

GENETIC ALGORITHMS AND SMOOTHING FILTERS IN SOLVING THE GEOPHYSICAL INVERSION PROBLEM

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Abstract: The combination of genetic algorithms, smoothing filters and geophysical tomography is used in solving the geophysical inversion problem. This hybrid technique is developed to improve the results obtained by using genetic algorithms only. The application of smoothing filters can improve the performance of GA implementation for solving the geophysical inversion problem. Some test-examples and the obtained comparative results are presented.

Keywords: Genetic algorithms, smoothing filters, tomography.

1. INTRODUCTION

Tomographic reconstruction techniques are used in different fields: geophysical exploration, medical imaging, astronomy etc. By using geophysical tomography, the characteristics of an underground region between two boreholes can be determined. The tomographic technique provides a means of estimating P-wave velocity in the region between two boreholes. In the time measuring procedure, wave transmitters are placed in one borehole and receivers in the other one in order to collect the first-arrival traveltime data. By using the first-arrival traveltime data, the distribution of wave velocities along a plan between two boreholes is calculated and displayed as a digital picture. Because of that, this problem is known as Image Reconstruction from Projections. The standard tomographic procedures for solving this problem are based on the decomposition of the cross-hole area into a number of cells and an assumption of straight raypaths. The details about the application of

geophysical tomography and basic notions related to this method can be found in: De Franco and Cavagna [4], Dines and Lytle [5], Ivanson [12], and Peterson et al. [17].

1.1. Problem formulation

The geophysical inversion problem is a problem of determining the characteristics of an underground region by using measurement data. The solution to this problem is often based on applying the tomographic reconstruction techniques.

The examined underground region, called the cross-hole region, is some rectangular area between two boreholes. The problem is to estimate the seismic velocities from first-arrival traveltimes in a cross-hole region using a straight-line ray model. To discretise the geophysical inversion problem, the usual tomographic procedure (Dines and Lytle [5], Peterson et al. [17]) is used. The cross-hole region is divided into a grid of cells (see Fig.1). Some constant, initial velocity $v(x,y)$ is assumed in every cell. Traveltime and velocity structure are interrelated in the relation

$$t_k = \int_{R_k} \frac{ds}{v(x,y)} \quad (1)$$

where t_k is the traveltime of the k^{th} ray, ds is the differential raypath length of the k^{th} ray, $v(x,y)$ is the two-dimensional velocity function and R_k is the raypath trajectory of the k^{th} ray (Dines and Lytle [5], Peterson et al. [17]).

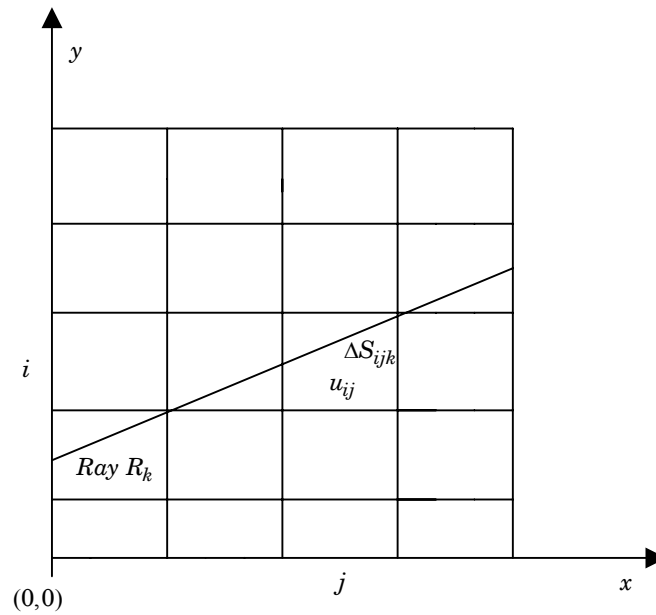


Figure 1: Discrete model of the cross-hole region

It is convenient to denote the reciprocal value $1/v(x,y)$ with $u(x,y)$, so we deal with slowness instead of velocity. Then the equation (1) becomes

$$t_k = \int_{R_k} u(x,y) ds \quad (2)$$

When the cross-hole region is divided into a grid of cells $(m \times n)$, the equation (2) can be approximated as

$$t_k = \sum_{i=1}^m \sum_{j=1}^n u_{ij} \Delta s_{ijk} \quad (3)$$

where u_{ij} is the unknown slowness in the (i,j) -cell, Δs_{ijk} is the length of a ray segment that intercepts the (i,j) -cell, m is the number of the vertical cells and n is the number of the horizontal cells. It is understood that $\Delta s_{ijk} = 0$ for those i and j values for which the associated cell is not intercepted by the ray R_k . A detailed explanation about geophysical tomography can be seen in Dines and Lytle [5], Herman [10] and Ivanson [12].

1.2. Genetic algorithms

Genetic algorithms are a kind of a mathematical simulation for Darwin's theory of evolution. The basic notions and description of genetic algorithms may be found in Goldberg [8]. The starting point is a formation of the initial population either using some particular method or at random. The elements of the initial population, individuals, are points from the searching space for a giving problem. Every individual is uniquely determined by its genetic material. The adaptation of every individual has to be found according to the fact how good a solution that individual is. To that end, the appropriated value of the fitness function is assigned to every individual. Using selection and the values of the fitness function, "the best fitted" individuals are being chosen, while a new population is formed using crossover and mutation. The rules of genetics and evolution imply a greater probability that the new population have a better genetic material. Iterating this procedure, from generation to generation, the genetic material of the population becomes better and the process should converge to the optimal solution of a given problem. If the specified number of generation is reached or some criteria of convergence are fulfilled, the procedure stops.

More information about GA can be found in: Beasley et al. [2], Beasley et al. [3], Goldberg [8], Ribeiro-Filho et al. [20], Srinivas and Patnaik [23].

1.3. Smoothing filters

The application of the appropriated smoothing filters can improve the performance of the used iterative reconstruction algorithm (Herman [10], Herman and Lent[11]) and form smooth areas in regions of the image. That implies better-defined boundaries in a reconstructed image (Balanis and Bentley [1]). The possibility of

achieving these improvements makes the combination of filters and the iterative reconstruction algorithm attractive (Herman [10], Herman and Lent[11]). Usually, the smoothing filter is implemented between the iterative steps of the reconstruction algorithm (Balanis and Bentley [1]). All cells in the image go through filter. For every cell the window of the filter is formed consisting of that particular "center" cell and its neighboring cells. Some often-used shapes of window can be seen at Fig. 2. The value of the center cell is replaced with a new value and it is calculated using the values of cells within the window of that center cell. The new values become actual after all cells in the image have been passed through the filter. (More details are exposed in section 3.1, 3.2 and 3.3.)

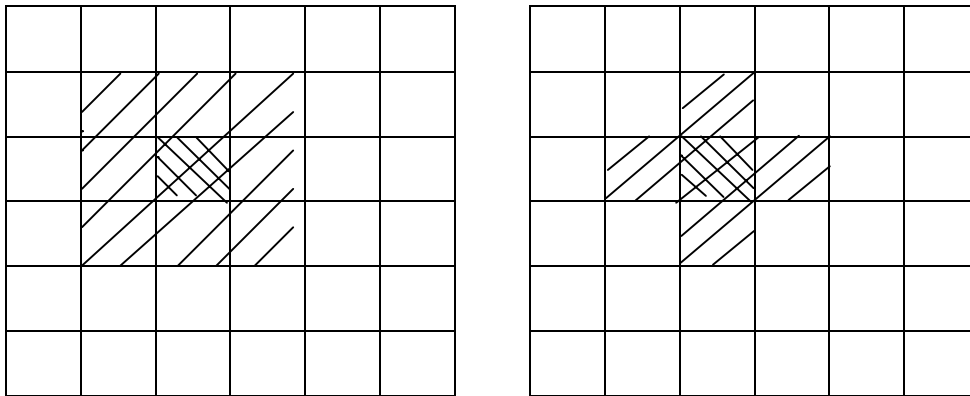


Figure 2: Square-shaped window and cross-shaped window

2. SOLVING THE GEOPHYSICAL INVERSION PROBLEM USING GENETIC ALGORITHMS

This section contains a short review of solving the geophysical inversion problem with the use of genetic algorithms. Detailed explanation is presented in Šešum et al [24].

The encoding mechanism is a very important issue for the structure of a genetic algorithm. The genes representing coded numerical value of velocity in a cell of cross-hole region have the same fixed length. In the described implementation that length is 5 bits (this information has to be given in advance). The value of velocity is obtained by

$$v = \min v + dv \cdot tmp, \quad (4)$$

where dv is the velocity partition,

$$dv = (\max v - \min v) / 2^{nbit} \quad (5)$$

$\min v$ is the minimum velocity, $\max v$ is the maximum velocity, tmp is the value of the binary string that represents velocity over a cell and $nbit$ is given in advance.

The objective function is

$$\delta = \sqrt{\frac{\sum_{i=1}^m \sum_{j=1}^n (t_{ij}^T - t_{ij}^C)^2}{k^2}} \quad (6)$$

where t_{ij}^T is the measured traveltime, t_{ij}^C is the computed traveltime and k is the number of transmitters/receivers. The computed traveltimes are calculated from (3) and measured traveltimes are obtained after the measurement in a borehole. The objective function (6) arises as a measure of differences between the computed and measured traveltimes. The goal is to minimize this function. So, we have here a typical optimization problem and a genetic algorithm could be applied.

The following genetic operators are used: rank-based selection, one-point crossover and mutation. The crossover rate is $p_{cross} = 0.85$ and the mutation rate is in segment $[0.001, 0.03]$, depending on the number of cells in the cross-hole region.

The population size is 150 individuals and the maximum number of generations is 5000.

The proposed method is tested on synthetic examples, where the dimensions of the cross-hole regions are 100m×100m and 80m×60m. This gave promising results and provided quite good performances. (More details about the application of genetic algorithms can be found in: Beasley at al. [2], Beasley at al. [3] and Goldberg [8].)

3. SOLVING THE GEOPHYSICAL INVERSION PROBLEM USING SMOOTHING FILTERS AND GENETIC ALGORITHMS

This section describes a new hybrid technique - the combination of smoothing filters and genetic algorithms. The purpose of developing this technique was to improve the results obtained using genetic algorithm only.

Experiences in researching filters and their application in geophysical tomography (Balanis and Bentley[1]) showed that the following filters gave relatively good results:

- minimum variance partitioning-averaging, MVP-AVG
- minimum variance partitioning-median filtering, MVP-MED
- selective smoothing

3.1. MVP-AVG

Instead of using all neighboring cells in the window (see Fig 2.), it is suggested to select a group of neighbors in the window (Balanis and Bentley [1]), in order to obtain better results. The selection for a group of neighbors can be done with MVP technique (Hall[9]).

For a given set of numbers, MVP technique finds the partition of set with minimum variance. The set of number is divided into several groups (usually two or three). To reduce the number of possible partitions, the numbers from a given set were previously sorted increasingly. For n numbers, there are $n - 1$ partitions and every partition has a fixed (in advance) number of groups. Each partition is processed in the following way:

- for every partition's group, finding its mean $m_i, i = 1, \dots, \text{number_of_groups}$
- calculating

$$S_l = \sum_{i=1}^{\text{numberofgroups}} \left[\sum_{u_j \in \text{group}_i} |u_j - m_i|^2 \right], \quad l = 1, \dots, n - 1 \quad (7)$$

Finally, the minimum S_{\min} in $(S_l)_{l=1}^{n-1}$ is found and the partition that is joined with the value S_{\min} is the minimum variance partitioning for that set of numbers. This way provides an objective mechanism for determining the neighborhood for a given center cell. Then, a new value of the center cell can be determined in various ways.

Example. Suppose that the following values are in the cells: 2,5,3,9,7 and the number of groups is 2. After the sorting, we have: 2,3,5,7,9. The partitions are:

$$S_1 : \{2\} \{3,5,7,9\}, \quad m_1 = 2.00, \quad m_2 = 6.00$$

$$S_2 : \{2,3\} \{5,7,9\}, \quad m_1 = 2.50, \quad m_2 = 7.00$$

$$S_3 : \{2,3,5\} \{7,9\}, \quad m_1 = 3.33, \quad m_2 = 8.00$$

$$S_4 : \{2,3,5,7\} \{9\}, \quad m_1 = 4.25, \quad m_2 = 9.00$$

Applying the formula (7), we get: $S_1 = 20, S_2 = 8.5, S_3 = 7.01, S_4 = 14.75$. The minimum is S_3 and the selected partition is $S_3 : \{2,3,5\} \{7,9\}$.

One approach is to replace the value of the center cell with the average value of group that contains the center cell. This technique is called MVP-AVG.

More information about MVP technique, averaging technique and MVP-AVG can be found in: Balanis and Bentley [1], Rosenfeld [21], Radcliff et al. [19], Schowengerdt [22] and Hall [9].

3.2. MVP-MED

After determining the neighborhood for a given center cell using MVP technique, as described in 3.1, this technique replaces the value of the center cell with the median value of the group that contains the center cell (Balanis and Bentley [1], Ekstrom [7], Rosenfeld [21]).

Example: Let us consider the square-shaped window from Fig 2. The center cell will take on the fifth highest value of the cells within the window.

3.3. Selective smoothing

New value of the center cell is formed by that cell and those neighboring cells with values that satisfy the condition, implied by the threshold level. If the difference between the value u_1 of the center cell and the value u_i of the neighboring cell is less or equal to the threshold level, that neighboring cell will be included in calculating a new value of the center cell (Radcliff and Balanis [18], Herman [10], Balanis and Bentley [1]).

Example: Let us consider the square-shaped window from Fig. 2 with dimension 3×3 which is the neighborhood of the center cell u_1 . The new value of that center cell can be obtained from

$$u_1^I = \frac{w_1 u_1 + w_2 \sum_{i=2}^5 f_i u_i + w_3 \sum_{i=6}^9 f_i u_i}{w_1 + w_2 \sum_{i=2}^5 f_i + w_3 \sum_{i=6}^9 f_i} \quad (8)$$

where

$$f_i = \begin{cases} 1, & \text{for } |u_i - u_1| \leq t \\ 0, & \text{else} \end{cases} \quad i = 2, \dots, 9 \quad (9)$$

w_1, w_2, w_3 are smoothing weights (chosen in advance), t is threshold level.

The successful application of this filter and the achievement satisfactory results are conditioned by the choice of the optimal values for w_1, w_2, w_3 and t (Radcliff and Balanis [18]).

3.4. Genetic algorithms and filters

The filters described in the previous section are implemented in the GA application for solving the geophysical inversion problem to improve the reconstruction of profile. Applying the appropriated filter between every iteration in the reconstruction algorithm (i.e. GA generation) can improve the convergency of that algorithm, if the system of equations (3) is consistent. Otherwise, this approach does not give good results (Balanis and Bentley [1]). In that case, the improvement can be obtained by using filter after a certain number of iterations, when the picture reaches some development (Balanis and Bentley [1]).

A lot of numerical experiments are made with a different number of generations and different filters. In the following variants of the hybrid technique, the optimal number of generations (obtained by experiments) and optimal filters are used.

- a) 4000 GA generations were applied to the test-profile; then such partially reconstructed profile is subjected to 100 passings through filter after every next GA generation;
- b) after every 100 GA generations, a partially reconstructed profile is subjected to filtering for 10 times;

- c) 3000 GA generations were applied to the test-profile; then such partially reconstructed profile is subjected to 1 passing through filter after every next GA generation;
- d) after applying 5000 GA generations to the test-profile, the reconstructed profile (obtained by running of the pure GA) is subjected to filtering.
Two-group MVP is used.

Some conclusions about the parameters (presented below) in selective smoothing are accepted, as suggested in Radcliff and Balanis [18]. The investigated region may have one or more anomalies where values differ from background's values (see Fig 3.). For profiles that have high values in the anomaly's cells and low values in the background's cells, $1/6$ of the difference between the highest value and the lowest value in the profile is assigned to the variable t . Reversely, for profiles that have low values in the anomaly's cells and high values in the background's cells, $2/5$ of the difference between the highest value and the lowest value in the profile is assigned to the variable t . The values w_1, w_2 and w_3 are chosen to satisfy the relation $w_1 = w_2 = 2w_3$ for every profile.

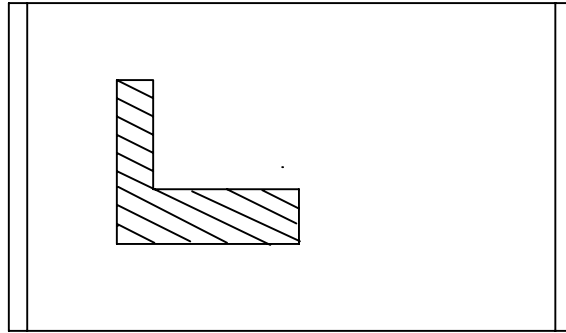


Figure 3: A hypothetical 'L' shaped anomaly in the background between two boreholes

The implementation of MVP-AVG filtering technique, described in section 3.1. in the hybrid algorithm, can be described with Pascal pseudo-code:

```
(* MVP-AVG variant(a) *)
begin
if (current_generation > 4000) then
  begin
  for filt := 1 to 100 do
    begin
    for i := 1 to m do
      for j := 1 to n do
        begin
          sort in ascending order cells in window that contains cell (i,j);
          apply MVP-AVG filter to the window of cell (i,j);
          new value of cell (i,j) put in mat(i,j);
        end;
      end;
    end;
  end;
end;
```


replace old values of cells with the new ones from matrix mat;
end

end

end.

The other variants (b), (c) and (d) of MVP-AVG filter as well as MVP-MED filters and selective smoothings are implemented analogously.

4. RESULTS

4.1. Test-examples

Syntetic test-examples, where the examined profile consists of part by part constant regions, are generated (see Fig. 4). The number of cells is 30 (6×5). In every borehole there are 10 transmitters/receivers. The distance between boreholes is 5.0m and the borehole's depth is 6.0m. The threshold level $t = 0.1$, because in these test-examples high values are in the anomaly's cells and low values are in the background's cells.

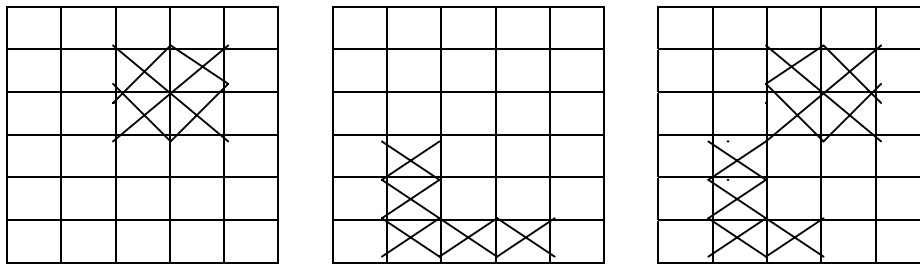


Figure4: Profiles of test-instances P_1 , P_2 and P_3 where the values in anomaly's cells are 0.7 and the values in background's cells are 0.1

The testing has been done using PC compatible computer AMD 80486 at 120MHz with 8 MB of memory. Since genetic operators are nondeterministic, every test example was running 10 times and we have computed the average value.

4.2. Numerical results

Table 1 contains results obtained by the following methods:

- ART (Algebraic Reconstruction Technique)(see Dines and Lytle [4]) and different versions of ART i.e. M-ART (Modified-ART), DT-ART (Distance Transposed-ART), DTM-ART (Distance Transposed Modified-ART) and MDT-ART (Modified Distance Transposed-ART). These results are taken from Balanis and Bentley [1]. A detailed explanation about these techniques can be found in Balanis and Bentley [1].

- GA (Šešum et al [24]).

The measure of error δ_2 given by relation:

$$\delta_2 = \sqrt{\frac{\sum_{i=1}^m \sum_{j=1}^n (u_{ij}^T - u_{ij}^C)^2}{m \times n}} \quad (10)$$

is used, where u_{ij}^T is the true slowness, u_{ij}^C is the computed slowness and $m \times n$ is the total number of cells in the profile.

Table 1.

	P_1	P_2	P_3
$\delta_2(GA)$	0.02901	0.03221	0.17492
$\delta_2(ART)$	0.13300	0.20100	0.22300
$\delta_2(DT - ART)$	0.08300	0.15600	0.81200
$\delta_2(M - ART)$	0.08100	0.16500	0.17600
$\delta_2(MDT - ART)$	0.08300	0.14800	0.17300
$\delta_2(DTM - ART)$	0.09200	0.14600	0.17900

The variants (a), (b), (c) and (d) of the implemented filters and the GA combinations are described in section 3.4, while the obtained results are given in Table 2.

Table 2.

	P_1	P_2	P_3
$\delta_2(MVP - AVG(a))$	0.03009	0.01686	0.16008
$\delta_2(MVP - AVG(b))$	0.03053	0.22923	0.31102
$\delta_2(MVP - AVG(c))$	0.03356	0.23989	0.35823
$\delta_2(MVP - MED(a))$	0.05073	0.07503	0.41323
$\delta_2(MVP - MED(b))$	0.02984	0.05272	0.38797
$\delta_2(MVP - MED(c))$	0.03607	0.16241	0.24384
$\delta_2(\text{selective smooth. (a)})$	0.02640	0.04177	0.24936
$\delta_2(\text{selective smooth. (b)})$	0.03008	0.12567	0.37428
$\delta_2(\text{selective smooth. (c)})$	0.02778	0.09194	0.26610
$\delta_2(GA / MVP - AVG(d))$	0.00657	0.00933	0.14037
$\delta_2(GA / MVP - MED(d))$	0.00730	0.00000	0.14715
$\delta_2(GA / \text{selective smooth. (d)})$	0.02746	0.02992	0.16970

Table 3 contains results of the application of the most successful ART modification, MDT-ART and filters (Balanis and Bentley [1]).

Table 3.

	P_1	P_2	P_3
$\delta_2(MDT - ART / MVP - AVG)$	0.050	0.187	0.195
$\delta_2(MDT - ART / MVP - MED)$	0.062	0.182	0.155
$\delta_2(MDT - ART / selective\ smooth.)$	0.076	0.142	0.170

From the comparison of the results obtained using GA with the results obtained using ART (see Table 1) we can conclude that GA give better results (in case of P_1 and P_2) and almost the same result (in case of P_3).

The comparison of the results obtained using the combination of GA and filters with the results obtained using the combination of ART and filters (from Table 2. and Table 3) shows that the combination of GA and filters (variant(d)) gives better results for all three test-examples.

The values presented in Tables 1-3 confirm that the combination of GA and filters improves results obtained using GA only.

5. CONCLUSION

Generally, GAs are very suitable for hybridizing with other methods. In this paper the new hybrid method (GA and filters) for solving the geophysical inversion problem is presented. The intention was to develop the technique that would improve the results obtained by using genetic algorithms only. The applying of smoothing filters can improve the performances of a main iterative reconstruction algorithm as well as the quality of reconstructed picture. The applying of new programming techniques, in this case, is not requested. The chosen GA structure provided relatively good results. We used the following filters: MVP-AVG, MVP-MED and selective smoothing. All of them showed good features in combination with GA.

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