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AN EPQ MODEL FOR DELAY DETERIORATING PRODUCTS WITH PRICE, FRESHNESS AND GREENING EFFORTS DEPENDENT DEMAND UNDER MARKDOWN STRATEGY

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Abstract: Consumers today use green, fresh, perishable products because of their freshness, healthfulness, and sustainability. In this article, we developed the continuous production inventory model for the producer who produces and sells fresh perishable products with the input of green efforts. There are two distinct kinds of product decay to take into account: products whose physical condition gradually deteriorates over time at a constant rate, and products whose freshness quality declines with time. Product demand is influenced by the selling price of the product, its freshness level, and its greening efforts. In order to increase the sales of inventory and enhance the profit from clearing stocks at the end of their life, we have adopted the markdown policy. Due to freshness degradation, a markdown strategy is adopted after a period of product deterioration to boost demand. The objectives of the study are to find out the optimal period for replenishment cycle time, the optimal value of greening efforts, and the optimal markdown percentage such that the producer's total profit is maximum. A numerical example is used to validate a mathematical formulation that reflects real-world circumstances. A sensitivity analysis of the parameters is done to determine the model's reliability. Some significant managerial results are provided, and the article concludes with a consideration of the future scope of related research.

Keywords: Inventory, EPQ model, freshness level, green efforts, deterioration, Markdown, Perishable products.

MSC: 90B05.

1. INTRODUCTION

Perishable products, including fruits, vegetables, meat, dairy products, different beverages etc., are currently purchased based on price, health concerns, and freshness considerations. Manufacturers and retailers with a strong dedication to going green have undertaken a number of initiatives. Greening strategies will reduce expenses, encourage new customers, and benefit the environment. Consumers are becoming more invested in environmentally friendly products. For example, green packaged fruit juices, green packaged plain drinking water, etc. are eco-friendly beverages whose freshness level decreases with time. Green farming produces perishable products that are preferred over products created with pesticides and inorganic fertilizers because organic products contain antioxidants and nutrients that improve the human body's resistance. The dairy and beverage sectors use eco-friendly technologies to mitigate the impact of greenhouse gases. For example, they use eco-friendly packaging for their dairy and beverage goods and have taken numerous additional actions. Buyers desire to purchase products according to their freshness, so the demand is significantly impacted by the product's freshness level. Perishable products may not have a greater impact on deterioration at the start of their shelf life, and the freshness of the product remains unchanged during this time frame. As a result of physical deterioration and qualitative degradation brought on by prolonged storage in the facility, items such as fruits, vegetables, dairy products, and drinks have lost their uniqueness and quality. The freshness of the product reduces with time due to deterioration. The producer adopts a markdown policy to make up for losses brought on by deterioration. The product loses usability over time as it begins to deteriorate from its initial state. The longer an item is kept in inventory, the higher the deterioration cost. Manufacturers occasionally employ the markdown policy for clearance at the end of the season or to sell off out-of-date goods during the product's life in order to boost profits and eliminate inventory. An EPQ model for perishable products is developed taking into account all of these factors, where the demand rate is a function of the selling price, product age (freshness level), and greening effort.

1.1 Significance of the study

A decision-maker or producer adopts many business strategies to increase their profit. Our study focuses especially on the perishable green product, which is produced and sold by producers. The freshness and deterioration of the product are major factors that influence market demand. Consumers are moving towards green and sustainable goods. Product freshness is an important component of its quality, and as a result, consumers' purchasing decisions are influenced by the freshness of green products. The deterioration effect is negligible at the start of their production period for perishable products, and the freshness of the product does not decrease as noticeably during this time. i.e., the product is fully fresh. After production stops, the effect of deterioration starts. Due to the effect of deterioration, the product loses its freshness continuously, so market demand decreases, and hence policymakers adopt a markdown strategy after some time of deterioration to stimulate demand. Greening efforts are an action taken to minimize the impact trade has on the ecosystem and ensure sustainable products. Taking into account all of these factors, demand for perishable products is a function of selling price, age of the product (freshness), and greening efforts. The main challenges of the proposed problem are: (i) what is the optimum value of the cycle time, greening level, and markdown percentages such that the producer's total profit is the maximum? (ii) When to start and stop production? When does the decision-maker apply the markdown offer? (iii) How does product freshness affect total profit? (iv) What is the role of green investment in terms of order quantity and profit? (v) What is the contribution of the markdown policy? Previously, none of the literature combined product freshness and greening strategies. We will discuss the product freshness and greening efforts with the markdown policy and answer the above questions. The purpose of the study is to determine the optimum cycle time, cost of greening efforts, and markdown percentage to maximize the producer's total revenue in the EPQ model with a markdown strategy. A numerical example and sensitivity analysis authenticate this model.

The rest of the article is formatted in the manner listed here. The study of literature along with research gap analysis is provided in Section 2. The notations and assumptions of model are described in Section 3. In Section 4, Mathematical formulations and solution methodology mentioned. Real and Numerical example, graphical representation of convexity is carried out in Section 5. Section 6, the sensitivity of parameters and derived some managerial insights is derived and finally, draws conclusions and indicates future research directions.

2. LITERATURE REVIEW

We developed a deterministic production inventory model for perishable green items in order to precisely reflect consumer preferences with regard to freshness. Several researchers have suggested in their research that the majority of perishable product inventory models created follow a random lifespan of products, their freshness remains unchanged, and their freshness does not affect demand before they have expired. The first study to look at how demand is impacted by a product's freshness was done by Fujiwara and Parera [1]. According to study by Bai and Kendall [2], the demand for fresh produce is reliant on both its freshness and the inventory that is on display. In an EOQ model for fresh produce, where demand influences the freshness expiration date and inventory levels, Chen et al. [3] examined this concept. Dobson et al. [4] developed an inventory model in which products freshness reduces demand rate linearly. Agi and Soni [5] presented a deterministic model for jointly optimising pricing and inventory control of a perishable product subject to both physical deterioration and freshness condition degradation in which the demand for the product depends on its price, the current inventory level and the freshness condition. The major literatures can be distributed as per following subsections:

Businesses face difficulty throughout the cycle time in that they must figure out a balance between the requirement to maximize their profit and the need to clear out end-of-life inventory. The next step is the establishment of a markdown policy to decrease waste as fresh products get closer to the end of their selling time. Urban and Baker [6] looked at how much of a discount on the selling price may be applied throughout the deterioration phase in order to increase profit per unit of time as well as how a pre-deterioration markdown might impact the unit profit. Widyadana and Wee [7] developed a deteriorating inventory model with nonlinear price-dependent demand under a markdown policy to increase the profit. Srivastava and Gupta [8] designed an EPQ deteriorating model with price and time-dependent demand under a markdown policy to reduce inventory and increase profit. In the context of a markdown policy, Kamaruzaman and Omar [9] created an inventory model for a fresh product whose demand is influenced by its price, level of inventory, freshness, and expiration date.

Producers and retailers have recently launched a number of efforts to show how serious they are about being green. Reviewing Walmart's green initiatives, Plambeck [10] asserted that "being a good steward of the environment and being profitable are the same." Coca-Cola, one of the largest beverage businesses in the world, has started recycling and reprocessing used bottles through incentives in developing countries like India due to environmental concerns. Swami and Shah [11] suggested channel coordination in a supply chain with greening investment considerations. Li et al. [12] outlined pricing policies for a competitive dual channel green supply chain. Raza and Faisal [13] developed inventory models for joint pricing and greening effort decisions with discounts. Meemken and Qaim [14] gave an annual report on organic agriculture, food security, and environmental issues. They investigated soil quality, biodiversity, and environmental effects on organic agriculture from 2000 to 2015. Shah et al. [15] formulated an inventory model based on perishable products for price and stock-dependent demand rates, along with greening efforts. Ji et al. [16] studied how green credit financing at a discounted rate motivates the supplier to improve the reduction in carbon emission levels. Shah et al. [17] obtained pricing decisions with the effect of advertisement and greening efforts for a greengrocer.

Deterioration has been one of the top concerns for businesses ever since the inventory process began. Ghare and Schrader [18] first introduced the idea of deterioration in inventory systems. Existing study articles can be divided into two categories based on whether the deterioration is instantaneous or non-instantaneous. Goyal and Giri [19] analysed a variety of publications that looked at deteriorating inventory going back to the early 1900s. Products like fresh fruit or vegetables do not degrade during the initial stages of production or storage, but degrade gradually after a certain amount of time. By contrast, packaged food and liquids, medicines, and beauty products are all likely to be completely unusable once they have passed their expiration dates. Huang [20] developed a production quantity model that which considered cash discounts and permissible delay payment concepts. Yadav and Swami [21] presented an EPQ model with time-varying holding cost with the assumption that partial backlog shortages and Weibull distribution deterioration rate. Shah and Vaghela [22] constructed an EPQ model for deteriorating items with price dependent demand and two level trade credit financing. Mashud et al. [23] worked on non-instantaneous deteriorating items with different demand rates, allowing partial backlogs. Sundararajan and Palanivel [24] discussed an EOQ model of non-instantaneous deteriorating items with price, time-dependent demand, and backlogs. Singh and Rani [25] developed a deteriorating EPQ model with multivariate demand under a markdown policy. Shee and Chakrabarti [26] studied an EPQ model for an imperfect production system with deteriorating items, price-dependent demand, rework, and lead time under a markdown policy.

One of the most important factors in determining a product's level of popularity is its selling price. The impact of pricing on demand has been the subject of numerous studies. The inventory models based on price-dependent demand were designed by Wee [27], Abad [28], Mukhopadhyay et al. [29], etc. Maihami and Kamalabadi [30], Farughi et al. [31], considered in their study for non-instantaneous deterioration of products with price and time- dependent demand. Mashud et al. [32] created a joint pricing deteriorating inventory model considering the product life cycle and advance payment with a discount facility. Shaikh et al. [33] designed inventory with price sensitive quadratic demand under the advance-cash-credit payment scheme but did not consider a markdown pricing scheme. Bhaula et al. [34] developed an inventory model for perishable products with optimized selling price and freshness to optimize net profit under subsequent price discounts. Soni [35] developed a model in which price and freshness demand were combined with a price discount facility. Raiya and Mittal [37] developed an EOQ model considering constant demand and imperfect quality items under carbon caps, carbon tax policies, and the green investment concept not taken by Raiya and Mittal [37] in their study. Khedlekar et al. [38] incorporated price sensitive demand, shortages, and promotional efforts into their study. Katariya and Shukla [39] developed a SEPQ model, including green investment and pricing policies.

2.1. Research gap and our contribution

The earlier literature has a strong emphasis on several techniques that have been written about in the field of green inventory management. The market demand is significantly influenced by the product's freshness, price, and environmental efforts. Following a review of the published literature, none of the researchers adopted the concept of greening efforts with freshness and price-related demand. The demand for perishable goods such as organic agricultural products, green packaged beverages, and green dairy products was influenced by consumers' preferences for greenness in addition to freshness and price. Another innovative concept is that perishable goods shouldn't physically or qualitatively degrade more quickly throughout production and that their freshness should be assumed to be 100%. Hence, demand at the beginning of inventory cycle time is price- and green-efforts-dependent because $f(t) \to 1$ and $\theta \to 0$. After production stops, products affected by deterioration of both type and product value degrade continuously, during this period demand pattern depend on freshness, price and greening efforts; Due to freshness degradation, a markdown strategy is adopted after a period of product deterioration to boost demand. Green investment concept in the EPQ model with freshness and markdown strategy, which is another new idea of this study. The current study looks to maximize the producer's total profit by taking all factors into account. This differentiates the current research from previous, related research works. The highlights of the research gap are mentioned in Table 1.

Table 1: Literatures related to proposed model and analysis of research gap

Author(s)	Demand Modelling	Deterioration	Markdown Policy	Greening Investment
Urban and Baker [6]	SPD,SD,TD	No	Yes	No
Widyadana and Wee [7]	SPD	Yes	Yes	No
Srivastava and Gupta [8]	SP,TD	Yes	Yes	No
Chen et al.[3]	FD,SD	No	No	No
Dobson et al.[4]	FD	No	No	No

Raza and Faisal [13]	SPD,GED	No	No	Yes
Mashud et al. [23]	SPD	Yes	No	No
Shah and Vaghela(22)	SPD	Yes	No	No
Bhaula et al. [33]	SPD	Yes	No	No
Agi and Soni [5]	SPD,FD,SD	Yes	No	No
Kamaruzaman and Omar [9]	SPD,FD,SD	No	Yes	No
Soni [35]	SPD,FD	Yes	No	No
Shah et al.[15]	SPD,GED	Yes	No	Yes
Shah et al.[17]	SPD,GED	Yes	No	Yes
Shee and Chakrabarti [26]	SPD	Yes	Yes	No
Shaikh et al. [33]	SPD	No	No	No
Raiya and Mittal [37]	CD	No	No	No
Khedlekar et al. [38]	SPD	Yes	No	No
Proposed Model	**SPD,FD, GED**	**Yes**	**Yes**	**Yes**

Note: SPD: Selling price dependent, FD: Freshness dependent, GED: Greening efforts dependent, SD: Stock dependent, TD: Time dependent, CD: Constant demand

3. NOTATIONS AND ASSUMPTIONS

3.1. Notations

Table 2: Notations for proposed model

Parameters:	
A	Set up cost per cycle (in \$/order)
h	Constant holding cost (\$/unit/unit time).
p	Original price of product (\$/unit)
r	Markdown rate; defined as the percentage decrease of an original price of product.
P	Production rate proportional to demand (unit/year).
k	Production cost per unit per cycle. (\$/unit)

δ	Production percentage.
τ	Maximum life-time of the product beyond which no consumer will buy the product. $0 < T \le \tau$
λ	Greening efforts effectiveness parameter.($\lambda > 0$)
θ	Rate of deterioration of product. $0 < \theta < 1$.
C_d	Deterioration cost (unit/year)
t_1	Production period; (in years)
$t_1 + t_2$	Markdown offering at time; (in years)
t_3	Markdown period; (in years)
l	proportional factor of production rate and demand, $l \ge 1$
Decision variables:	
T	Cycle time; (in years); where $t_1 + t_2 + t_3 = T \le \tau$
g_e	Cost of greening efforts; (in \$/unit).
m_p	Markdown percentage.
Expressions and functions:	
TP	Retailer's/firm's total profit per cycle (in \$).
$R(p,t,g_e)$	Demand function at time t.
$I_1(t)$	Inventory level at time t during $0 \le t \le t_1$.
$I_2(t)$	Inventory level at time t during $0 \le t \le t_2$.
$I_3(t)$	Inventory level at time t during $0 \le t \le t_3$.
Q_1	Inventory level (total production) at time t_1 (units).
Q_2	Total quantity under markdown offered after time t_2 (units). $Q_2 \le Q_1$.

3.2. Assumptions

- A single type of perishable product is considered over a specific cycle time.
- The rate of production is proportional to demand i.e. $P = l \cdot R(p,t,g_e), 0 \le t \le t_1$ and production stops at time t_1. After a product unit has been produced, it must be sold.
- Production time is proportional to the cycle time which is equivalent to $t_1 = \delta T$.
- A product starts to deteriorate after production stops, and a markdown is offered after some time of product deterioration. Products have two kinds of deterioration: physical deterioration at a constant rate of existing inventory and deterioration in the freshness quality of the product.
- Deteriorated products cannot be repaired or replaced, and they have no salvage value.
- Markdown pricing is applied only once during the inventory cycle.
- The markdown price is known, in advance.
- The product's freshness may be impacted by a variety of variables, including duration in stock, heating rate, humidity, and storage, among several others. Getting a product's explicit freshness rating seems to be impossible. Despite this,

it goes without saying that every product's freshness ultimately deteriorates and expires over time. Therefore, we can assume that the freshness index is 1 at time 0 and gradually decreases over time until it eventually reaches 0 as the product gets closer to its expire (i.e., it cannot be sold). The freshness index is defines as $f(t)=\frac{\tau-t}{\tau}, 0 \le t \le \tau$. (Chen et al [3], Dobson [4], Agi and Soni [5], Soni [35]).

- The replenishment cycle time is shorter than the longest possible product shelf life. i.e. $T \le \tau$.
- The producer’s greening effort requires a capital investment in greening efforts over a certain time frame rather than raising the unit price of the product. The producer uses green investments to produce or maintain organic or green products.
- Total greening investments per unit time t is $\int_0^t \int_0^{g_e} \lambda \cdot g_e \, dg_e dt = \frac{\lambda \cdot g_e^2 \cdot t}{2}$. So, the total investment for the time duration T is $\frac{\lambda \cdot g_e^2 \cdot T}{2}$. (Swami and Shah [11], Raza and Faisal [13] , Shah et al.(15))).
- The demand rate is based on selling price of product, freshness index, i.e. age of product as well as green efforts. Demand rate function mathematically defined as, $R(p,t,g_e)=\begin{cases} r(p)+\gamma g_e, & 0 \le t \le t_1 \\ r(p) f(t)+\gamma g_e, 0 \le t \le t_2 \\ r(p_r) f(t)+\gamma g_e, 0 \le t \le t_3 \end{cases}$ in this expression $r(p)=\alpha-\beta p$ and $r(p_r)=\alpha-\beta p(1-r)$, are any non-negative, continuous, convex decreasing functions of selling price, and $\alpha>0$ represent the constant market demand , $\beta>0$ is the price elasticity factor, $\gamma>0$ is greening investments effectiveness scale and $t_1+t_2+t_3=T$.
- The lead time is negligible and the replenishment rate infinite and shortages are not permissible.

4. MATHEMATICAL MODEL FORMULATION

Based on the assumptions and notations, the behaviour of the inventory system depicted in Figure 1. From the Figure 1, at the time $t=0$ the inventory level is zero. Production and supply of the fresh green product begin concurrently, and production ends at $t=t_1$ at which maximum inventory level Q_1 is reached. During $0 \le t \le t_1$, there is no physical deterioration and freshness index of product is $f(t) \to 1$, i.e. product is fully fresh.

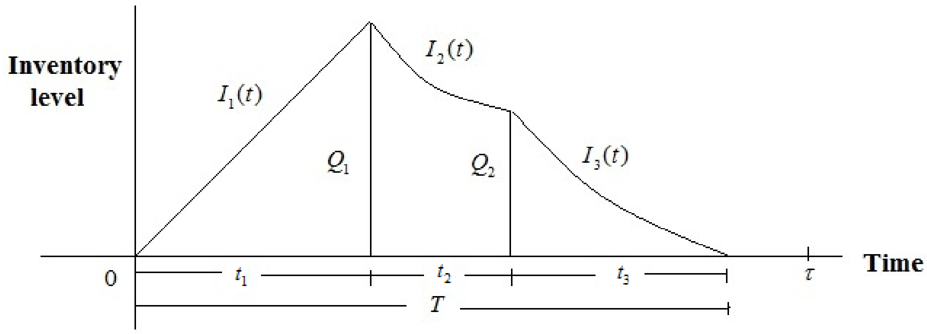

Figure 1: Graphical representation of the inventory behavior

The inventory level during $0 \le t \le t_1$ can be represented by the following differential equation,

$$\frac{dI_1(t)}{dt} = P - (r(p) + \gamma g_e) = (l-1)(r(p) + \gamma g_e), 0 \le t \le t_1 \tag{1}$$

With the condition $I_1(0) = 0$, the solution of Eq. (1) is,

$$I_1(t) = (l-1)(r(p) + \gamma g_e)t \tag{2}$$

The inventory level at time $t = t_1$ is i.e. $I_1(t_1) = Q_1$;

$$Q_1 = (l-1)(r(p) + \gamma g_e)t_1 \tag{3}$$

After time $t = t_1$, inventory level decrease due to demand and physical deterioration as well as freshness degradation of product. The inventory level on $0 \le t \le t_2$ is,

$$\frac{dI_2(t)}{dt} + \theta I_2(t) = -(r(p)f(t) + \gamma g_e), 0 \le t \le t_2 \tag{4}$$

The solution of eq. (4) at $I_2(0) = Q_1$ is given by;

$$I_2(t) = Q_1 e^{-\theta t} + \left(e^{-\theta t} - 1\right)\left(\frac{\gamma g_e}{\theta} + \frac{r(p)}{\theta} + \frac{r(p)}{\tau\theta^2}\right) + \frac{r(p)t}{\tau\theta} \tag{5}$$

Likewise, at $0 \le t \le t_3$, level of inventory declines due to joint effect of demand and deterioration. The demand is also decrease with time. Markdown policy is applied during this interval to boost demand through a reduction in the original selling price. Thus, we have

$$\frac{dI_3(t)}{dt} + \theta I_3(t) = -(r(p_r)f(t) + \gamma g_e), 0 \le t \le t_3 \tag{6}$$

Finally, the solution of Eq. (6) using boundary condition $I_3(0) = Q_2$,

$$I_3(t) = Q_2 e^{-\theta t} + \left(e^{-\theta t} - 1\right)\left(\frac{\gamma g_e}{\theta} + \frac{r(p_r)}{\theta} + \frac{r(p_r)}{\tau\theta^2}\right) + \frac{r(p_r)t}{\tau\theta} \tag{7}$$

Noted that $I_3(t_3)=0$; the inventory level at $t=t_2$ is,

$$Q_2=\left(e^{\theta t_3}-1\right)\left(\frac{\gamma g_e}{\theta}+\frac{r(p_r)}{\theta}+\frac{r(p_r)}{\tau\theta^2}\right)-\frac{r(p_r)t_3}{\tau\theta}e^{\theta t_3} \tag{8}$$

Now, calculate the different inventory costs and sales revenue, to find the total profit of producer.

Annual fixed setup cost is , $SC=\frac{A}{T}$ (9)

Production cost per unit time is, $PC=\frac{k}{T}\int_0^{t_1} l\cdot(r(p)+\gamma g_e)dt$ (10)

The holding cost of holding inventory per unit time is,

$$HC=\frac{h}{T}\left[\int_0^{t_1}I_1(t)dt+\int_0^{t_2}I_2(t)dt++\int_0^{t_3}I_3(t)dt\right] \tag{11}$$

Deterioration cost per unit time be

$$DC=\frac{C_d}{T}\left[\left(Q_1-\int_0^{t_2}I_2(t)dt\right)+\left(Q_2-\int_0^{t_3}I_3(t)dt\right)\right] \tag{12}$$

Depending on the product's level of greening, producers may have to incur extra expenses to increase the product's quality. Greening efforts investment per cycle is,

$$GEI=\frac{1}{T}\int_0^T\int_0^{g_e}\lambda\cdot g_e\,dg_e dt=\frac{1}{T}\frac{\lambda\cdot g_e^2\cdot T}{2}=\frac{\lambda\cdot g_e^2}{2} \tag{13}$$

The revenue before markdown and the revenue after markdown are combined to form the total revenue. Hence, the total sales revenue generated by each cycle is given as,

$$SR=\frac{p}{T}\left(\int_0^{t_1}(r(p)+\gamma g_e)dt+\int_0^{t_2}(r(p)f(t)+\gamma g_e)dt\right)+\frac{p(1-r)}{T}\int_0^{t_3}(r(p_r)f(t)+\gamma g_e))dt \tag{14}$$

The total profit of the retailer per cycle time T is formulated as,

$$TP_1(t_1,t_2,t_3,g_e,m_p)=SR-(SC+PC+HC+DC+GEI) \tag{15}$$

The eq. (15) is the form of t_1, t_2 and t_3 but as per Srivastava and Gupta (2014) [8] the relations of t_1, t_2, t_3 with δ, m_p and T defines as,

$$t_1=\delta T\ ,$$
$$t_2=m_p(T-t_1)=m_p(1-\delta)T \text{ and}$$
$$t_3=T-(t_1+t_2)=(1-\delta)(1-m_p)T$$

Substitute above relations in eq. (15), it can be rewritten as in form of T is,

$$TP(T,g_e,m_p)=SR-(SC+PC+HC+DC+GEI) \tag{16}$$

Due to complexity of nonlinear form of Eq.(16), to find the value of decision variables T^* , g_e^* , m_p^* , and to prove the concavity of total profit function $TP(T^*, g_e^*, m_p^*)$, we adopted following solution procedure.

4.1. Solution methodology to find the optimal solution and concavity

In this section, we determine the optimal value of decision variables T^* , g_e^* , m_p^* which maximize the total profit $TP(T^*, g_e^*, m_p^*)$ per cycle. The necessary conditions for maximize of the total profit function given by (16) are,

$$\frac{\partial TP}{\partial T} = 0, \frac{\partial TP}{\partial g_e} = 0, \frac{\partial TP}{\partial m_p} = 0 \tag{17}$$

Use the Hessian matrix method, to prove the concavity of total profit function $TP(T^*, g_e^*, m_p^*)$ at the value of decision variables T^* , g_e^* , m_p^* , Let's take third order Hessian matrix,

$$H(T^*, g_e^*, m_p^*) = \begin{bmatrix} \frac{\partial^2 TP(T^*, g_e^*, m_p^*)}{\partial T^2} & \frac{\partial^2 TP(T^*, g_e^*, m_p^*)}{\partial T \partial g_e} & \frac{\partial^2 TP(T^*, g_e^*, m_p^*)}{\partial T \partial m_p} \\ \frac{\partial^2 TP(T^*, g_e^*, m_p^*)}{\partial g_e \partial T} & \frac{\partial^2 TP(T^*, g_e^*, m_p^*)}{\partial g_e^{\,2}} & \frac{\partial^2 TP(T^*, g_e^*, m_p^*)}{\partial g_e \partial m_p} \\ \frac{\partial^2 TP(T^*, g_e^*, m_p^*)}{\partial m_p \partial T} & \frac{\partial^2 TP(T^*, g_e^*, m_p^*)}{\partial m_p \partial g_e} & \frac{\partial^2 \pi(T^*, g_e^*, m_p^*)}{\partial m_p^2} \end{bmatrix} \tag{18}$$

Conditions for concavity at (T^*, g_e^*, m_p^*) are,

$$\frac{\partial^2 TP}{\partial T^2} < 0 \, , \frac{\partial^2 TP}{\partial T^2} \cdot \frac{\partial^2 TP}{\partial g_e^{\,2}} - \left(\frac{\partial^2 TP}{\partial T \partial g_e} \right)^2 > 0 \text{ and } \det(H) < 0 \tag{19}$$

Furthermore, to verify the all Eigen values of (18) are negative then $TP(T^*, g_e^*, m_p^*)$ maximize. (Cárdenas-Barrón and Sana [36]).

To obtain an optimal solution of decision variables and optimal profit function, follow the steps mentioned below:

Step 1: First allocate value of inventory parameters with proper unit other than decision variables.

Step 2: Take the partial derivative of eq.(16) with respect to T , g_e and m_p and equating to zero.

Step 3: Solving the equations stated in eq. (17) simultaneously using the mathematical software Maple XVIII, to find T^* and g_e^* and m_p^* .

Step 4: Verify the sufficient conditions stated in eq. (19) at T^* and g_e^* and m_p^* , if not satisfied go to step 1 and choose other value of parameters in step 1, repeat process till Eq. (19) satisfied.

Step 5: Using Eq. (16), find $TP(T^*, g_e^*, m_p^*)$ value.

Step 6: Using Eq. (3) and Eq. (8) , find Q_1^* and Q_2^* value.

Step 7: stop

5. REAL EXAMPLE WITH NUMERICAL RESULTS

5.1. Real example

The proposed model concerns the producer, who produces and sells the fresh, green, perishable product. Let's take the real example: a beverage industry produced a fresh, nutrient-dense juice with green packaging. At the stage of production, packaged juice is fully fresh and does not show signs of deterioration. After some time, packaged juice may show signs of deterioration, and the producer may apply a markdown policy to boost demand. Juice producers apply a "best before date' policy. Green technology to improve the healthiness of fruit and vegetable beverages. Other real example is, organic agricultural products are produced and sold by farmers without using inorganic fertilizers and harmful pesticides and by investing as a green investment in nature-based organic fertilizers and bio-pesticides to meet customers' demands to produce organic fresh products. Green processing technology was applied to dairy products, and green packaging was used for perishable dairy products. This model may be relevant to perishable goods with low market demand relative to other goods and ongoing quality decline over time.

5.2. Numerical example

To demonstrate the outcome of the proposed model, the following parameter values are used in a numerical example.

$\alpha = 450$ units, $\beta = 3.3$, $\gamma = 0.5$, l =1.5, A =\$200/order, C_d =\$10/unit, h =\$3/unit, k =\$2/unit, $\delta = 0.4$, r =5%, $\tau = \frac{180}{365}$ year, p =\$100/unit, $\lambda = 5$, $\theta = 0.08$. Optimal results derived as per the steps mentioned in previous section. The optimal value of decision variables is T^* = **0.28411 year**, green efforts cost is g_e^* =**\$9.60/unit** time, optimal markdown percentage is $m_p^* = 0.53197$ .The optimal **total profit** $TP(T^*, g_e^*, m_p^*)$ is **\$10805.76** per cycle time, optimal **production period** is t_1^* =0.1136 year. Optimal **markdown offering time** from $t_1^* + t_2^* = 0.2043$ year and **markdown period** $t_3^* = 0.0798$ year. Now from (18) and (19), Hessian matrix at optimal value of decision variables is,

$$H(T^*, g_e^*, m_p^*) = \begin{bmatrix} -17433.9 & -0.309320 & -662.939832 \\ -0.309320 & -5 & 4.571022540 \\ -662.939832 & 4.571022540 & -5433.546669 \end{bmatrix},$$

$$\frac{\partial^2 TP}{\partial T^2} = -17433.9 < 0 \quad \frac{\partial^2 TP}{\partial T^2} \cdot \frac{\partial^2 TP}{\partial g_e^2} - \left(\frac{\partial^2 TP}{\partial T \partial g_e} \right)^2 = 87169.40 > 0, \quad \det(H) = -4.7979 < 0$$

and Eigen values of Hessian matrix are

$\lambda_1 = -17470.41 < 0, \lambda_2 = -4.99 < 0, \lambda_3 = -5397.03 < 0$. Hence, it is proved that the optimal value of decision variables satisfied sufficient conditions of concavity.
Here noticed that, the original selling price (p =\$100/unit) and markdown rate (r =5%) are known, it means that markdown price ($p(1-r)$ = **\$95)** is also known.

The graphical representation of concavity of the objective function as mentioned in Figure 2, Figure 3, Figure 4:

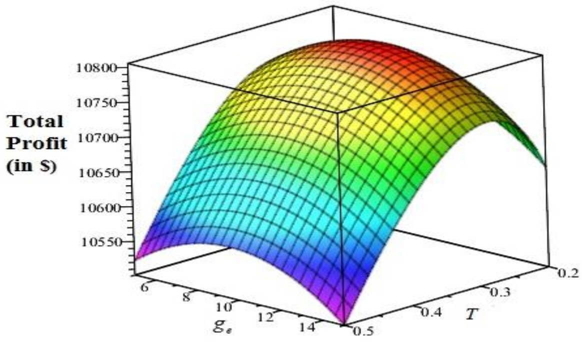

Figure 2: Concavity of total profit function with respect to greening efforts and cycle time

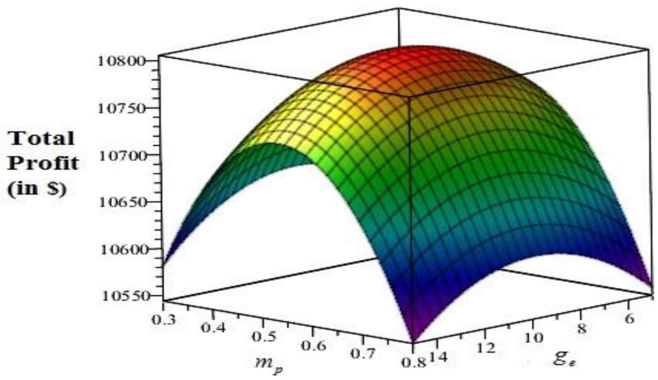

Figure 3: Concavity of total profit function with respect to markdown percentage and greening efforts

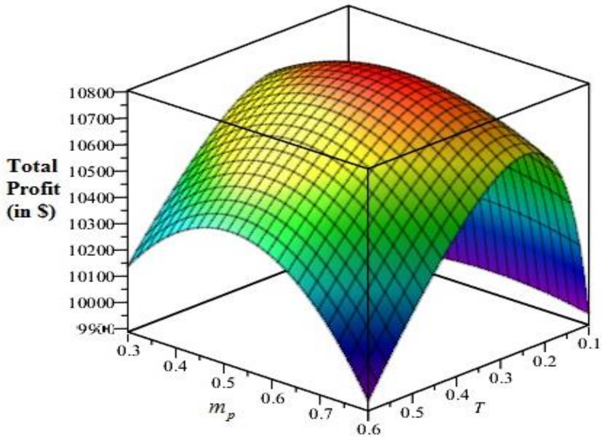

Figure 4: Concavity of total profit function with respect to markdown percentage and cycle time

6. SENSITIVITY ANALYSIS AND DISCUSSION

The suggested inventory model's ideal solutions are examined using sensitivity analysis, which changes each parameter from -40% to +40% individually while leaving the rest untouched.

Effect of markdown rate r

From Table **3**, we observed that, if we increase the markdown rate by -40% to +40%, then total profit increases 1% to 2%, markdown percentage and production quantity reduces decreases slightly. Markdown offering time $(t_1 + t_2)$ will be reduced and markdown period t_3 slightly increases with number of markdown quantities increases. Markdown percentage decreases with increases of markdown rate.

Table 3: Variations effect of markdown rate on decisions variables and total profit

r	T^*	g_e^*	m_p^*	Q_1^*	Q_2^*	$TP*$	$t_1^* + t_2^*$	t_3^*
0.03	0.2837	9.68	0.5653	14.17	9.28	10716.92	0.2097	0.0740
0.04	0.2841	9.64	0.5482	14.17	9.86	10761.41	0.2071	0.0770
0.05	0.2841	9.60	0.5320	14.18	10.42	10805.76	0.2043	0.0798
0.06	0.2837	9.56	0.5166	14.19	10.97	10849.78	0.2014	0.0823
0.07	0.2829	9.51	0.5020	14.20	11.50	10893.27	0.1984	0.0845

Effect of production percentage δ

Table 4 indicated that, if we increase the production percentage by -40% to +40%, then it is obvious that production quantity increases with total profit decreases and total cycle time increases. Mark down percentage and green efforts will be reduces with higher value of δ.

Table 4: Variations effect of production percentage on decisions variables and total profit

δ	T^*	g_e^*	m_p^*	Q_1^*	Q_2^*	$TP*$	$t_1^*+t_2^*$	t_3^*
0.24	0.2264	9.8848	0.5343	-	-	10904.34	0.1463	0.0801
0.32	0.2521	9.7434	0.5332	10.70	10.46	10855.65	0.1721	0.0800
0.4	0.28411	9.6012	0.5319	14.18	10.42	10805.76	0.2043	0.0798
0.48	0.3249	9.4576	0.5304	19.45	10.37	10754.15	0.2456	0.0896
0.56	0.3784	9.3121	0.5283	26.42	10.26	10699.88	0.2999	0.0879

Effect of maximum life time τ

Table 5 depicted that if we increase the life of product (freshness) then total profit will be increases. The production quantity and markdown offering quantity increases with increases the life of product. Markdown offering time will be late because product freshness is increasing. Green efforts cost increases with increases the value of τ .

Table 5: Variations effect of maximum life time on decisions variables and total profit

τ	T^*	g_e^*	m_p^*	Q_1^*	Q_2^*	$TP*$	$t_1^*+t_2^*$	t_3^*
0.2959	0.2217	9.5968	0.5229	11.06	8.06	10407.91	0.0887	0.0696
0.3945	0.2551	9.5990	0.5276	12.73	9.34	10644.65	0.1020	0.0808
0.4932	0.2841	9.6012	0.5320	14.18	10.42	10805.76	0.1136	0.0907
0.5918	0.3101	9.6032	0.5360	15.48	11.38	10924.41	0.1240	0.0997
0.6904	0.3337	9.6053	0.5398	16.66	12.22	11016.38	0.1335	0.1081

Effect of constant demand α , price elasticity factor β , and green investment effectiveness scale γ

Table 6 and Table 8, shown that α and γ increases by -40% to -40% then total profit increases. The production quantity and markdown offering quantity also increases with increases α and γ . Markdown offering time and markdown duration reduces with total cycle time reduces as α increases but Markdown offering time increases and markdown duration decreases with total cycle time reduces as γ increases. Greening efforts cost and markdown percentage increases due to increases α and γ .

As per the Table 7 Total profit is negative proportional to the price elasticity factor β . The higher value of β indicated the late markdown offer time, higher markdown period. Replenishment cycle time and markdown quantity will be increases with β but greening efforts cost and markdown percentage decreases.

Table 6: Variations effect of constant demand on decisions variables and total profit

α	T^*	g_e^*	m_p^*	Q_1^*	Q_2^*	$TP*$	$t_1^*+t_2^*$	t_3^*
270	0.48253	9.415253	0.344365	6.70	8.08	2844.26	0.2927	0.1898
360	0.352416	9.535272	0.464781	11.24	9.75	6764.70	0.2392	0.1132
450	0.28411	9.601177	0.53197	14.18	10.42	10805.76	0.20433	0.0798
540	0.209662	9.695342	0.630369	18.02	10.90	19060.96	0.1631	0.0465
630	0.166498	9.774247	0.71459	20.31	11.01	27455.17	0.1380	0.0285

Table 7: Variations effect of price elasticity factor on decisions variables and total profit

β	T^*	g_e^*	m_p^*	Q_1^*	Q_2^*	TP^*	$t_1^*+t_2^*$	t_3^*
1.98	0.1799	9.7862	0.7285	18.48	7.60	22854.91	0.1506	0.0293
2.64	0.2225	9.6979	0.6340	16.99	9.51	16775.48	0.1737	0.0489
3.3	0.2841	9.6012	0.5320	14.18	10.42	10805.76	0.2043	0.0798
3.96	0.3870	9.4553	0.3804	-	-	5009.61	0.2431	0.1439
4.62	0.4142	9.2151	0.1226	-	-	2260.80	0.2961	0.2181

Table 8: Variations effects of green investment effectiveness scale on decisions variables and total profit

γ	T^*	g_e^*	m_p^*	Q_1^*	Q_2^*	TP^*	$t_1^*+t_2^*$	t_3^*
0.3	0.2844	5.7578	0.5268	13.85	10.29	10658.34	0.2036	0.0807
0.4	0.2843	7.67880	0.52910	13.99	10.35	10722.82	0.2039	0.0803
0.5	0.2841	9.6012	0.5320	14.18	10.42	10805.76	0.2043	0.0798
0.6	0.2840	11.5253	0.5356	14.41	10.51	10907.20	0.2048	0.0791
0.7	0.2836	13.4517	0.5398	14.68	10.61	11027.17	0.2053	0.0783

Effect of cost parameters A, k, h, C_d

As per the table 9 and table 10, observed that the total profit slightly decreases with cost parameters A and C_d. Production quantity increases with A but decreases if increases C_d. Markdown offer time proportional to the A and C_d both but markdown period increases with A but decreases with C_d. A higher value A causes higher values of T^* but a higher value C_d causes lower values of T^*. Total profit, production quantity and mark down quantity decreases with the increases the value of k & h.

Table 9: Variations effect of production set up cost on decisions variables and total profit

A	T^*	g_e^*	m_p^*	Q_1^*	Q_2^*	TP^*	$t_1^*+t_2^*$	t_3^*
120	0.219711	9.614363	0.542071	10.97	8.05	11123.34	0.1593	0.060367
160	0.253956	9.606791	0.536077	12.68	9.32	10954.44	0.1833	0.070689
200	0.28411	9.601177	0.53197	14.18	10.42	10805.76	0.2043	0.079783
240	0.311363	9.596702	0.528923	15.54	11.40	10671.42	0.2234	0.088005
280	0.336421	9.592965	0.526543	16.79	12.28	10547.92	0.2409	0.095569

Table 10: Variations effect of deterioration cost on decisions variables and total profit

C_d	T^*	g_e^*	m_p^*	Q_1^*	Q_2^*	TP^*	$t_1^*+t_2^*$	t_3^*
6	0.287212	9.602465	0.48559	14.34	11.48	10866.52	0.1986	0.088647
8	0.286062	9.599046	0.508842	14.28	10.96	10834.67	0.2018	0.084301
10	0.28411	9.601177	0.53197	14.18	10.42	10805.76	0.2043	0.079783
12	0.281384	9.609057	0.555216	14.05	9.86	10779.79	0.2063	0.075093
14	0.277903	9.622996	0.578817	13.87	9.27	10756.79	0.2077	0.070229

Effect of rate of deterioration θ

From Table **11**, increases the percentage of deterioration rate then total profit decreases. Other decision variables value almost unchanged with fluctisonous in θ .

Table 11: Variations effect of deterioration rate on decisions variables and total profit

θ	T^*	g_e^*	m_p^*	Q_1^*	Q_2^*	$TP*$	$t_1^*+t_2^*$	t_3^*
0.048	0.2842	9.6012	0.5316	14.19	10.42	10806.17	0.2043	0.0799
0.064	0.2842	9.6012	0.5318	14.19	10.42	10805.97	0.2043	0.0799
0.08	0.2842	9.6012	0.5320	14.18	10.42	10805.76	0.2043	0.0798
0.096	0.2841	9.6012	0.5322	14.18	10.43	10805.54	0.2043	0.0797
0.112	0.2840	9.6012	0.5323	14.18	10.43	10805.32	0.2043	0.0797

Effect of Markdown price

Table 12 gives the decision policy with respect to markdown price. Higher markdown price resulted to lower profit but lower markdown price gives to higher markdown offer quantities.

Table 12: Effect of Markdown price in decision strategy

$p(1-r)$	T^*	g_e^*	m_p^*	Q_1^*	Q_2^*	$TP*$	$t_1^*+t_2^*$	t_3^*
75	0.2536	8.62	0.3303	12.61	19.01	11453.31	0.1517	0.1018
85	0.2702	9.13	0.4052	**13.46**	**15.24**	11206.52	0.1738	0.0964
95	**0.2841**	**9.60**	**0.5320**	**14.18**	**10.42**	**10805.76**	**0.2043**	**0.0798**
97	0.2837	9.68	0.5653	14.17	9.27	10716.92	0.2097	0.0739

Figure 5 show that that the total profit highly increases with increases the constant market demand and highly decreases if the selling price elasticity parameter increases. Markdown rate increases then total profit slightly increases. Product freshness increases then total profit also increases. Cost related parameters inversely proportional to the profit.

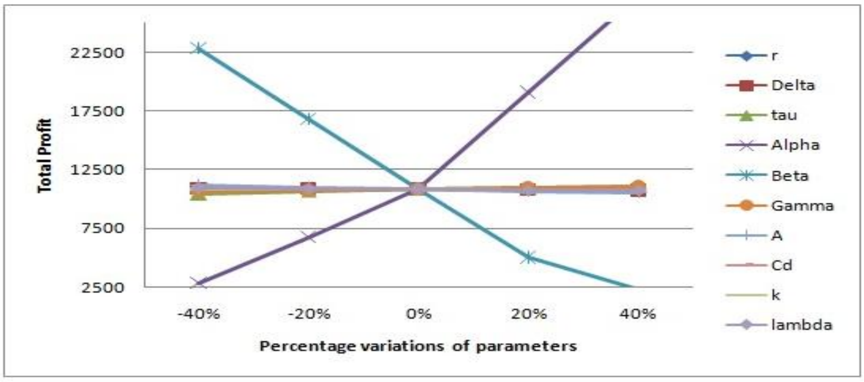

Figure 5: Total profit Vs Variation of parameters

6.1. Managerial insights (Significance results of study)

From the above sensitivity and mathematical analysis, following managerial insights summarized:

- The optimum value of cycle time gives the optimal value of production time, markdown offering time, markdown period, and product deterioration duration. A decision-maker can decide when to start and stop production, when to apply the markdown policy, and how much time to markdown.
- The optimal value of ordering a quantity of fresh product; and product with markdown, suggests to the decision-maker how much quantity should be replenishment per cycle such that total profit is maximized.
- A higher markdown rate results in a higher profit. If the markdown price applies to more quantity, then total profit also increases, subject to the markdown offering time and markdown period. (Table 3). A decision-maker should take a higher markdown rate for gain more profit.
- The producer should maintain constant demand for the fresh product, higher constant demand results in higher profit with the lowest cycle time and higher product quantity. (Table 6)
- Product freshness is the key determinant of market demand; our investigation revealed that if a product has a longer shelf life, the producer will make more revenue by selling more of it. (Table 5)
- Product freshness is based on the pace of degradation; at the beginning, deterioration has no impact. The products have a higher level of freshness when the non-deterioration period is longer (i.e., when the markdown offer is made later). So, the producer chooses to sell products whose non deterioration period is longer.
- Markdown percentage optimization helps the decision maker decide when to apply the markdown policy and how long the markdown period is so that total profit is maximized.
- Higher green investment results in green or organic products, which are more nutritious and sustainable. This is measured by the success of greening activities. Our research demonstrates that more investments in greening result in larger profits, which are increased by greater product volumes and higher markdown percentages. (Table 8)
- Higher production setup costs, deterioration costs, holding costs, and production costs increase the total cost of the system and reduce profit. A decision maker should try to control the different costs, ensure the system works smoothly, and increase the total profit.
- The rate of physical deterioration of the product plays an important role; a higher rate of physical deterioration slightly reduces the profit. (Table 11)

7. CONCLUSION

An economic production quantity model with price, freshness level, and greening efforts dependent demand has been designed in this paper under the markdown policy. The optimal replenishment time, optimal production quantities, optimal markdown offer quantities, optimal production period, optimal markdown offering time, and optimal profit have been determined. Perishable products' freshness and greening level can be considered the major elements that influence a buyer's purchasing behaviour. The novelty of the proposed model is the concept of greening efforts with freshness and price-related demand is considered. The demand for perishable goods such as green packaged beverages, dairy products, and organic farming products was influenced by consumers' preferences for greenness in addition to freshness and price. Another novel idea is that perishable goods shouldn't physically or qualitatively degrade more quickly throughout production and that their freshness should be assumed to be 100%. Hence, demand at the beginning of inventory cycle time is price- and green-efforts-dependent. After production stops, products affected by deterioration of both type and product value degrade continuously, during this period demand pattern depend on freshness, price and greening efforts; Due to freshness degradation, a markdown strategy is adopted after a period of product deterioration to boost demand. Green investment concept in the EPQ model with freshness and markdown strategy, which is another new idea of this study. The results show that markdown offering time and markdown rate make important contributions to maximum total profit, and decision-makers must be very precise when figuring out markdown offering time and markdown rate because the markdown offering should not be too early or too late in order to help in maximising the total profit. The markdown technique is a key method for clearing out stock before it reaches its maximum lifespan. Markdown percentage optimization helps to decision maker to decide the markdown period, hence maximize the profit. The results indicated that more green investments and products with longer shelf lives and shorter deterioration periods boost overall profit with a markdown strategy, which distinguished the previous literature. The problem has been turned into a mathematical model for the purpose of model justification, and a solution process was provided along with an example. To illustrate the analytical results and offer significant managerial implications as a conclusion, we make use of sensitivity analysis.

This model could be extended by taking preservation techniques into account to slow down deterioration. Different payment methods and concepts for carbon policies, such as carbon tax, cap and trade, and carbon limit policies, may be added to the model in the future. Also, by including shortages, the suggested model can be made more inclusive.

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