gamma_1}}(\xi)/\Lambda_{\xi_1})$$

and

$$\Gamma_2 = (\Theta_{\sigma_{\Gamma_2}}(\xi)/\eth_{\xi_2}, \delta_{\sigma_{\Gamma_2}}(\xi)/\Omega_{\xi_2}, \psi_{\sigma_{\Gamma_2}}(\xi)/\Lambda_{\xi_2})$$

be two SV-NPHFNs. The following are the basic set theoretic operations:

(1) $\Gamma_1 \cup \Gamma_2 = \left\{ \bigcup_{\substack{\Theta_1 \in \Theta_{\sigma_{\Gamma_1}}, \eth_1 \in \eth_{\xi_1} \\ \Theta_2 \in \Theta_{\sigma_{\Gamma_2}}, \eth_2 \in \eth_{\xi_2}}} \max(\Theta_1, \Theta_2), \bigcup_{\substack{\delta_1 \in \delta_{\sigma_{\Gamma_1}}, \Omega_1 \in \Omega_{\xi_1} \\ \delta_2 \in \delta_{\sigma_{\Gamma_2}}, \Omega_2 \in \Omega_{\xi_2}}} \min(\delta_1, \delta_2), \bigcup_{\substack{\lambda_1 \in \psi_{\sigma_{\Gamma_1}}, \Lambda_1 \in \Omega_{\xi_1} \\ \lambda_2 \in \psi_{\sigma_{\Gamma_2}}, \Lambda_2 \in \Omega_{\xi_2}}} \min(\lambda_1, \lambda_2) \right\};$

(2) $\Gamma_1 \cap \Gamma_2 = \left\{ \bigcup_{\substack{\Theta_1 \in \Theta_{\sigma_{\Gamma_1}}, \eth_1 \in \eth_{\xi_1} \\ \Theta_2 \in \Theta_{\sigma_{\Gamma_2}}, \eth_2 \in \eth_{\xi_2}}} \min(\Theta_1, \Theta_2), \bigcup_{\substack{\delta_1 \in \delta_{\sigma_{\Gamma_1}}, \Omega_1 \in \Omega_{\xi_1} \\ \delta_2 \in \delta_{\sigma_{\Gamma_2}}, \Omega_2 \in \Omega_{\xi_2}}} \max(\delta_1, \delta_2), \bigcup_{\substack{\lambda_1 \in \psi_{\sigma_{\Gamma_1}}, \Lambda_1 \in \Omega_{\xi_1} \\ \lambda_2 \in \psi_{\sigma_{\Gamma_2}}, \Lambda_2 \in \Omega_{\xi_2}}} \max(\lambda_1, \lambda_2) \right\};$

(3) $\Gamma_1^c = \left\{ \psi_{\sigma_{\Gamma_1}}(\xi)/\Lambda_{\xi_1}, \delta_{\sigma_{\Gamma_1}}(\xi)/\Omega_{\xi_1}, \Theta_{\sigma_{\Gamma_1}}(\xi)/\eth_{\xi_1} \right\}.$

Definition 10. *[42] Let* ζ *be the universal set and* $\hbar \subseteq \zeta \times \zeta$ *be a (crisp) relation. Then*
(1) $\hbar$ *is reflexive if* $(\delta, \delta) \in \hbar$, *for each* $\delta \in \zeta$;
(3) $\hbar$ *is symmetric if* $\forall \delta, a \in \zeta$, $(\delta, a) \in \hbar$, *then* $(a, \delta) \in \hbar$;
(4) $\hbar$ *is transitive if* $\forall\ \delta, a, b \in \zeta, (\delta, a) \in \zeta$ *and* $(a, b) \in \hbar \longrightarrow (\delta, b) \in \hbar$.

3. SINGLE-VALUED NAUTROSOPHIC PROBABILISTIC HESITANT FUZZY DOMBI ARITHMETIC OPERATORS

Definition 11. *Suppose that α and β are two real numbers with $\sigma(\xi)$ hesitant function and $\Omega_{\sigma(\xi)}$ is . Then, the Dombi $T-norm$ and $T-conorm$ between α and β is defined as:*

$$O_D(\alpha,\beta) = \left(\frac{1}{1+\left\{ (\frac{1-\alpha_{\sigma(\xi)}}{\alpha_{\sigma(\xi)}})^{\Upsilon} + (\frac{1-\beta_{\sigma(\xi)}}{\beta_{\sigma(\xi)}})^{\Upsilon} \right\}^{1/\Upsilon}} \right) / \Omega_{\sigma(\xi_i)},$$

$$O_D^c(\alpha,\beta) = \left(1 - \frac{1}{1+\left\{ \frac{1-\alpha_{\sigma(\xi)}}{\alpha_{\sigma(\xi)}})^{\Upsilon} + (\frac{1-\beta_{\sigma(\xi)}}{\beta_{\sigma(\xi)}})^{\Upsilon} \right\}^{1/\Upsilon'}} \right) / \Omega'_{\sigma(\xi_i)}$$

where possibilities with the property that $\oplus_{i=1}^{s}\Omega_{\sigma_i} = 1$, $\Upsilon \geq 1$ and $(\alpha,\beta) \in [0,1] \times [0,1]$.

Here, some Dombi operators for single valued neutrosophic probabilistic hesitant fuzzy sets (SV-NPHFSs) are defined using T-norm and T-conorm.

Definition 12. *Suppose $\Gamma_1 = (\Theta_1, \delta_1, \psi_1)$ and $\Gamma_2 = (\Theta_2, \delta_2, \psi_2)$ are two single valued neutrosophic numbers (SV-NNs) with hasitancy $\sigma(\xi)$,and probability functions $\Omega_{\sigma(\xi)}, \Lambda_{\sigma(\xi)}, \omega_{\sigma(\xi)}$ satisfying $\Omega_{\sigma(\xi)} + \Lambda_{\sigma(\xi)} + \omega_{\sigma(\xi)} = 1$, $\Upsilon \geq 1$ and $\lambda > 0$. Then, the Dombi T-norm and T-conorm operations of SV-NPHFNs are defined as:*

$$\mathbf{1}\ \ \Gamma_1 \oplus \Gamma_2 = \left\{ \begin{array}{c} \left\langle 1 - \frac{1}{1+\left\{ \left(\frac{\Theta_{1_{\sigma(\xi)}}}{1-\Theta_{1_{\sigma(\xi)}}} \right)^{\Upsilon} + \left(\frac{\Theta_{2_{\sigma(\xi)}}}{1-\Theta_{2_{\sigma(\xi)}}} \right)^{\Upsilon} \right\}^{1/\Upsilon}} \right\rangle / \Omega_{\sigma(\xi)}, \\ \left\langle \frac{1}{1+\left\{ \left(\frac{1-\delta_{1_{\sigma(\xi)}}}{\delta_{1_{\sigma(\xi)}}} \right)^{\Upsilon} + \left(\frac{1-\delta_{2_{\sigma(\xi)}}}{\delta_{2_{\sigma(\xi)}}} \right)^{\Upsilon} \right\}^{1/\Upsilon}} \right\rangle / \Lambda_{\sigma(\xi)}, \\ \left\langle \frac{1}{1+\left\{ \left(\frac{1-v_{1_{\sigma(\xi)}}}{v_{1_{\sigma(\xi)}}} \right)^{\Upsilon} + \left(\frac{1-v_{2_{\sigma(\xi)}}}{v_{2_{\sigma(\xi)}}} \right)^{\Upsilon} \right\}^{1/\Upsilon}} \right\rangle / \omega_{\sigma(\xi)} \end{array} \right\};$$

$$\mathbf{2}\ \ \Gamma_1 \otimes \Gamma_2 = \left\{ \begin{array}{c} \left\langle \frac{1}{1+\left\{ \left(\frac{\Theta_{1_{\sigma(\xi)}}}{1-\Theta_{1_{\sigma(\xi)}}} \right)^{\rho} + \left(\frac{\Theta_{2_{\sigma(\xi)}}}{1-\Theta_{2_{\sigma(\xi)}}} \right)^{\Upsilon} \right\}^{1/\Upsilon}} \right\rangle / \Omega_{\sigma(\xi)}, \\ \left\langle 1 - \frac{1}{1+\left\{ \left(\frac{1-\delta_{1_{\sigma(\xi)}}}{\delta_{1_{\sigma(\xi)}}} \right)^{\Upsilon} + \left(\frac{1-\delta_{2_{\sigma(\xi)}}}{\delta_{2_{\sigma(\xi)}}} \right)^{\Upsilon} \right\}^{1/\Upsilon}} \right\rangle / \Lambda_{\sigma(\xi)}, \\ \left\langle 1 - \frac{1}{1+\left\{ \left(\frac{1-v_{1_{\sigma(\xi)}}}{v_{1_{\sigma(\xi)}}} \right)^{\Upsilon} + \left(\frac{1-v_{2_{\sigma(\xi)}}}{v_{2_{\sigma(\xi)}}} \right)^{\Upsilon} \right\}^{1/\Upsilon}} \right\rangle / \omega_{\sigma(\xi)} \end{array} \right\};$$

$$\mathbf{3}\ \ \lambda\Gamma_1 = \left\{ \begin{array}{c} \left\langle 1 - \frac{1}{1+\left\{\lambda\left(\frac{\Theta_{1_{\sigma(\xi)}}}{1-\Theta_{1_{\sigma(\xi)}}}\right)^{\Upsilon}\right\}^{1/\Upsilon}} \right\rangle / \Omega_{\sigma(\xi)}, \\ \left\langle \frac{1}{1+\left\{\lambda\left(\frac{1-\delta_{1_{\sigma(\xi)}}}{\delta_{1_{\sigma(\xi)}}}\right)^{\Upsilon}\right\}^{1/\Upsilon}} \right\rangle / \Lambda_{\sigma(\xi)}, \\ \left\langle \frac{1}{1+\left\{\lambda\left(\frac{1-\psi_{1_{\sigma(\xi)}}}{\psi_{1_{\sigma(\xi)}}}\right)^{\Upsilon}\right\}^{1/\Upsilon}} \right\rangle / \omega_{\sigma(\xi)} \end{array} \right\};$$

$$\mathbf{4}\ \ \Gamma_1^{\lambda} = \left\{ \begin{array}{c} \left\langle \frac{1}{1+\left\{\lambda\left(\frac{\Theta_{1_{\sigma(\xi)}}}{1-\Theta_{1_{\sigma(\xi)}}}\right)^{\Upsilon}\right\}^{1/\Upsilon}} \right\rangle / \Omega_{\sigma(\xi)}, \\ \left\langle 1 - \frac{1}{1+\left\{\lambda\left(\frac{1-\delta_{1_{\sigma(\xi)}}}{\delta_{1_{\sigma(\xi)}}}\right)^{\Upsilon}\right\}^{1/\Upsilon}} \right\rangle / \Lambda_{\sigma(\xi)}, \\ \left\langle 1 - \frac{1}{1+\left\{\lambda\left(\frac{1-\psi_{1_{\sigma(\xi)}}}{\psi_{1_{\sigma(\xi)}}}\right)^{\Upsilon}\right\}^{1/\Upsilon}} \right\rangle / \omega_{\sigma(\xi)} \end{array} \right\}.$$

3.1. Dombi weighted arithmetic aggregation operations of single valued neutrosophic probabilistic hesitant fuzzy Numbers

In this section two dombi operators fro single valued neutrosophic probabilistic hesitant fuzzy numbers (SV-NPHFNs), single-valued neutrosophic probabilistic hesitant fuzzy Dombi weighted arithmetic average (SV-NPHFDWAA) operator and single-valued neutrosophic probabilistic hesitant fuzzy Dombi weighted arithmetic geometric (SV-NPHFDWAG) operator are defined. Also we defined some properties for these operators.

Definition 13. $\Gamma_\tau = \left(\Theta_{\tau_{\sigma_\tau(\xi)}}/\Omega_{\sigma_\tau(\xi)}, \delta_{\tau_{\sigma_\tau(\xi)}}/\Lambda_{\sigma_\tau(\xi)}, \psi_{\tau_{\sigma_\tau(\xi)}}/\omega_{\sigma_\tau(\xi)}\right), \tau = 1, 2, ..., n,$ *is a collection of SV-NPHFNs and* $\daleth = (\daleth_1, \daleth_2, ..., \daleth_n)$ *are the weight vector for* Γ_τ *with* $\daleth_\tau \in [0, 1]$ *and* $\sum_{\tau=1}^{n} \daleth_\tau = 1$ *and* $\Omega_{\sigma(\xi)} + \Lambda_{\sigma(\xi)} + \omega_{\sigma(\xi)} = 1$. *Then, the SV-NPHFDWAA and SV-NPHFDWAG operators are defined, as follows:*

$$SV - NPHFDWAA\ (\Gamma_1, \Gamma_2, ..., \Gamma_n) = \bigoplus_{\tau=1}^{n} \daleth_\tau \Gamma_\tau\ ;$$

$$SV - NPHFDWAG(\Gamma_1, \Gamma_2, ..., \Gamma_n) = \bigoplus_{\tau=1}^{n} \Gamma_\tau^{\daleth_\tau}$$

Theorem 1. *Suppose that* $\Gamma_\tau = \left(\Theta_{\tau_{\sigma_\tau(\xi)}}/\Omega_{\sigma_\tau(\xi)}, \delta_{\tau_{\sigma_\tau(\xi)}}/\Lambda_{\sigma_\tau(\xi)}, \psi_{\tau_{\sigma_\tau(\xi)}}/\omega_{\sigma_\tau(\xi)}\right)$, $\tau = 1, 2, ..., n$, *is the collection of SV-NPHFNs and* $\daleth = (\daleth_1, \daleth_2, ..., \daleth_n)$ *are the weight vector for* Γ_τ *with* $\daleth_\tau \in [0, 1]$ *and* $\sum_{\tau=1}^{n} \daleth_\tau = 1$ *and* $\Omega_{\sigma(\xi)} + \Lambda_{\sigma(\xi)} + \omega_{\sigma(\xi)} = 1$.

Then, the aggregated value for the SV-NPHFDWAA operator is a SV-NPHFN, that can be calculated as following:

Theorem 2.

$$SV-NPHFDWAA(\Gamma_1,\Gamma_2,...,\Gamma_n)$$

$$= \left\{ \begin{array}{c} \left\langle \frac{1}{1+\left\{ \sum_{\tau=1}^{n} \beth_\tau \left(\frac{\Theta_{\tau_{\sigma_\tau(\xi)}}}{1-\Theta_{\tau_{\sigma_\tau(\xi)}}} \right)^{\Upsilon} \right\}^{1/\Upsilon'}} \right\rangle / \sum_{\tau=1}^{n} \Omega_{\sigma_\tau(\xi)}, \\ \left\langle 1 - \frac{1}{1+\left\{ \sum_{\tau=1}^{n} \beth_\tau \left(\frac{1-\delta_{\tau_{\sigma_\tau(\xi)}}}{\delta_{\tau_{\sigma_\tau(\xi)}}} \right)^{\Upsilon} \right\}^{1/\Upsilon'}} \right\rangle / \sum_{\tau=1}^{n} \Lambda_{\sigma_\tau(\xi)}, \\ \left\langle 1 - \frac{1}{1+\left\{ \sum_{\tau=1}^{n} \beth_\tau \left(\frac{1-\psi_{\tau_{\sigma_\tau(\xi)}}}{\psi_{\tau_{\sigma_\tau(\xi)}}} \right)^{\Upsilon} \right\}^{1/\Upsilon}} \right\rangle / \sum_{\tau=1}^{n} \omega_{\sigma_\tau(\xi)} \end{array} \right\}$$

using the mathematical induction method, we can prove the above Theorem 1.

Proof. At $n = 2$, Dombi operators for SV-NPHFNs are obtained as following:

$$SV-NPHFDWAG(\Gamma_1,\Gamma_2,...,\Gamma_n) = \Gamma_1 \oplus \Gamma_2$$

$$= \left\{ \begin{array}{c} \left\langle 1 - \frac{1}{1+\left\{ \beth_1 \left(\frac{\Theta_{1_{\sigma_1(\xi)}}}{1-\Theta_{1_{\sigma_1(\xi)}}} \right)^{\Upsilon} + \beth_2 \left(\frac{\Theta_{2_{\sigma_2(\xi)}}}{1-\Theta_{2_{\sigma_2(\xi)}}} \right)^{\Upsilon} \right\}^{1/\Upsilon'}} \right\rangle / \sum_{\tau=1}^{2} \Omega_{\sigma_\tau(\xi)}, \\ \left\langle \frac{1}{1+\left\{ \beth_1 \left(\frac{1-\delta_{1_{\sigma_1(\xi)}}}{\delta_{1_{\sigma_1(\xi)}}} \right)^{\Upsilon} + \beth_2 \left(\frac{1-\delta_{2_{\sigma_2(\xi)}}}{\delta_{2_{\sigma_2(\xi)}}} \right)^{\Upsilon} \right\}^{1/\Upsilon'}} \right\rangle / \sum_{\tau=1}^{2} \Lambda_{\sigma_\tau(\xi)}, \\ \left\langle \frac{1}{1+\left\{ \beth_1 \left(\frac{1-\psi_{1_{\sigma_1(\xi)}}}{\psi_{1_{\sigma_1(\xi)}}} \right)^{\Upsilon} + \beth_2 \left(\frac{1-\psi_{2_{\sigma_2(\xi)}}}{\psi_{2_{\sigma_2(\xi)}}} \right)^{\Upsilon} \right\}^{1/\Upsilon}} \right\rangle / \sum_{\tau=1}^{2} \omega_{\sigma_\tau(\xi)} \end{array} \right\}$$

At $n = \kappa$, we obtained the following equations:

$$SV-NPHFDWAA\ (\Gamma_1,\Gamma_2,...,\Gamma_n)$$

$$= \left\{ \begin{array}{c} \left\langle 1 - \frac{1}{1+\left\{ \sum_{\tau=1}^{\kappa} \beth_\tau \left(\frac{\Theta_{\tau_{\sigma_\tau(\xi)}}}{1-\Theta_{\tau_{\sigma_\tau(\xi)}}} \right)^{\Upsilon} \right\}^{1/\Upsilon'}} \right\rangle / \sum_{\tau=1}^{\kappa} \Omega_{\sigma_\tau(\xi)}, \\ \left\langle \frac{1}{1+\left\{ \sum_{\tau=1}^{\kappa} \beth_\tau \left(\frac{1-\delta_{\tau_{\sigma_\tau(\xi)}}}{\delta_{\tau_{\sigma_\tau(\xi)}}} \right)^{\Upsilon} \right\}^{1/\Upsilon'}} \right\rangle / \sum_{\tau=1}^{\kappa} \Lambda_{\sigma_\tau(\xi)}, \\ \left\langle \frac{1}{1+\left\{ \sum_{\tau=1}^{\kappa} \beth_\tau \left(\frac{1-\psi_{\tau_{\sigma_\tau(\xi)}}}{\psi_{\tau_{\sigma_\tau(\xi)}}} \right)^{\Upsilon} \right\}^{1/\Upsilon}} \right\rangle / \sum_{\tau=1}^{\kappa} \omega_{\sigma_\tau(\xi)} \end{array} \right\}.$$

At $n = \kappa + 1$, we obtained the following result:
$SV - NPHFDWAA(\Gamma_1, \Gamma_2, ..., \Gamma_n)$

$$= \left\{ \begin{array}{l} \left\langle 1 - \frac{1}{1+\left\{ \sum_{\tau=1}^{\kappa} \beth_\tau \left(\frac{\Theta_{\tau_{\sigma_\tau(\xi)}}}{1-\Theta_{\tau_{\sigma_\tau(\xi)}}} \right)^{\Upsilon} \right\}^{1/\Upsilon'}} \right\rangle / \sum_{\tau=1}^{\kappa} \Omega_{\sigma_\tau(\xi)}, \\ \left\langle \frac{1}{1+\left\{ \sum_{\tau=1}^{\kappa} \beth_\tau \left(\frac{1-\delta_{\tau_{\sigma_\tau(\xi)}}}{\delta_{\tau_{\sigma_\tau(\xi)}}} \right)^{\Upsilon} \right\}^{1/\Upsilon'}} \right\rangle / \sum_{\tau=1}^{\kappa} \Lambda_{\sigma_\tau(\xi)}, \\ \left\langle \frac{1}{1+\left\{ \sum_{\tau=1}^{\kappa} \beth_\tau \left(\frac{1-\psi_{\tau_{\sigma_\tau(\xi)}}}{\psi_{\tau_{\sigma_\tau(\xi)}}} \right)^{\Upsilon} \right\}^{1/\Upsilon}} \right\rangle / \sum_{\tau=1}^{\kappa} \omega_{\sigma_\tau(\xi)} \end{array} \right\} \oplus \beth_\kappa + \Gamma_\kappa + 1$$

$$= \left\{ \begin{array}{l} \left\langle 1 - \frac{1}{1+\left\{ \sum_{\tau=1}^{\kappa} \beth_\tau \left(\frac{\Theta_{\tau_{\sigma_\tau(\xi)}}}{1-\Theta_{\tau_{\sigma_\tau(\xi)}}} \right)^{\Upsilon} \right\}^{1/\Upsilon'}} \right\rangle / \sum_{\tau=1}^{\kappa} \Omega_{\sigma_\tau(\xi)}, \\ \left\langle \frac{1}{1+\left\{ \sum_{\tau=1}^{\kappa} \beth_\tau \left(\frac{1-\delta_{\tau_{\sigma_\tau(\xi)}}}{\delta_{\tau_{\sigma_\tau(\xi)}}} \right)^{\Upsilon} \right\}^{1/\Upsilon'}} \right\rangle / \sum_{\tau=1}^{\kappa} \Lambda_{\sigma_\tau(\xi)}, \\ \left\langle \frac{1}{1+\left\{ \sum_{\tau=1}^{\kappa} \beth_\tau \left(\frac{1-\psi_{\tau_{\sigma_\tau(\xi)}}}{\psi_{\tau_{\sigma_\tau(\xi)}}} \right)^{\Upsilon} \right\}^{1/\Upsilon}} \right\rangle / \sum_{\tau=1}^{\kappa} \omega_{\sigma_\tau(\xi)} \end{array} \right\}$$

Hence, prove that theorem is true for $n = \kappa + 1$. Thus, it holds for all n. $\square$

Then, the SV-NPHFDWAA operator contains the following properties:
(1) Reducibility: When $\beth = (1/n, 1/n, ..., 1/n)$, There are obviously exceptions.

$SV - NPHFDWAA\ (\Gamma_1, \Gamma_2, ..., \Gamma_n)$

$$= \left\{ \begin{array}{l} \left\langle 1 - \frac{1}{1+\left\{ \sum_{\tau=1}^{\kappa} \frac{1}{n} \left(\frac{\Theta_{\tau_{\sigma_\tau(\xi)}}}{1-\Theta_{\tau_{\sigma_\tau(\xi)}}} \right)^{\Upsilon} \right\}^{1/\Upsilon'}} \right\rangle / \sum_{\tau=1}^{\kappa} \Omega_{\sigma_\tau(\xi)}, \\ \left\langle \frac{1}{1+\left\{ \sum_{\tau=1}^{\kappa} \frac{1}{n} \left(\frac{1-\delta_{\tau_{\sigma_\tau(\xi)}}}{\delta_{\tau_{\sigma_\tau(\xi)}}} \right)^{\Upsilon} \right\}^{1/\Upsilon'}} \right\rangle / \sum_{\tau=1}^{\kappa} \Lambda_{\sigma_\tau(\xi)}, \\ \left\langle \frac{1}{1+\left\{ \sum_{\tau=1}^{\kappa} \frac{1}{n} \left(\frac{1-\psi_{\tau_{\sigma_\tau(\xi)}}}{\psi_{\tau_{\sigma_\tau(\xi)}}} \right)^{\Upsilon} \right\}^{1/\Upsilon}} \right\rangle / \sum_{\tau=1}^{\kappa} \omega_{\sigma_\tau(\xi)} \end{array} \right\}.$$

(2) Indempotency: Suppose that all the SV-NPHFNs are
$\Gamma_\tau = \left(\Theta_{\tau_{\sigma_\tau(\xi)}}/\Omega_{\sigma_\tau(\xi)}, \delta_{\tau_{\sigma_\tau(\xi)}}/\Lambda_{\sigma_\tau(\xi)}, \psi_{\tau_{\sigma_\tau(\xi)}}/\omega_{\sigma_\tau(\xi)}\right)$, $\tau = 1, 2, ..., n$, is the collection of

SV-NPHFNs and $\beth = (\beth_1, \beth_2, ..., \beth_n)$ are the weight vector for Γ_τ with $\beth_\tau \in [0,1]$ and $\sum_{\tau=1}^{n} \beth_\tau = 1$ and $\Omega_{\sigma(\xi)} + \Lambda_{\sigma(\xi)} + \omega_{\sigma(\xi)} = 1$.

Then, SV-NPHFDWAA $(\Gamma_1, \Gamma_2, ..., \Gamma_n) = \Gamma$.

(3) Commutativity: Let the SV-NPHFS $(\Gamma_1^{'}, \Gamma_{2'}, ..., \Gamma_n^{'})$ be any permutation of $(\Gamma_1, \Gamma_2, ..., \Gamma_n)$. Then there is $SV - NNPHFDWAA(\Gamma_1^{'}, \Gamma_{2'}, ..., \Gamma_n^{'}) = SV - NPHFDWAA(\Gamma_1, \Gamma_2, ..., \Gamma_n)$.

(4) Boundedness: Let $\Gamma_{\min} = \min(\Gamma_1, \Gamma_2, ..., \Gamma_n)$ and

$\Gamma_{\max} = \max(\Gamma_1, \Gamma_2, ..., \Gamma_n)$. Then, $\Gamma_{\min} \leq SV - NPHFDWAA(\Gamma_1, \Gamma_2, ..., \Gamma_n) \leq \Gamma_{\max}$.

Proof. (1) According to the Theorem 1, the property is apparent.

(2) Since $\Gamma_\tau = (\Theta_\tau, \delta_\tau, \psi_\tau)$; $\tau = 1, 2, ..., n$. Then, by using Theorem 1, we can obtained the result as following:

$$SV - NPHFDWAA(\Gamma_1, \Gamma_2, ..., \Gamma_n)$$

$$= \left\{ \begin{array}{c} \left\langle 1 - \frac{1}{1+\left\{ \sum_{\tau=1}^{n} \beth_\tau \left(\frac{\Theta_{\tau_{\sigma_\tau(\xi)}}}{1-\Theta_{\tau_{\sigma_\tau(\xi)}}} \right)^{\Upsilon} \right\}^{1/\Upsilon'}} \right\rangle / \sum_{\tau=1}^{n} \Omega_{\sigma_\tau(\xi)}, \\ \left\langle \frac{1}{1+\left\{ \sum_{\tau=1}^{n} \beth_\tau \left(\frac{1-\delta_{\tau_{\sigma_\tau(\xi)}}}{\delta_{\tau_{\sigma_\tau(\xi)}}} \right)^{\Upsilon} \right\}^{1/\Upsilon'}} \right\rangle / \sum_{\tau=1}^{n} \Lambda_{\sigma_\tau(\xi)}, \\ \left\langle \frac{1}{1+\left\{ \sum_{\tau=1}^{n} \beth_\tau \left(\frac{1-\psi_{\tau_{\sigma_\tau(\xi)}}}{\psi_{\tau_{\sigma_\tau(\xi)}}} \right)^{\Upsilon} \right\}^{1/\Upsilon}} \right\rangle / \sum_{\tau=1}^{n} \omega_{\sigma_\tau(\xi)} \end{array} \right\}$$

$$= \left\{ \begin{array}{c} \left\langle \frac{1}{1+\left\{ \left(\frac{\Theta_{\tau_{\sigma_\tau(\xi)}}}{1-\Theta_{\tau_{\sigma_\tau(\xi)}}} \right)^{\Upsilon} \right\}^{1/\Upsilon'}} \right\rangle / \sum_{\tau=1}^{n} \Omega_{\sigma_\tau(\xi)}, \\ \left\langle 1 - \frac{1}{1+\left\{ \left(\frac{1-\delta_{\tau_{\sigma_\tau(\xi)}}}{\delta_{\tau_{\sigma_\tau(\xi)}}} \right)^{\Upsilon} \right\}^{1/\Upsilon'}} \right\rangle / \sum_{\tau=1}^{n} \Omega_{\sigma_\tau(\xi)}, \\ \left\langle 1 - \frac{1}{1+\left\{ \lambda \left(\frac{1-\psi_{\tau_{\sigma_\tau(\xi)}}}{\psi_{\tau_{\sigma_\tau(\xi)}}} \right)^{\Upsilon} \right\}^{1/\Upsilon}} \right\rangle / \sum_{\tau=1}^{n} \Omega_{\sigma_\tau(\xi)} \end{array} \right\}$$

$$= \left\{ \begin{array}{c} \left(1 - \frac{1}{1+\frac{\Theta_{\sigma(\xi)}}{1-\Theta_{\sigma(\xi)}}} \right) / \Omega_{\sigma(\xi)}, \\ \left(\frac{1}{1+\frac{\delta_{\sigma(\xi)}}{\delta_{\sigma(\xi)}-\Theta_{\sigma(\xi)}}} \right) / \Lambda_{\sigma(\xi)}, \\ \left(\frac{1}{1+\frac{\psi_{\sigma(\xi)}}{\psi_{\sigma(\xi)}-\Theta_{\sigma(\xi)}}} \right) / \omega_{\sigma(\xi)} \end{array} \right\} = \langle \Theta, \delta, \psi \rangle = \Gamma.$$

Hence $SV-NPHFDWAA(\Gamma_1,\Gamma_2,...,\Gamma_n)=\Gamma$ holds (3) The property is apparent. (4) Suppose $\Gamma_{\min}=\min(\Gamma_1,\Gamma_2,...,\Gamma_n)=\langle\Theta^-,\delta^-,\psi^-\rangle$ and $\Gamma_{\max}=\max(\Gamma_1,\Gamma_2,...,\Gamma_n)=\langle\Theta^+,\delta^+,\psi^+\rangle$. Then, we have $\Theta^-=\min\limits_{\tau}(\Theta_\tau),\delta^-=\max\limits_{\tau}(\delta_\tau),\psi^-=\max\limits_{\tau}(\psi_\tau),\Theta^+=\max\limits_{\tau}(\Theta_\tau),\delta^+=\min\limits_{\tau}(\delta_\tau)$ and $\psi^+=\max\limits_{\tau}(\delta_\tau)$. □

Thus, the results are as following inequalities:

$$\left\{\begin{array}{c}\left(1-\frac{1}{1+\{\sum\limits_{\tau=1}^{n}\beth_\tau(\frac{\Theta^-_{\tau_{\sigma_\tau(\xi)}}}{1-\Theta^-_{\tau_{\sigma_\tau(\xi)}}})^\Upsilon\}^{1/\Upsilon}}\right)/\sum\limits_{\tau=1}^{n}\Omega_{\sigma_\tau(\xi)}\leq\left(1-\frac{1}{1+\{\sum\limits_{\tau=1}^{n}\beth_\tau(\frac{\Theta_{\tau_{\sigma_\tau(\xi)}}^{\tau}}{1-\Theta^{\tau}_{\tau_{\sigma_\tau(\xi)}}})^\Upsilon\}^{1/\Upsilon}}\right)/\sum\limits_{\tau=1}^{n}\Lambda_{\sigma_\tau(\xi)}\\ \leq\left(1-\frac{1}{1+\{\sum\limits_{\tau=1}^{n}\beth_\tau(\frac{\Theta^+_{\tau_{\sigma_\tau(\xi)}}}{1-\Theta^+_{\tau_{\sigma_\tau(\xi)}}})^\Upsilon\}^{1/\Upsilon'}}\right)/\sum\limits_{\tau=1}^{n}\omega_{\sigma_\tau(\xi)}\end{array}\right\};$$

$$\left\{\begin{array}{c}\left(\frac{1}{1+\{\sum\limits_{\tau=1}^{n}\beth_\tau(\frac{1-\delta^+_{\tau_{\sigma_\tau(\xi)}}}{\delta^+_{\tau_{\sigma_\tau(\xi)}}})^\Upsilon\}^{1/\Upsilon}}\right)/\sum\limits_{\tau=1}^{n}\Omega_{\sigma_\tau(\xi)}\leq\left(\frac{1}{1+\{\sum\limits_{\tau=1}^{n}\beth_\tau(\frac{1-\delta_{\tau_{\sigma_\tau(\xi)}}}{\delta_{\tau_{\sigma_\tau(\xi)}}})^\Upsilon\}^{1/\Upsilon}}\right)/\sum\limits_{\tau=1}^{n}\Lambda_{\sigma_\tau(\xi)}\\ \leq\left(\frac{1}{1+\{\sum\limits_{\tau=1}^{n}\beth_\tau(\frac{1-\delta^-_{\tau_{\sigma_\tau(\xi)}}}{\delta^-_{\tau_{\sigma_\tau(\xi)}}})^\Upsilon\}^{1/\Upsilon'}}\right)/\sum\limits_{\tau=1}^{n}\omega_{\sigma_\tau(\xi)}\end{array}\right\}$$

$$\left\{\begin{array}{c}\left(\frac{1}{1+\{\sum\limits_{\tau=1}^{n}\beth_\tau(\frac{1-\psi^+_{\tau_{\sigma_\tau(\xi)}}}{\psi^+_{\tau_{\sigma_\tau(\xi)}}})^\Upsilon\}^{1/\Upsilon}}\right)/\sum\limits_{\tau=1}^{n}\Omega_{\sigma_\tau(\xi)}\leq\left(\frac{1}{1+\{\sum\limits_{\tau=1}^{n}\beth_\tau(\frac{1-\psi_{\tau_{\sigma_\tau(\xi)}}}{\psi_{\tau_{\sigma_\tau(\xi)}}})^\Upsilon\}^{1/\Upsilon}}\right)/\sum\limits_{\tau=1}^{n}\Lambda_{\sigma_\tau(\xi)}\\ \leq\left(\frac{1}{1+\{\sum\limits_{\tau=1}^{n}\beth_\tau(\frac{1-\psi^-_{\tau_{\sigma_\tau(\xi)}}}{\psi^-_{\tau_{\sigma_\tau(\xi)}}})^\Upsilon\}^{1/\Upsilon}}\right)/\sum\limits_{\tau=1}^{n}\omega_{\sigma_\tau(\xi)}\end{array}\right\}.$$

Hence $\Gamma_{\min}\leq SV-NPHFDWAA(\Gamma_1,\Gamma_2,...,\Gamma_n)\leq\Gamma_{\max}$ holds.

Theorem 3. *Suppose* $\Gamma_\tau=<\Theta_\tau,\delta_\tau,\psi_\tau>$ $(\tau=1,2,...,n)$ *be a collection of SV-NPHFNs and* $\beth=(\beth_1,\beth_2,...,\beth_n)$ *are the weight vector for* Γ_τ *with* $\beth_\tau\in[0,1]$ *and* $\sum\limits_{\tau=1}^{n}\beth_\tau=1$*. Then, the aggregated value of the SV-NDWAA operator is still a SV-NN, which is calculated by the following formula:*

(1) Reducibility: When the weight vector is $\beth=(1/n,1/n,...,1/n)$, it is apparent that there exists the following result:

$$SV-NPHFDWAG(\Gamma_1,\Gamma_2,...,\Gamma_n)$$

$$= \left\{ \begin{array}{c} \left(1 - \frac{1}{1+\{\sum_{\tau=1}^{\kappa} \frac{1}{n}(\frac{1-\Theta_{\tau_{\sigma_\tau(\xi)}}}{\Theta_{\tau_{\sigma_\tau(\xi)}}})^{\Upsilon}\}^{1/\Upsilon'}} \right) / \sum_{\tau=1}^{\kappa} \Omega_{\sigma_\tau(\xi)}, \\ \left(\frac{1}{1+\{\sum_{\tau=1}^{\kappa} \frac{1}{n}(\frac{\delta_{\tau_{\sigma_\tau(\xi)}}}{1-\delta_{\tau_{\sigma_\tau(\xi)}}})^{\Upsilon}\}^{1/\Upsilon'}} \right) / \sum_{\tau=1}^{\kappa} \Lambda_{\sigma_\tau(\xi)}, \\ \left(\frac{1}{1+\{\sum_{\tau=1}^{\kappa} \frac{1}{n}(\frac{\psi_{\tau_{\sigma_\tau(\xi)}}}{1-\psi_{\tau_{\sigma_\tau(\xi)}}})^{\Upsilon}\}^{1/\Upsilon}} \right) / \sum_{\tau=1}^{\kappa} \omega_{\sigma_\tau(\xi)}. \end{array} \right\}.$$

The proof of Theorem 2 is as for Theorem 1. Thus, it is as following.

(2) Indempotency: Suppose all the SV-NPHFNs be $\Gamma_\tau = (\Theta_\tau, \delta_\tau, \psi_\tau)$; $\tau = 1, 2, ..., n$. Then,

$$SV - NPHFDWAG(\Gamma_1, \Gamma_2, ..., \Gamma_n) = \Gamma.$$

(3) Commutativity: Suppose the SV-NPHFS $(\Gamma_1^{'}, \Gamma_{2'}, ..., \Gamma_n^{'})$ is any permutation for $(\Gamma_1, \Gamma_2, ..., \Gamma_n)$. Then,

$$SV - NPHFDWAG(\Gamma_1^{'}, \Gamma_{2'}, ..., \Gamma_n^{'}) = SV - NPHFDWAG(\Gamma_1, \Gamma_2, ..., \Gamma_n).$$

(4) Boundedness: Suppose that $\Gamma_{\min} = \min(\Gamma_1, \Gamma_2, ..., \Gamma_n)$ and $\Gamma_{\max} = \max(\Gamma_1, \Gamma_2, ..., \Gamma_n)$. Then,

$$\Gamma_{\min} \leq SV - NPHFDWAG(\Gamma_1, \Gamma_2, ..., \Gamma_n) \leq \Gamma_{\max}.$$

The proving process for these properties is the same as for the SV-NPHFDWAA operator's properties. As a result, they are not repeated here.

4. MADM METHOD USING THE SV-NDWAA OPERATOR AND THE SV-NDWAG OPERATOR

To resolve MADM issues with SV-NPHFNs in this area, we will make use of a MADM mentioned by the SV-NPHFDWAA and SV-NPHFDWAG operators. In order to give a handling strategy for MADM circumstances involving SV-NPHFNs, we use the SV-NPHFDWAA and SV-NPHFDWAG operators. This handling method can be summarised as follows:

The following are the main steps for MADM:

Step-1 Build the experts' assessment matrices as

$$(E)^{\hat{\jmath}} = \begin{bmatrix} \sigma(\xi_{11}^{\hat{\jmath}}) & \sigma(\xi_{12}^{\hat{\jmath}}) & \cdots & \sigma(\xi_{1j}^{\hat{\jmath}}) \\ \sigma(\xi_{21}^{\hat{\jmath}}) & \sigma(\xi_{22}^{\hat{\jmath}}) & \cdots & \sigma(\xi_{2j}^{\hat{\jmath}}) \\ \sigma(\xi_{31}^{\hat{\jmath}}) & \sigma(\xi_{32}^{\hat{\jmath}}) & \cdots & \sigma(\xi_{3j}^{\hat{\jmath}}) \\ \vdots & \vdots & \ddots & \vdots \\ \sigma(\xi_{i1}^{\hat{\jmath}}) & \sigma(\xi_{i2}^{\hat{\jmath}}) & \cdots & \sigma(\xi_{ij}^{\hat{\jmath}}) \end{bmatrix}$$

where $\hat{\jmath}$ shows the experts.

Step-2 Review the normalised experts matices $(\mathbb{N})^{\hat{j}}$, as follows:

$$(\mathbb{N})^{\hat{j}} = \begin{cases} \sigma(\xi_{ij}) = \sigma\left(\xi_{ij}\right) & \text{if} \quad \text{For benefit} \\ \left(\sigma(\xi_{ij})\right)^c = \left(\sigma\left(\xi_{ij}\right)\right)^c & \text{if} \quad \text{For non-benefit} \end{cases}$$

Step-3 Utilize the SV-NPHFDWAA and SV-NPHFDWAA aggregation operations to analyze the gathered SV-neutrosophic hesitant fuzzy data of decision makers.

$$\Gamma_i = SV - NPHFDWAA(\Gamma_{i1}, \Gamma_{i2}, ..., \Gamma_{in})$$

$$= \left\{ \begin{array}{c} \left(1 - \frac{1}{1+\{\sum\limits_{\tau=1}^{n} \beth_\tau (\frac{\Theta_{i\tau_{\sigma_{i\tau}(\xi)}}}{1-\Theta_{i\tau_{\sigma_{i\tau}(\xi)}}})^{\Upsilon}\}^{1/\Upsilon'}}\right) / \sum\limits_{\tau=1}^{n} \Omega_{\sigma_\tau(\xi)}, \left(\frac{1}{1+\{\sum\limits_{\tau=1}^{n} \beth_\tau (\frac{1-\delta_{i\tau_{\sigma_{i\tau}(\xi)}}}{\delta_{i\tau_{\sigma_{i\tau}(\xi)}}})^{\Upsilon}\}^{1/\Upsilon'}}\right) / \sum\limits_{\tau=1}^{n} \Lambda_{\sigma_\tau(\xi)}, \\ \left(\frac{1}{1+\{\sum\limits_{\tau=1}^{n} \beth_\tau (\frac{1-\psi_{i\tau_{\sigma_{i\tau}(\xi)}}}{\psi_{i\tau_{\sigma_{i\tau}(\xi)}}})^{\Upsilon}\}^{1/\Upsilon}}\right) / \sum\limits_{\tau=1}^{n} \omega_{\sigma_\tau(\xi)} \end{array} \right\},$$

$$\Gamma_i = SV - NPHFDWAG(\Gamma_{i1}, \Gamma_{i2}, ..., \Gamma_{in})$$

$$= \left\{ \begin{array}{c} \left(\frac{1}{1+\{\sum\limits_{\tau=1}^{n} \beth_\tau (\frac{1-\Theta_{i\tau_{\sigma_{i\tau}(\xi)}}}{\Theta_{i\tau_{\sigma_{i\tau}(\xi)}}})^{\Upsilon}\}^{1/\Upsilon'}}\right) / \sum\limits_{\tau=1}^{n} \Omega_{\sigma_\tau(\xi)}, \left(1 - \frac{1}{1+\{\sum\limits_{\tau=1}^{n} \beth_\tau (\frac{\delta_{i\tau_{\sigma_{i\tau}(\xi)}}}{1-\delta_{i\tau_{\sigma_{i\tau}(\xi)}}})^{\Upsilon}\}^{1/\Upsilon'}}\right) / \sum\limits_{\tau=1}^{n} \Lambda_{\sigma_\tau(\xi)}, \\ \left(\frac{1}{1+\{\sum\limits_{\tau=1}^{n} \beth_\tau (\frac{\psi_{i\tau_{\sigma_{i\tau}(\xi)}}}{1-\psi_{i\tau_{\sigma_{i\tau}(\xi)}}})^{\Upsilon}\}^{1/\Upsilon}}\right) / \sum\limits_{\tau=1}^{n} \omega_{\sigma_\tau(\xi)} \end{array} \right\},$$

where $\beth = (\beth_1, \beth_2, ..., \beth_n)$ is weight vector $\beth_\tau \in [0,1]$ and $\sum\limits_{\tau=1}^{n} \beth_\tau = 1$.

Step-4 Using the suggested aggregation information, assess the aggregated SV-NPHFNs for each alternative that is being investigated in relation to the provided set of requirements.

Step-5 To rank the alternatives based on the scoring function, enter,

$$\Delta(\sigma(\xi)) = \frac{1}{3} \left(\begin{array}{c} 2 + \left(\frac{1}{Y_\zeta} \sum\limits_{\Theta_{\sigma_\xi} \in \Theta_{\sigma_{\sigma(\xi)}}, \Omega_{\sigma_\xi} \in \Omega_{\sigma_\xi}} (\Theta_{\sigma_\xi} \times \Omega_{\sigma_\xi}) \right) - \\ \left(\frac{1}{Y_\zeta} \sum\limits_{\delta_{\sigma_\xi} \in \delta_{\sigma_{\sigma(\xi)}}, \Lambda_{\sigma_\xi} \in \Lambda_{\sigma_\xi}} (\delta_{\sigma_\xi} \times \Lambda_{\sigma_\xi}) \right) - \\ \left(\frac{1}{Y_\zeta} \sum\limits_{\eta_{\sigma_\xi} \in \psi_{\sigma_{\sigma(\xi)}}, \Omega_{\sigma_\xi} \in \Omega_{\sigma_\xi}} (\eta_{\sigma_\xi} \times \omega_{\sigma_\xi}) \right) \end{array} \right),$$

Step-6 Sort all of the alternate results in decreasing order. The superior or ideal alternative will be the one with greater value.

All steps of the algorithm are shown in Figure 1.

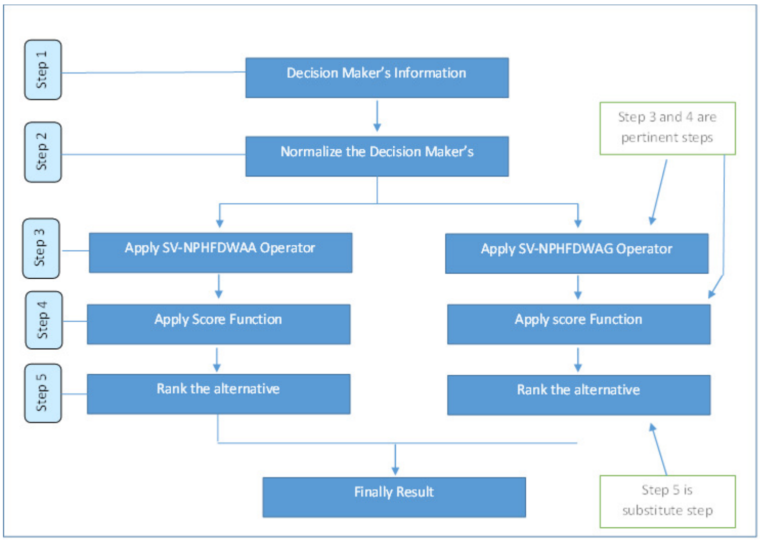

Figure 1: Graphical overview of the algorithm

5. CASE STUDY

Social media refers to websites and programmes that priorities cooperation, content sharing, engagement, and community-based feedback. People connect and communicate with their friends, family, and other communities using social media. It is a helpful tool for talking with individuals locally and internationally as well as for generating, sharing, and spreading information. Social media has the power to influence consumer buying decisions through reviews, advertising, and marketing tactics. Due to social media's recent technological development, little research has been done to ascertain whether using it might have favourable or unfavourable long-term repercussions. But a significant link has been found by multiple research between excessive social media use and an increased risk of depression, anxiety, loneliness, self-harm, and even suicidal ideation.

This is a straightforward method of making money on social media and is referred to as "sponsored posting." When a user clicks on the sponsored post that a business has sponsored to appear on a relevant page, you get paid. This is an effective strategy to use a sizable social media following. Most social networking sites include a framework for sponsored posts. Even the straight sign-up for Twitter's sponsored postings is available. To match your business with pertinent topics and begin making money with each appropriate tweet, visit Sponsored Tweets. With the help of its Amazon Associate programme, Amazon makes it simple to promote affiliate products. Amazon will give you a small percentage for any sale that results from the link you post. Visit their website for more details on how to sign up for the Amazon affiliate programme.

It's not about how many people "like" your postings that social media use in the classroom is on the rise. Students' ability to develop their creative, critical thinking, and communication skills can be accelerated in some ways when using social media because of the collaborative setting and open forum it promotes as well as the quick information exchange it allows. Social media stimulates independent thinking and inquiry, preparing students for life after school. When these social media skills are strengthened in a classroom environment, they can be guided and enhanced to produce superior learning outcomes and critical awareness.

Due to the fact that so many kids are accustomed to social media and technology, incorporating their use into the classroom is more natural than ever. Every social media site has a variety of educational applications, including sharing announcements, hosting live lectures, and much more. First, social media gives students, instructors, and parents a smoother, more direct way to communicate. They can check in and ask or answer questions. There are more chances for e-learning thanks to social media. Social networking may assist in teaching students how to work remotely, which is a vital lesson as remote employment and online education grow in popularity. Before adopting social media in education, it's critical to understand its effects, but we firmly believe that it will increase students' technological proficiency.

5.1. Numerical example

Someone is interested to take survey about social media. He wants to check how this media platforms are affecting our youth and provide earnings as well as Learning different knowledge for improving themselves for our youngster. He decide to make a group of four experts for different social media platforms (δ_1 =YouTube, δ_2 =Instagram, δ_3 = Facebook, δ_4 =Twitter) for taking a views about these platforms. This group create parameters for comparisons of different platforms with their earning and learning point of view to check which platform is doing his best to provide best opportunities for earnings as well as learning. We set the four parameters ($\Phi_1^{\mathfrak{z}}$ =Earning, $\Phi_2^{\mathfrak{z}}$ =Learning, $\Phi_3^{\mathfrak{z}}$ =Fun, $\Phi_4^{\mathfrak{z}}$ = Advertising) and collect the data from experts.

Step-1 The information of professional expert is given in Table 1(a)-1(d) in the form of SV-NPHFNs.

Table 1(a): Expert information

	$\Phi_1^{\mathfrak{z}}$	$\Phi_2^{\mathfrak{z}}$
δ_1	$\begin{pmatrix} (0.5/0.5, 0.2/0.5), \\ (0.6/0.6, 0.3/0.4), \\ (0.3/1.0) \end{pmatrix}$	$\begin{pmatrix} (0.3/1.0), \\ (0.9/1.0), \\ (0.1/0.3, 0.7/0.7) \end{pmatrix}$
δ_2	$\begin{pmatrix} (0.4/0.1, 0.6/0.9), \\ (0.7/1.0), \\ (0.8/0.4, 0.9/0.6) \end{pmatrix}$	$\begin{pmatrix} (0.8/1.0), \\ (0.2/0.5, 0.5/0.5), \\ (0.9/1.0) \end{pmatrix}$

Table 1(b): Expert information

	$\Phi_3^\imath$	$\Phi_4^\imath$
δ_1	$\left(\begin{array}{c}(0.4/0.8, 0.5/0.2), \\ (0.6/1.0), \\ (0.4/1.0)\end{array}\right)$	$\left(\left(\begin{array}{c}(0.2/1.0), \\ (0.3/1.0), \\ (0.2/0.4, 0.3/0.6)\end{array}\right)\right)$
δ_2	$\left(\begin{array}{c}(0.2/1.0), \\ (0.6/0.7, 0.3/0.3), \\ (0.4/1.0)\end{array}\right)$	$\left(\begin{array}{c}(0.9/1.0), \\ (0.4/1.0), \\ (0.4/1.0)\end{array}\right)$

Table 1(c): Expert information

	$\Phi_1^\imath$	$\Phi_2^\imath$
δ_3	$\left(\begin{array}{c}(0.5/1.0), \\ (0.6/0.6, 0.3/0.4), \\ (0.3/0.7, 0.4/0.3\end{array}\right)$	$\left(\begin{array}{c}(0.5/0.7, 0.2/0.2, 0.9/0.1), \\ (0.3/1.0), \\ (0.3/0.2, 0.4/0.8)\end{array}\right)$
δ_4	$\left(\begin{array}{c}(0.2/1.0), \\ (0.8/1.0), \\ (0.4/1.0)\end{array}\right)$	$\left(\begin{array}{c}(0.2/1.0), \\ (0.6/0.4, 0.4/0.4, 0.3/0.2), \\ (0.4/1.0)\end{array}\right)$

Table 1(d): Expert information

	$\Phi_3^\imath$	$\Phi_4^\imath$
δ_3	$\left(\begin{array}{c}(0.9/1.0), \\ (0.4/1.0), \\ (0.1/1.0)\end{array}\right)$	$\left(\begin{array}{c}(0.5/1.0), \\ (0.3/1.0), \\ (0.4/1.0)\end{array}\right)$
δ_4	$\left(\begin{array}{c}(0.2/1.0), \\ (0.1/1.0), \\ (0.5/0.3, 0.3/0.2, 0.4/0.5)\end{array}\right)$	$\left(\begin{array}{c}(0.2/0.8, 0.5/0.1, 0.2/0.1), \\ (0.6/1.0), \\ (0.1/1.0)\end{array}\right)$

Step-2 Expert opinion is of the beneficiary type. As a result, we don't need to normalize the SV-NPHFNs in this context.

Step-3 The following aggregation operators are used to evaluate the alternative's aggregation details under the given list of attributes:

Case 1: Table 2 shows the results of aggregation using the SV-NPHFDWAA and SV-NPHFDWAG operator.

Table 2(a): Aggregated details using $SV - NPHFDWAA$

δ_1	$\left(\begin{array}{c}\{0.2787/0.4, 0.3921/0.1, 0.3250/0.4, 0.3432/0.1\}, \\ \{0.4797/0.6, 0.4524/0.4\}, \\ \{0.2631/0.12, 0.2371/0.18, 0.3841/0.28, 0.4123/0.42\}\end{array}\right)$
δ_2	$\left(\begin{array}{c}\{0.5619/0.1, 0.5353/0.9\}, \\ \{0.3441/0.35, 0.3268/0.15, 0.4330/0.35, 0.4431/0.15\}, \\ \{0.3578/0.4, 0.4563/0.6\}\end{array}\right)$

Table 2(b): Aggregated information using $SV-NPHFDWAA$

δ_3	$\begin{pmatrix} \{0.6383/0.7, 0.5956/0.2, 0.7271/0.1, \}, \\ \{0.2974/0.6, 0.3466/0.4\}, \\ \{0.2367/0.14, 0.3172/0.56, 0.3184/0.06, 0.2976/0.24\}, \end{pmatrix}$
δ_4	$\begin{pmatrix} \{0.2329/0.8, 0.2148/0.1, 0.2231/0.1\}, \\ \{0.2816/0.4, 0.2472/0.4, 0.2552/0.2\}, \\ \{0.3112/0.3, 0.2601/0.2, 0.3132/0.5\} \end{pmatrix}$

Case-2: Applying the SV-NPHFDWAG operator to aggregate information as stated above, followed by the scoring function.

Step-5 Table 3 displays the score values for all alternatives with established aggregation operations.

Table 3: Score Values

Operators	$\Delta(\delta_1)$	$\Delta(\delta_2)$	$\Delta(\delta_3)$	$\Delta(\delta_4)$
$SV-NPHFDWAA$	0.3527	0.3154	0.3291	0.3412
$SV-NPHFDWAG$	0.4162	0.3419	0.3954	0.3578

Step-6 Rank the alternatives $\delta_\kappa(\kappa = 1, 2, ..., 4)$ is enclosed in Table 4.

Table 4: Ranking of the alternatives

Operators	Score	Best Alternative
$SV-NPHFDWAA$	$\Delta(\delta_1) > \Delta(\delta_4) > \Delta(\delta_3) > \Delta(\delta_2)$	δ_1
$SV-NPHFDWAG$	$\Delta(\delta_1) > \Delta(\delta_3) > \Delta(\delta_4) > \Delta(\delta_2)$	δ_1

From the aforementioned computational procedure, we deduced that δ_1 is the best option out of the bunch and is therefore strongly advised. Figure 2 shows the ranking.

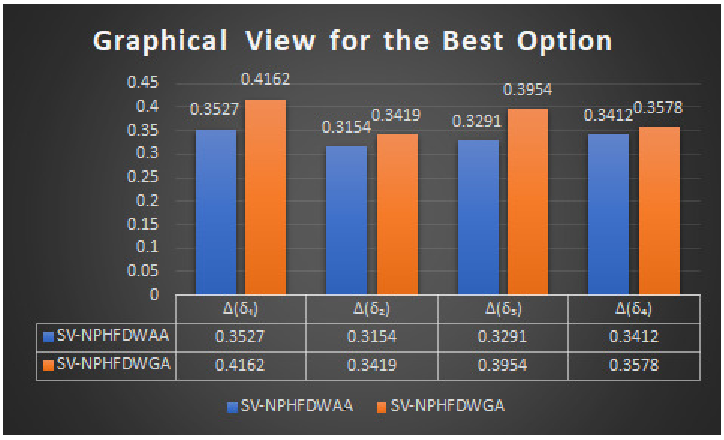

Figure 2: Numerical Comparison

6. RELIABILITY AND VALIDITY TEST

Choosing the best choice from the group's evaluation criteria is a difficult process in reality. Wang and Triantaphyllou proposed a method for evaluating the reliability and validity of DM systems [45]. The following is the testing technique.
Test Step-1: The right and effective MAGDM strategy is to provide the appropriate alternative with no change and also without modifying the similar position of each choice criterion in order to replace the weaker element of the alternative with the normalised element.
Test Step-2: The transitive property must be fulfilled using a MAGDM technique that is both efficient and accurate.
Test Step-3: when a significant MAGDM problem is lowered to a small problem. The un-decomposed problem's original rating should be reflected in the combined alternative rating. To rank the alternative, we employ the same strategies employed in the MAGDM problem on smaller difficulties.
The MAGDM issue was reduced to a smaller one and the same suggested DM technique was used in order to achieve the best results. We may apply the same strategy to a little issue and obtain the same result as with the MAGDM problem, making MAGDM an appropriate and useful technique.

6.1. Validity test the proposed DM methodology

In this section [45], We examine the adequacy and validity of our established strategy using the proficiency and validity of the abovementioned test. The SV-NPHF information is enclosed in the Table 5 as follows:

Table 5(a): Expert information

	$\Phi_1^{\imath}$	$\Phi_2^{\imath}$
δ_1	$\begin{pmatrix} (0.5/0.5, 0.2/0.5), \\ (0.6/0.6, 0.3/0.4), \\ (0.3/1.0) \end{pmatrix}$	$\begin{pmatrix} (0.3/1.0), \\ (0.9/1.0), \\ (0.1/0.3, 0.7/0.7) \end{pmatrix}$
δ_2	$\begin{pmatrix} (0.4/0.1, 0.6/0.9), \\ (0.7/1.0), \\ (0.8/0.4, 0.9/0.6) \end{pmatrix}$	$\begin{pmatrix} (0.8/1.0), \\ (0.2/0.5, 0.5/0.5), \\ (0.9/1.0) \end{pmatrix}$

Table 5(b): Expert information

	$\Phi_3^{\imath}$	$\Phi_4^{\imath}$
δ_1	$\begin{pmatrix} (0.4/0.8, 0.5/0.2), \\ (0.6/1.0), \\ (0.4/1.0) \end{pmatrix}$	$\begin{pmatrix} (0.2/1.0), \\ (0.3/1.0), \\ (0.2/0.4, 0.3/0.6) \end{pmatrix}$
δ_2	$\begin{pmatrix} (0.2/1.0), \\ (0.6/0.7, 0.3/0.3), \\ (0.4/1.0) \end{pmatrix}$	$\begin{pmatrix} (0.9/1.0), \\ (0.4/1.0), \\ (0.4/1.0) \end{pmatrix}$

Table 5(c): Expert information

	$\Phi_1^{\imath}$	$\Phi_2^{\imath}$
δ_3	$\begin{pmatrix} (0.5/1.0), \\ (0.6/0.6, 0.3/0.4), \\ (0.3/0.7, 0.4/0.3 \end{pmatrix}$	$\begin{pmatrix} (0.5/0.7, 0.2/0.2, 0.9/0.1), \\ (0.3/1.0), \\ (0.3/0.2, 0.4/0.8) \end{pmatrix}$
δ_4	$\begin{pmatrix} (0.2/1.0), \\ (0.8/1.0), \\ (0.4/1.0) \end{pmatrix}$	$\begin{pmatrix} (0.2/1.0), \\ (0.6/0.4, 0.4/0.4, 0.3/0.2), \\ (0.4/1.0) \end{pmatrix}$

Table 5(d): Expert information

	$\Phi_3^{\imath}$	$\Phi_4^{\imath}$
δ_3	$\begin{pmatrix} (0.9/1.0), \\ (0.4/1.0), \\ (0.1/1.0) \end{pmatrix}$	$\begin{pmatrix} (0.5/1.0), \\ (0.3/1.0), \\ (0.4/1.0) \end{pmatrix}$
δ_4	$\begin{pmatrix} (0.2/1.0), \\ (0.1/1.0), \\ (0.5/0.3, 0.3/0.2, 0.4/0.5) \end{pmatrix}$	$\begin{pmatrix} (0.2/0.8, 0.5/0.1, 0.2/0.1), \\ (0.6/1.0), \\ (0.1/1.0) \end{pmatrix}$

Test Step 1:-In this stage, we replace the poorer part of the alternative by providing the appropriate alternative with no changes and no changes to the comparable positions of each selection criterion. Table 6 enclosed the updated decision matrix

Table 6(a): Update Expert information

	$\Phi_1^{\imath}$	$\Phi_2^{\imath}$
δ_1	$\begin{pmatrix} (0.3/1.0), \\ (0.6/0.6, 0.3/0.4), \\ (0.5/0.5, 0.2/0.5) \end{pmatrix}$	$\begin{pmatrix} (0.3/1.0), \\ (0.9/1.0), \\ (0.1/0.3, 0.7/0.7) \end{pmatrix}$
δ_2	$\begin{pmatrix} (0.8/0.4, 0.9/0.6), \\ (0.7/1.0), \\ (0.4/0.1, 0.6/0.9) \end{pmatrix}$	$\begin{pmatrix} (0.8/1.0), \\ (0.2/0.5, 0.5/0.5), \\ (0.9/1.0) \end{pmatrix}$

Table 6(b): Update Expert information

	$\Phi_3^{\imath}$	$\Phi_4^{\imath}$
δ_1	$\begin{pmatrix} (0.4/1.0), \\ (0.6/1.0), \\ (0.4/0.8, 0.5/0.2) \end{pmatrix}$	$\begin{pmatrix} (0.2/1.0), \\ (0.3/1.0), \\ (0.2/0.4, 0.3/0.6) \end{pmatrix}$
δ_2	$\begin{pmatrix} (0.4/1.0), \\ (0.6/0.7, 0.3/0.3), \\ (0.2/1.0) \end{pmatrix}$	$\begin{pmatrix} (0.9/1.0), \\ (0.4/1.0), \\ (0.4/1.0) \end{pmatrix}$

Table 6(c): Update Expert information

	Φ_1^ι	Φ_2^ι
δ_3	$\begin{pmatrix} (0.5/1.0), \\ (0.6/0.6, 0.3/0.4), \\ (0.3/0.7, 0.4/0.3 \end{pmatrix}$	$\begin{pmatrix} (0.3/0.2, 0.4/0.8), \\ (0.3/1.0), \\ (0.5/0.7, 0.2/0.2, 0.9/0.1) \end{pmatrix}$
δ_4	$\begin{pmatrix} (0.2/1.0), \\ (0.8/1.0), \\ (0.4/1.0) \end{pmatrix}$	$\begin{pmatrix} (0.4/1.0), \\ (0.6/0.4, 0.4/0.4, 0.3/0.2), \\ (0.2/1.0) \end{pmatrix}$

Table 6(d): Update Expert information

	Φ_3^ι	Φ_4^ι
δ_3	$\begin{pmatrix} (0.9/1.0), \\ (0.4/1.0), \\ (0.1/1.0) \end{pmatrix}$	$\begin{pmatrix} (0.4/1.0), \\ (0.3/1.0), \\ (0.5/1.0) \end{pmatrix}$
δ_4	$\begin{pmatrix} (0.2/1.0), \\ (0.1/1.0), \\ (0.5/0.3, 0.3/0.2, 0.4/0.5) \end{pmatrix}$	$\begin{pmatrix} (0.1/1.0), \\ (0.6/1.0), \\ (0.2/0.8, 0.5/0.1, 0.2/0.1) \end{pmatrix}$

Using the suggested set of SV-neutrosophic probabilistic hesitant fuzzy Dombi aggregation operators, we now calculate the combined preference values of each alternative under the criterion weight $w = (0.1, 0.3, 0.4, 0.2)^T$ as follows:

Case-I: Aggregated information using SV-NPHFDWAA and SV-NPHFDWAG operators. Now Apply the score function and we can obtained the result as given in Table 7.

Table 7: Score Values

Operators	$\Delta(\delta_1)$	$\Delta(\delta_2)$	$\Delta(\delta_3)$	$\Delta(\delta_4)$
SV-NPHFDWAA(updated)	0.3566	0.3106	0.3512	0.3481
SV-NPHFDWAG(updated)	0.4193	0.3142	0.3491	0.3241

Rank the alternatives $\delta_\kappa(\kappa = 1, 2, ..., 4)$ is enclosed in Table 8.

Table 8: Ranking of the alternatives

Operators	Score	Best Altern.
SV-NPHFDWAA(updated)	$\Delta(\delta_1) > \Delta(\delta_3) > \Delta(\delta_4) > \Delta(\delta_2)$	δ_1
SV-NPHFDWAG(updated)	$\Delta(\delta_1) > \Delta(\delta_3) > \Delta(\delta_4) > \Delta(\delta_2)$	δ_1

We get again the same alternative δ_1 by using the test step-1, which is also obtained by applying of our suggested method. Numerical comparison for validity test is shown in Figure-3.

Test Step-2 & 3 We are now testing the validity test steps-2 & 3 to demonstrate that the proposed approach is reliable and relevant. To this end, we first transformed the MAGDM problem into three smaller sub-problems such as $\{\delta_2, \delta_1, \delta_4\}$, $\{\delta_1, \delta_4, \delta_3\}$ and $\{\delta_2, \delta_4, \delta_3\}$. We now implement our suggested decision-making approach to the smaller problems that have been transformed and give us the ranking

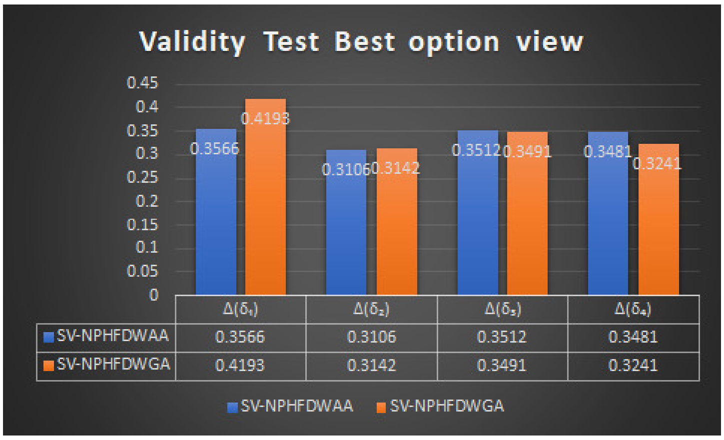

Figure 3: Numerical Comparison for Validity Test

of alternatives as: $\delta_1 > \delta_3 > \delta_2$, $\delta_1 > \delta_3 > \delta_4$ and $\delta_3 > \delta_4 > \delta_2$ respectively. We analyzed that $\delta_1 > \delta_3 > \delta_4 > \delta_2$ is the same as the standard decision-making approach results when assigning a detailed ranking.

6.2. Advantages

This section explains the benefits of the proposed work over the current work. These are the benefits of our work:

- In order to overcome the issue of MCDM information representation, SV-NPHFSs combined SV-NSs and PHFSs. Hence, SV-NPHFSs were essential in explaining unclear and incomplete MCDM results.
- Starting on probability theory and Bayesian processes, probability with reluctant fuzzy sets shown fault tolerance to address the challenge of MCDM information processing. PHFSs are therefore essential for dealing with incomplete and noisy data, and they may be thought of as a useful tool for thorough MCDM information analysis.
- Evaluation professionals are frequently sought for two pieces of information when dealing with real-world decision-making challenges: their expertise of the evaluation domains and the effectiveness of the assessment objects. All of the methods in use today don't trust the expertise of the experts and only take into account positive facts. The challenges were present, but our suggested approachSV-NPHF Dombi arithmatic AOswas successful and surmounted them.
- As a realistic and helpful technique for modelling various uncertainties in typical MCDM scenarios, SV-NPHFSs were used. By breaking the idea of Dombi arithmetic into three parts, it is possible to define indefinite and incomplete MCDM information with precision.

- Dombi arithmetic can be used to significantly increase the computational effectiveness of information fusion in MCDM information fusion algorithms. Moreover, it might be possible to properly estimate the choice risks connected to information fusion processes.

7. CONCLUSION

We deal with intricate and advanced data every day. In order to work more effectively and compute thorough information, we developed methodologies and tools for this type of data. A fundamental cost of aggregation is the expense of reducing the amount of data to a single value. For situations when each item has a range of possible values dictated by MD, indeterminacy, and non-MD, the SV-NPHFS was created as a potent fusion of an SV-NS and PHFS. Both SV-NPHFWAA and SV-NPHFWAG operators were recommended. The SV-NPHFWAA and SV-NPHFWAG operators were used as the foundation for a unique MADM technique. Below we go into more depth about the advantages of these techniques.

- Initially, the important properties of the idempotency, commutativity, boundedness, and monotonicity of the SV-NPHFWAG and SV-NPHFWAA operators are explored.
- Second, it was shown that the operators we suggested are more flexible than the prior operators, and we compared the suggested AOs' flexibility to the earlier AOs.
- Third, when compared to other existing techniques for MCDM problems in an SV-NPHF environment, the results produced by the SV-NPHFWAA and SV-NPHFWAG operators are accurate and dependable, proving their usefulness in real-world applications.
- The MCDM techniques proposed in this paper are also capable of recognizing more correlation between attributes and alternatives, demonstrating that they have a higher accuracy and a larger setpoint than the existing methodologies, which are unable to take into account the inter-relationships of attributes in practical uses. This shows that by using the MCDM procedures outlined in this paper, many additional links between features may be discovered.
- Future studies on personalized individual consistency control consensus problems, consensus building with non-cooperative behaviour management decision-making problems, and two-sided matching decision-making with multi-granular and incomplete criteria weight information may make use of the proposed AOs. This analysis of the restrictions imposed by recommended AOs disregards the levels of participation, abstention, and non-membership. On this side of the proposed AOs, a new hybrid structure of prioritised, interactive AOs is being put into place.
- We will study the theoretical underpinnings of SV-NPHFSs for Einstein operations in upcoming work using state-of-the-art decision-making methodologies like TOPSIS, VIKOR, TODAM, GRA, and EDAS. We'll also go

over the ways in which these techniques are used in a number of disciplines, including soft computing, robotics, horticulture, intelligent systems, social sciences, finance, and human resource management.

Acknowledgements. The authors would like to thank the Deanship of Scientific Research at Umm Al-Qura University for supporting this work by Grant Code: 22UQU4310396DSR57.

Funding. This research received no external funding.

REFERENCES

[1] L.A. Zadeh, "Fuzzy sets", *Information and Control*, vol. 8, no. 3, pp. 338-353, 1965.

[2] K.T. Atanassov, "Intuitionistic fuzzy sets", *Fuzzy Sets and Systems*, vol. 20, no. 1, pp. 87-96, 1986.

[3] Z. Xu, "Intuitionistic fuzzy aggregation operators", *IEEE Transactions on Fuzzy Systems*, vol. 15, no. 6, pp. 1179-1187, 2007.

[4] Z. Xu, & R.R. Yager, "Some geometric aggregation operators based on intuitionistic fuzzy sets",, *International Journal of General Systems*, vol. 35, no. 4, pp. 417-433, 2006.

[5] T. Mahmood, U. ur Rehman, and Z. Ali, "Analysis and application of Aczel-Alsina aggregation operators based on bipolar complex fuzzy information in multiple attribute decision making", *Information Sciences*, vol. 619, pp. 817-833, 2023.

[6] Z. Xu, and R.R. Yager, "Some geometric aggregation operators based on intuitionistic fuzzy sets", *International Journal of General Systems*, vol. 35, no. 4, pp. 417-433, 2006.

[7] M. Saad, T. Mahmood, K. Ullah, and N. Jan, "Computing shortest path in a single valued neutrosophic hesitant fuzzy network", *The Nucleus*, vol. 56, no. 3, pp. 123-130, 2019.

[8] V. Torra, "Hesitant fuzzy sets", *International Journal of Intelligent Systems*, vol. 25, no. 6, pp. 529-539, 2010.

[9] S. Ashraf, S. Abdullah, S. Zeng, H. Jin, and F. Ghani, "Fuzzy decision support modeling for hydrogen power plant selection based on single valued neutrosophic sine trigonometric aggregation operators", *Symmetry*, vol. 12, no. 2, p. 298, 2020.

[10] T. Mahmood, and U. Ur Rehman, "A novel approach towards bipolar complex fuzzy sets and their applications in generalized similarity measures", *International Journal of Intelligent Systems*, vol. 37, no. 1, pp. 535-567, 2022.

[11] J. Liu, & M. Sun, "Generalized power average operator of hesitant fuzzy numbers and its application in multiple attribute decision making", *Journal of Computational Information Systems*, vol. 9, no. 8, pp. 3051-3058, 2013.

[12] M. Xia, & Z. Xu, "Hesitant fuzzy information aggregation in decision making", *International Journal of Approximate Reasoning*, vol. 52, no. 3, pp. 395-407, 2011.

[13] H. Liao, & Z. Xu, "Extended hesitant fuzzy hybrid weighted aggregation operators and their application in decision making", *Soft Computing*, vol. 19, no. 9, pp. 2551-2564, 2015.

[14] Z. Xu, and W. Zhou, "Consensus building with a group of decision makers under the hesitant probabilistic fuzzy environment", *Fuzzy Optimization and Decision Making*, vol. 16, no. 4, pp. 481-503, 2017.

[15] M. Kamran, S. Ashraf, N. Salamat, M. Naeem, and T. Botmart, "Cyber security control selection based decision support algorithm under single valued neutrosophic hesitant fuzzy Einstein aggregation information", *AIMS Mathematics*, vol. 8, no. 3, pp. 5551-5573, 2023.

[16] R. Krishankumar, K.S. Ravichandran, P. Liu, S. Kar, and A.H. Gandomi, "A decision framework under probabilistic hesitant fuzzy environment with probability estimation for multi-criteria decision making", *Neural Computing and Applications*, vol. 33, pp. 8417-8433, 2021.

[17] J. Ye, "Multicriteria decision-making method using the correlation coefficient under single-valued neutrosophic environment", *International Journal of General Systems*, vol. 42, no. 4, pp. 386-394, 2013.

[18] A.A. Khan, S. Ashraf, S. Abdullah, M. Qiyas, J. Luo, and S.U. Khan, "Pythagorean fuzzy Dombi aggregation operators and their application in decision support system", *Symmetry*, vol. 11, no. 3, pp.383, 2019.
[19] P. Rani, A.R. Mishra, R. Krishankumar, K.S. Ravichandran, and S. Kar, "Multi-criteria food waste treatment method selection using single-valued neutrosophic-CRITIC-MULTIMOORA framework", *Applied Soft Computing*, vol. 111, pp.107657, 2021.
[20] J. Ye, "A multicriteria decision-making method using aggregation operators for simplified neutrosophic sets", *Journal of Intelligent & Fuzzy Systems*, vol. 26, no. 5, pp. 2459-2466, 2014.
[21] P. Rani, J. Ali, R. Krishankumar, A.R. Mishra, F. Cavallaro, and K.S. Ravichandran, "An integrated single-valued neutrosophic combined compromise solution methodology for renewable energy resource selection problem", *Energies*, vol. 14, no. 15, pp. 4594, 2021.
[22] Y. Jin, M. Kamran, N. Salamat, S. Zeng, and R.H. Khan, "Novel Distance Measures for Single-Valued Neutrosophic Fuzzy Sets and Their Applications to Multicriteria Group Decision-Making Problem", *Journal of Function Spaces*, 2022.
[23] J. Dombi, "A general class of fuzzy operators, the DeMorgan class of fuzzy operators and fuzziness measures induced by fuzzy operators", *Fuzzy Sets and Systems*, vol. 8, no. 2, pp. 149-163, 1982.
[24] P. Liu, J. Liu, and S.M. Chen, "Some intuitionistic fuzzy Dombi Bonferroni mean operators and their application to multi-attribute group decision making", *Journal of the Operational Research Society*, vol. 69, no. 1, pp. 1-24, 2018.
[25] M.R. Seikh, and U. Mandal, "Intuitionistic fuzzy Dombi aggregation operators and their application to multiple attribute decision-making", *Granular Computing*, vol. 6, pp. 473-488, 2021.
[26] H. Garg, R. Krishankumar, and K.S. Ravichandran, "Decision framework with integrated methods for group decision-making under probabilistic hesitant fuzzy context and unknown weights", *Expert Systems with Applications*, vol. 200, pp. 117082, 2022.
[27] J. Chen, and J. Ye, "Some single-valued neutrosophic Dombi weighted aggregation operators for multiple attribute decision-making", *Symmetry*, vol. 9, no. 6, pp. 82, 2017.
[28] L. Shi, and J. Ye, "Dombi aggregation operators of neutrosophic cubic sets for multiple attribute decision-making", *Algorithms*, vol. 11, no. 3, pp. 29, 2018.
[29] C. Jana, and M. Pal, "Multi-criteria decision making process based on some single-valued neutrosophic Dombi power aggregation operators", *Soft Computing*, vol. 25, pp. 5055-5072, 2021.
[30] X. Lu, and J. Ye, "Dombi aggregation operators of linguistic cubic variables for multiple attribute decision making", *Information*, vol. 9, no. 8, pp. 188, 2018.
[31] X. He, "Typhoon disaster assessment based on Dombi hesitant fuzzy information aggregation operators", *Natural Hazards*, vol. 90, pp. 1153-1175, 2018.
[32] C. Jana, T. Senapati, M. Pal, and R.R. Yager, "Picture fuzzy Dombi aggregation operators: application to MADM process", *Applied Soft Computing*, vol. 74, pp.99-109, 2019.
[33] C. Jana, M. Pal, and J.Q. Wang, "Bipolar fuzzy Dombi aggregation operators and its application in multiple-attribute decision-making process", *Journal of Ambient Intelligence and Humanized Computing*, vol. 10, pp. 3533-3549, 2019.
[34] G. Wei, and Y. Wei, "Some single-valued neutrosophic dombi prioritized weighted aggregation operators in multiple attribute decision making", *Journal of Intelligent & Fuzzy Systems*, vol. 35, no. 2, pp. 2001-2013, 2018.
[35] X. Peng, and F. Smarandache, "Novel neutrosophic Dombi Bonferroni mean operators with mobile cloud computing industry evaluation", *Expert Systems*, vol. 36, no. 4, pp.e12411, 2019.
[36] A. Saha, D. Dutta, and S. Broumi, "Bulbul disaster assessment using single-valued spherical hesitant neutrosophic Dombi weighted aggregation operators", *Neutrosophic Operational Research: Methods and Applications*, pp. 221-243, 2021.
[37] V. Torra, and Y. Narukawa, "On hesitant fuzzy sets and decision", in:*IEEE International Conference on Fuzzy Systems*, Jeju, Korea (South), August, 2009, (pp. 1378-1382).
[38] W. Zhou, & Z. Xu, "Group consistency and group decision making under uncertain probabilistic hesitant fuzzy preference environment", *Information Sciences*, vol. 414, pp. 276-288,

2017.

[39] F. Smarandache, *A unifying field in logics, neutrosophy: Neutrosophic probability, Set and Logic*, American Research Press, Rehoboth, 1999.

[40] R. Sahin, and P. Liu, "Correlation coefficient of single-valued neutrosophic hesitant fuzzy sets and its applications in decision making", *Neural Computing and Applications*, vol. 28, pp. 1387-1395, 2017.

[41] S. Ashraf, S. Abdullah, S. Zeng, H. Jin, and F. Ghani, "Fuzzy decision support modeling for hydrogen power plant selection based on single valued neutrosophic sine trigonometric aggregation operators", *Symmetry*, vol. 12, no. 2, pp. 298, 2020.

[42] Z. Pawlak, "Rough sets", *International Journal of Computer & Information Sciences*, vol. 11, no. 5, pp. 341-356, 1982.

[43] R. Chinram, A. Hussain, T. Mahmood, & M.I. Ali, "EDAS method for multi-criteria group decision making based on intuitionistic fuzzy rough aggregation operators", *IEEE Access*, vol. 9, pp. 10199-10216, 2021.

[44] H. Wang, F. Smarandache, Y. Zhang, and R. Sunderraman, *Single Valued Neutrosophic Sets*, Infinite study, 2010.

[45] X. Wang, & E. Triantaphyllou, "Ranking irregularities when evaluating alternatives by using some ELECTRE methods", *Omega*, vol. 36, no. 1, pp. 45-63, 2008.