Yugoslav Journal of Operations Research
34 (2024), Number 2, 307–336
DOI: https://doi.org/10.2298/YJOR230915030C

RELIABILITY ANALYSIS AND ANFIS COMPUTATION FOR MULTI-SERVER REDUNDANT MACHINING SYSTEM WITH THE GENERALIZED TRIADIC POLICY

Parmeet KAUR CHAHAL
Central University of Jammu, Bagla, Samba, 181143, Jammu & Kashmir, India
parmeetcujammu@gamil.com

Kamlesh KUMAR*(corresponding author)
Central University of Jammu, Bagla, Samba, 181143, Jammu & Kashmir, India
kamlesh.math@cujammu.ac.in

Bhavneet Singh SOODAN
Chandigarh University, Kharar, Mohali, 140413, Punjab, India
bhavneet.e11868@cumail.in

Received: September 2023 / Accepted: November 2023

Abstract: The intent of this research is to have discussions about the transient behavior of a multi-server redundant machining system with a generalized Triadic control policy. Furthermore, the system facilitates a multiple working vacation policy for servers. The relevant rates of the associated birth-death process are chosen in order to frame the differential-difference equations for the system. Using the fourth order Runge-Kutta technique, time-dependent probabilities of the states are derived. Some performance metrics of system such as the average number of failed machines, average waiting time of failed machines, and reliability function are established using the states probabilities. Graphical and tabular representations demonstrate the behavior of these metrics with respect to a variety of system characteristics. Statistical hypothesis testing has been employed to compare the outcomes presented in tabular format. Beside these, numerical findings of the Runge-Kutta method are compared to those delivered by an adaptive neuro-fuzzy inference system (ANFIS) technique.

Keywords: Redundant machine repair system, multiple working vacation, Runge-Kutta's method, generalized triadic policy, pearson correlation coefficient and ANFIS..

MSC: 60J22, 60J28, 62M02, 65C30, 90B22, 90B25.

1. INTRODUCTION

Oftentimes, when we go through the machining systems, we sometimes see the server(s) sitting idle amid the absence of failed machines, or the failed machines queueing up for service owing to the non-availability of the server. Both of the aforementioned circumstances impede the economic viability of the machining system. By properly utilizing the accessible resources, the system economics can be enhanced. One strategy for sustaining the aforementioned goals requires controlling the active servers based on the size of the queue formed by the failed machines. Within the domain of modelling the machining systems by queueing models, several policies have been proposed to control the service mechanism. Yadin and Naor [1] devised one of aforesaid kind of policies, which they termed as the control N policy. As per the N-policy, the service is commenced by server only when $N(\geq 1)$ failed machines have accrued in the system, ceases the service when system becomes empty, and remains inactive until N failed machines have again aggregated.Recently, Chahal and Kumar[2] conducted an in-depth review and analysis of queueing modeling in machine repair problems, specifically focusing on control policies. Furthermore, the authors have provided mathematical formulations for queueing models of machining systems that incorporate various control policies. Bell [3] examined a service control policy for optimizing the two-server queueing model wherein the count of active servers can be governed by a threshold level. In order to control the service, Rhee and Sivazlian [4], for the first time introduced the triadic$(0, K, N, M)$ control policy for the two-server queueing system with removable service stations. The triadic$(0, K, N, M)$ control policy states that when the system boots up, the two-servers remain in a dormant state as long as the specified thresholds for the count of failed machines is achieved. Once the sum of failed machines equals the preset threshold value N, either of the two dormant servers will immediately be activated. Subsequently, whenever the count of failed machines meets another threshold $M(N < M)$, the other inactive server also gets activated. While two servers have become functional concurrently, if the count of failed machines drops to $Q(3 \leq Q < N)$, then the server that winds up service at that moment will be deactivated. Later, when the count of failed machines becomes zero the remaining server shifts to dormant mode. Until the prior specified thresholds are yet again acquired, both the servers remain dormant. Wang and Chang [5] studied a reliability statistics for a redundant two-server system.

Huang *et al.*[6] investigated a finite capacity two-server queueing system that controls the arrivals and the services while servers operates under triadic policy. The authors used genetic algorithm for obtaining the optimal value of arrival and service parameters. Liou *et al.*[7] evaluated the controlled two-server machine repair model functioning in accordance with the triadic (0,Q, N,M) strategy. Using a genetic algorithm, they developed a total expected cost function and determined the optimal operating (0,Q, N,M) policy and the service rate for minimum possible cost. Laxmi and Goswami *et al.*[8] studied a controllable machining system with removable servers functioning under a triadic strategy. Ketema *et al.*[9] analyzed triadic $(0, Q, N, M)$ policy for a machining system by incorporating multiple working vacation policy for servers. Utilizing a grid search technique, they optimized the service rate during working vacation and appropriate the thresholds $(0, Q, N, M)$ for reducing the predicted total cost. After reviewing the published literature on triadic policy for $M/M/2$ queueing models of machine repair systems, we have found that no generalized work have been done so far on triadic policy. We, in this paper, thus have generalized the triadic policy for multiple servers. In the operating systems, a machine failure always impair the capacity of the system and hinders its proper functioning. This contributes to the productivity loss as well as the system's unreliability. Supplying the system with additional backup machines, commonly referred to as standby support, offers a certain degree of improvement in the system efficiency. Enhancing system redundancy is a substantiated approach to boost the system availability and reliability. Whenever an active component machine fails, standby redundancy, which is comprised of inactive redundant components, is activated, thus enabling it to function as the substitution of the failed active component. Frameworks for queuing models that accommodate standbys have widespread applications that include computer networks, production system, and communication networks amongst many other systems. Redundant machining components can be classified as hot, cold, or warm. A hot standby fails at the same rate as an operating one whereas a warm standby fails at a lower rate than an operating one. Additionally, cold standbys are guaranteed to never fail and thus have failure rate of zero.

The terminology "redundant system" encompasses a sort of machining system that is furnished with provisions for standbys. Taylor and Jackson [10] pioneered the work in modelling the queueing systems with redundant machine repair. Gross *et al.* [11] studied the redundant machine repair system with the provision of standbys. Wang [12] performed the cost analysis of multi-server machining queueing system with mixed standbys. He also evaluated several system characteristics under the optimal operating conditions. Kolledath *et al.* [13] conducted a review of the research that has been done within the scope of queuing models with standby support. Gao and Wang [14] examined the mixed standby queueing system with an unreliable repair facility. They obtained the steady-state results for the availability of machining system and repair facility.Further, they also performed some numerical analysis to determine the effects of system parameters on reliability function, steady-state availability and MTTF. Kumar *et al.*[15] studied a queueing model incorporating threshold recovery policy for machining system with mixed standbys.

In real-world machining environments, it can be seen that the service facility is terminated as soon as it becomes empty, opting not to wait for broken machines for a prolonged duration. For the optimum use of a server, it either fully departs the service facility to take a vacation or remains in the service facility undertaking secondary tasks despite of providing principal repair services at reduced rates. The subsequent condition also persists for working vacation. The server resumes normal functioning only once a predefined count of failed machines are accumulated in the service facility. When the server returns from a vacation and finds that there is no failed machine accumulated for service, then he may either join the service facility or may take another vacation and it is termed as multiple vacation policy. Levy and Yechiali [16] were the initial researchers to publish the work on queueing models with service vacations. Chandrasekaran *et al.*[17] conducted in-depth review studies within the context of queueing systems with working vacations. Yang and Tsao [18] studied a reliability-based metrics of redundant repairable systems with retrials and working vacations.Bouchentouf and Yahiaouf[19] conducted a study on the single server queueing model with impatient consumers and a feedback policy, that focuses on multiple working vacations. The authors have derived concrete formulas for different performance indicators by utilizing probability generating functions. Recently Jain *et al.*[20] examined a retrial queue characterized by working vacations and impatient customers. They designed the cost function and optimized it by employing quasi-Newton method (QNM) and a genetic algorithm (GA). Gupta *et al.*[21] studied a two-server queuing system with working vacations and impatient customers. Zaid *et al.*[22] analyzed a multi-server queueing model with impatient consumers and variant working vacations interrupted by a Bernoulli schedule vacation. The authors determined the system's steady-state probabilities using the matrix-geometric method. Furthermore, the authors conducted a cost analysis for the system using a direct search approach.

For providing a numerical insight to the queueing modelling, the soft computing technique Adaptive Neuro-Fuzzy Inference System(ANFIS) have been effectively employed in communications networks, electronic components, automotive components with automatic transmissions, financial engineering, and many more. The ANFIS approach, characterized by a blend of neural networks and fuzzy logic, proves to be extremely useful for creating a fuzzy prediction model for a system that correlates the input data. Different sectors of the manufacturing industry are also benefited from this hybrid soft computing method, ANFIS. Lin and Liu [23] addressed the adaptive neuro-fuzzy in machine inference for estimating CMP machining system. Kumar and Jain [24] performed a numerical simulation of the $N-$ policy model for a multi-server machining system by incorporating redundancy, working vacations and contrasted the results taken by the SOR approach with those generated by the ANFIS technique. Jain and Meena [25] employed the ANFIS technique for the investigation of threshold recovery policy for a redundant machine repair model featuring two unreliable heterogeneous servers to estimate key performance indicators and contrasted them to the findings acquired through Runge-Kutta method. The study by Sethi *et al.* [26] integrated the impatient behavior of customers and a threshold recovery policy and then employed the ANFIS method to analyse the numerical results. Ahuja and Jain [27] undertook a transient examination of a single-server queueing system characterised by server unreliability, multiple-stage service, and working vacation. Outcomes from the ANFIS and the Runge-Kutta technique are also compared.

It is worthwhile to mention that as far as we are aware, no such service control strategy has been documented in the literature of queueing modelling for M/M/R system (here,R¿2), which administers the number of active servers in a service facility by specifying predetermined thresholds for server activation as well as server deactivation. After reviewing the published literature on triadic policy for M/M/2 queueing models of machine repair systems, we have found that no generalized work have been done so far on triadic policy. We, in this paper, thus have generalized the triadic policy for multiple servers. Furthermore, there are a limited number of papers available in the literatures that have conducted transient study for queueing models incorporating triadic policy. We have conducted a transient study for multi-server queueing models incorporating the generalized triadic policy. In addition, we have included some useful features such as redundancy and multiple vacation policy. The authors would propose that incorporating a generalized triadic policy into the queueing model of the multi-server machining system would result in more efficient utilization of available resources. The generalized triadic policy aims to control and maintain a balance between the number of busy servers and the number of failed machines in the system while dealing with multi-server system. This approach is highly beneficial in preventing congestion, which occurs when the number of failed machines increases due to a limited number of active servers in the system. It also addresses the issue of servers remaining idle for extended periods due to an empty system or a less number of failed machines available in the system. The remaining portion of this article is structured in the following manner: Section 2 provides a comprehensive explanation about the model and its justification in regard to the physical world. Section 3 constitutes a mathematical analysis of the model. Here, we establish the reliability function and the average number of failed machines in the system. We conduct numerical simulation in Section 4. We compare the Runge-Kutta's results with those of the hybrid soft computing technique as ANFIS in section 5. The outcomes and managerial inference are presented in section 6. The study concludes with a discussion of potential future developments in section 7.

2. MODEL SPECIFICATION

2.1. Formulation of the model

We examine a redundant machining system having a total of $L(= M + S + C)$ machines of which M machines are identical operating machines, S machines are warm standbys, C machines are cold standbys, and R servers. The system is deemed to be short if it has less than M operating machines. All R servers are in a passive mode at the outset when there are no failed machines accumulated in the system's service facility. Whenever the count of failed machines accumulated in the service facility attains a threshold level $N_i(1 \leq i \leq R)$ (such that $N_i < N_{i+1},\ ;\ 1 \leq i \leq R$), one of the $R-i+1$ inactive servers switches into active mode. In a similar manner, when the count of failed machines falls to $Q_i(1 \leq i \leq R-1)$ (such that $N_1 > Q_i > Q_{i+1},\ 1 \leq i \leq R-1$), then that one particular server among all the active servers that has just completed service becomes inactive. Furthermore, if the count of failed machines drops to zero then the last operational server also gets deactivated. We refer to this method of regulating service as generalized triadic policy for machine repair models. All the servers gets switched into working vacation mode once the system has become entirely empty. If the count of failed machines is N_1 or higher at an end of a working vacation, then one of the R servers is activated and the system shifts to regular busy period and then starts operating under generalized triadic policy; otherwise, they take another working vacation and continue to do so until the number of failed machines in the system is N_1 or greater at the end of a vacation period. The service times of R servers are considered to abide by exponential distributions with service rates μ and ϑ ($< \mu$) during regular busy periods and working vacation respectively. There exists an endless number of possible combinations for the generalized triadic policy's predetermined thresholds. In the present work, we provide an algorithm to determine acceptable threshold values which is shown in Table 1.

Table 1: Generalized Triadic policy threshold assignment algorithm

Algorithm to assign threshold values for Generalized Triadic policy
1. Set $(Q_1, \tau, \gamma \in \mathbb{N})$ such that $\tau \geq \gamma \geq 3$; $Q_1 - i\gamma \geq 1$ for $i = R-1$; $Q_1 + i\tau \leq L-2$ for $i = R$
2. Estimate the values of other threshold using the recursive relation given below
• $Q_i = Q_1 - (i-1)\gamma$; $2 \leq i \leq R-1$
• $N_i = Q_1 + i\tau$; $1 \leq i \leq R$
3. *For example:* If we consider number of servers, $R = 3, L = 25$ and assign, $Q_1 = 9$, $\tau = 4$, $\gamma = 3$
• Then the first server among 3 inactive servers will become active when number of failed machines acquire threshold $N_1 = 13$. Further when number of failed machines rises to $N_2 = 17$, one server among 2 inactive server will become active. When the count of failed machines rises to $N_3 = 21$, the remaining inactive server will be activated.
• While the system continues to function in regular busy period and the count of failed machines declines to $Q_1 = 9$, one server among the active servers will be deactivated. If the number of failed machines will further drops to $Q_2 = 6$, one more server will be deactivated. In case the system turns into empty, the remaining active server will be shifted to inactive mode and system will drift to the working vacation mode unless the pre-specified thresholds are attained.

Mathematically the model is formulated around the following presumptions:

- Operating machines, warm standbys, and cold standbys are susceptible to failure according to exponential distribution with parameters λ, α, and *zero* respectively. Each working machine and accessible standby (warm or cold) machine fails individually, irrespective of the status of all other machines.

- Each time an operating machine undergoes failure, it is supplanted with a warm standby provided any will be accessible, while if the warm standby fails, a cold standby machine substitutes it. A warm standby acquires the failure characteristics of an operating machine when it transitions into an operating state, and likewise for a cold standby when it transitions into a warm standby state. The transition from warm standby to operating mode and from cold standby to warm standby occurs instantaneously.

- When any machine gets fail, it is transferred to a service facility, wherein the machines are repaired in the sequence of their failure. Each of the R homogeneous servers provides repair service to failed machines under a generalized triadic control policy at an exponentially distributed service rate μ. Only one of the R servers will offer repair services to failed machines throughout the working vacation time at an exponentially distributed service rate ϑ ($\vartheta < \mu$).
- Each server can only serve a single machine at one time. When a machine that has failed arrives at the service facility and finds all the servers busy in rendering the service then it must enter the system queue until one becomes available.
- After undergoing servicing, a failed machine is same as a new one and enters cold standby mode, unless there are fewer than M machines in operating mode at that instance the repaired machine is moved into operating mode.
- Each server enters working vacation mode when he discovers that no failed machines have queued up for repair. The server's working vacation time is dispersed separately and evenly at the exponential rate ϱ.

2.2. Practicable implementations of the model

The machine repair model proposed in this paper can be installed on a material handling system assembly line as further described. As far as technological advancement is concerned, we can see that now-a-days robots are frequently used in material handling (MH) because of their inherent capacity to move heavy or bulky things. In the manufacturing world, robotic material handling is a standard practice. Robotic arms used for material handling transfer items on and off a conveyor belt or hold items in position during assembly. Throughout a wide variety of applications in industries, robotic material handling systems are in a high demand because of their productivity increments and consistencies. Let us consider a manufacturing system line for the material handling which consists of M identically operating robotic arms and S robotic arms as warm standby and C robotic arms as cold standby. The operating robotic arms can fail while transferring bulky items from one region of the structure to another. The failed robotic arm is promptly swapped by the available standby. The failed robotic arm is transferred to the service facility, where R skilled technicians (servers) are available to repair failed robotic arms. The queue length for failed robotic arms is in synchrony with the count of active servers at the service center i.e. the activation of dormant server is dependent on the number of failed robotic arms in the system. The entire service mechanism is governed by activation and deactivation of servers that have predefined threshold levels. All the R servers are initially disabled until N_1 failed robots accumulate in the system and one server gets activated to initiate the repair process. Moreover one among dormant $R-i+1$ servers becomes active when failed robotic machines facility queue size reaches a pre-set threshold value $N_i(1 \leq i \leq R)$. When the queue size drops to $Q_i(1 \leq i \leq R-1)$ one server among the active servers becomes inactive. When the number of defective robotic arms reaches zero, the single server is likewise disabled and all servers enter working vacation configuration. In this configuration, servers undertake secondary tasks. If the queue size exceeds the threshold amount N_1 at the completion of working vacation, one of the R server becomes active. If not, they continue to take working vacations until the queue size is equal to N_1 or higher. Here the triadic policy is generalized for multi-server queueing models of machining systems.

The proposed model can be potentially applied in some more multi-server machining systems as Inventory systems, Production system, computer networks, Telecommunication, among many more to simulate the real-time service control mechanism in efficient manner.

3. SYSTEM GOVERNING EQUATIONS

Now, we define each of the states for the redundant machine repair model outlined as a pair $\{\Psi(t), \Lambda(t)\}$. Here $\Lambda(t)$ indicates the count of failed machines at time t, whereas $\Psi(t)$ depicts the status of the service facility at time t.

$$\Psi(t) = \begin{cases} 0, & \text{when the system is in working vacation mode} \\ i, & \text{when the system is in regular busy mode with i active servers} \end{cases}$$

For each of the states of a repairable machining system, we define the system governing transient-state probabilities as:

$P_{0,n}(t) \equiv$ The probability of the system being in working vacation mode with n failed at time t machines, where $n = 0, 1, 2, ..., L$.

$P_{i,n}(t) \equiv$ The probability of the system being in the regular busy period mode with i active servers and n failed machines at time t, where $i = 1, 2, 3, ..., R-1$. Here for $i = 1,, n = 1, 2, 3, ..., N_2 - 1$ and for $i = 2, 3, ..., R-1$, $n = Q_{R-i+1} + 1, Q_{R-i+1} + 2, ..., Ni + 1 - 2, N_{i+1} - 1$.

$P_{R,n}(t) \equiv$ The probability of the system being in the regular busy period mode with at time t, R active servers and n failed machines, where $n = Q_1 + 1, Q_1 + 2, ..., L-1, L$.

The state dependent failure rate λ_n is defined as:

$$\lambda_n = \begin{cases} M\lambda + S\alpha & 1 \leq n < C \\ M\lambda + (C + S - n)\alpha & C \leq n < C + S \\ (L - n)\lambda & C + S \leq n < L \\ 0 & \text{otherwise} \end{cases} \tag{1}$$

When $t = 0$, the following initial condition is assumed:

$$P_{j,n}(t) = \begin{cases} 1 & ; \, j = 0 \\ 0 & ; \text{ otherwise} \end{cases} \tag{2}$$

3.1. Differential-difference equations for the Model

We structure the difference-differential equation for each of the system's state at time t utilizing outflow with negative sign and the inflow with positive sign for each state. Figure 1 present the state-transition diagram for redundant machining system with generalized Triadic control policy for $R = 3$.

For i=0, when the system is in working vacation mode

$$\frac{d}{dt}P_{0,0}(t) = -\lambda_0 P_{0,0}(t) + \vartheta P_{0,1}(t) + \mu P_{1,1}(t)\,, \tag{3}$$

$$\frac{d}{dt}P_{0,n}(t) = -\left[\lambda_n + \vartheta\right] P_{0,n}(t) + \vartheta P_{0,n+1}(t) + \lambda_{n-1} P_{0,n-1}(t)\,; \quad 1 \le n \le N_1 - 1\,, \tag{4}$$

$$\frac{d}{dt}P_{0,n}(t) = -\left[\lambda_n + \vartheta + \varrho\right] P_{0,n}(t) + \vartheta P_{0,n+1}(t) + \lambda_{n-1} P_{0,n-1}(t)\,; \quad N_1 \le n \le L - 1\,, \tag{5}$$

$$\frac{d}{dt}P_{0,L}(t) = -\left[\vartheta + \varrho\right] P_{0,L} + \lambda_{L-1} P_{0,L-1}(t)\,, \tag{6}$$

For i=1, the system is in regular busy mode with one active server

$$\frac{d}{dt}P_{1,1}(t) = -\left[\mu + \lambda_1\right] P_{1,1}(t) + \mu P_{1,2}(t)\,, \tag{7}$$

$$\frac{d}{dt}P_{1,n}(t) = -\left[\lambda_n + \mu\right] P_{1,n}(t) + \mu P_{1,n+1}(t) + \lambda_{n-1} P_{1,n-1}(t)\,, \quad 2 \le n \le Q_{R-1} - 1\,, \tag{8}$$

$$\frac{d}{dt}P_{1,Q_{R-1}}(t) = -\left[\lambda_{Q_{R-1}} + \mu\right] P_{1,Q_{R-1}}(t) + \mu P_{1,Q_{R-1}+1}(t) + \lambda_{Q_{R-1}-1} P_{1,Q_{R-1}-1}(t) + 2\mu P_{2,Q_{R-1}+1}(t)\,, \tag{9}$$

$$\frac{d}{dt}P_{1,n}(t) = -\left[\lambda_n + \mu\right] P_{1,n}(t) + \mu P_{1,n+1}(t) + \lambda_{n-1} P_{1,n-1}(t)\,, \quad Q_{R-1} + 1 \le n \le N_1 - 1\,, \tag{10}$$

$$\frac{d}{dt}P_{1,n}(t) = -\left[\lambda_n + \mu\right] P_{1,n}(t) + \mu P_{1,n+1}(t) + \lambda_{n-1} P_{1,n-1}(t) + \varrho P_{0,n}(t)\,, \quad N_1 \le n \le N_2 - 2\,, \tag{11}$$

$$\frac{d}{dt}P_{1,N_2-1}(t) = -\left[\lambda_{N_2-1} + \mu\right] P_{1,N_2-1}(t) + \lambda_{N_2-2} P_{1,N_2-2}(t) + \varrho P_{0,N_2-1}(t)\,, \tag{12}$$

For $2 \leq n \leq R-1$, the system is in regular busy mode with two active servers

$$\frac{d}{dt}P_{i,Q_{R-i+1}+1}(t) = -\left[\lambda_{Q_{R-i+1}+1} + i\mu\right] P_{i,Q_{R-i+1}+1}(t) + i\mu P_{i,Q_{R-i+1}+2}(t)\,, \tag{13}$$

$$\frac{d}{dt}P_{i,n}(t) = -\left[\lambda_n + i\mu\right] P_{i,n}(t) + i\mu P_{i,n+1}(t) + \lambda_{n-1}P_{i,n-1}(t)\,, \quad Q_{R-i+1} + 2 \leq n \leq Q_{R-i} - 1\,, \tag{14}$$

$$\frac{d}{dt}P_{i,Q_{R-i}}(t) = -\left[\lambda_{Q_{R-i}} + i\mu\right] P_{i,Q_{R-i}}(t) + i\mu P_{i,Q_{R-i}+1}(t) + \lambda_{Q_{R-i}-1}P_{i,Q_{R-i}-1}(t) + (i+1)\mu P_{i+1,Q_{R-i}+1}(t)\,, \tag{15}$$

$$\frac{d}{dt}P_{i,n}(t) = -\left[\lambda_n + i\mu\right] P_{i,n}(t) + i\mu P_{i,n+1}(t) + \lambda_{n-1}P_{i,n-1}(t)\,, \quad Q_{R-i} + 1 \leq n \leq N_i - 1\,, \tag{16}$$

$$\frac{d}{dt}P_{i,N_i}(t) = -\left[\lambda_{N_i} + i\mu\right] P_{i,N_i}(t) + i\mu P_{i,N_i+1}(t) + \lambda_{N_i-1}P_{i,N_i-1}(t) + \lambda_{N_i-1}P_{i-1,N_i-1}(t) + \varrho P_{0,N_i}(t)\,, \tag{17}$$

$$\frac{d}{dt}P_{i,n}(t) = -\left[\lambda_n + i\mu\right] P_{i,n}(t) + i\mu P_{i,n+1}(t) + \lambda_{n-1}P_{i,n-1}(t) + \varrho P_{0,n}(t)\,, \quad N_i + 1 \leq n \leq N_{i+1} - 2\,, \tag{18}$$

$$\frac{d}{dt}P_{i,N_{i+1}-1}(t) = -\left[\lambda_{N_{i+1}-1}(t) + i\mu\right] P_{i,N_{i+1}-1}(t) + \lambda_{N_{i+1}-2}P_{i,N_{i+1}-2}(t) + \varrho P_{0,N_{i+1}-1}(t)\,, \tag{19}$$

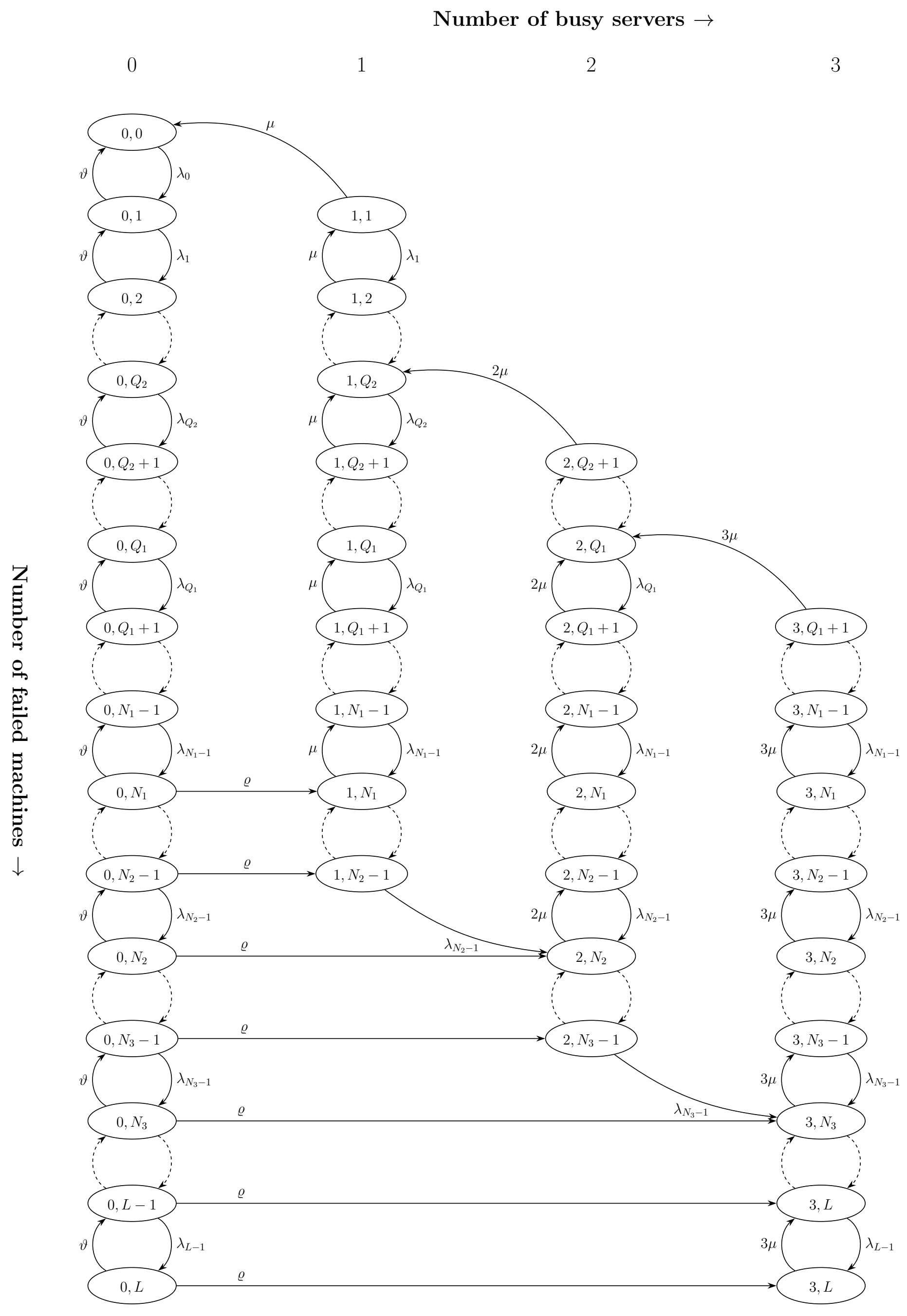

Figure 1: State transition diagram of machine repair problem with working vacation policy under triadic control policy generalized for 3−server model

For i=R, the system is in regular busy mode with R active servers

$$\frac{d}{dt}P_{R,Q_1+1}(t) = -\left[\lambda_{Q_1+1} + R\mu\right] P_{R,Q_1+1}(t) + R\mu P_{R,Q_1+2}(t)\,, \tag{20}$$

$$\frac{d}{dt}P_{R,n}(t) = -\left[\lambda_n + R\mu\right] P_{R,n}(t) + R\mu P_{R,n+1}(t) + \lambda_{n-1}P_{R,n-1}(t)\,, \quad Q_1 + 2 \le n \le N_R - 1\,, \tag{21}$$

$$\frac{d}{dt}P_{R,N_R}(t) = -\left[\lambda_{N_R} + R\mu\right] P_{R,N_R}(t) + R\mu P_{R,N_R+1}(t) + \lambda_{N_R-1}P_{R,N_R-1}(t) + \lambda_{N_R-1}P_{R-1,N_R-1}(t) + \varrho P_{0,N_R}(t)\,, \tag{22}$$

$$\frac{d}{dt}P_{R,n}(t) = -\left[\lambda_n + R\mu\right] P_{R,n}(t) + R\mu P_{R,n+1}(t) + \lambda_{n-1}P_{R,n-1}(t) + \varrho P_{0,n}(t) \;\; ; \quad N_R + 1 \le n \le L - 1\,, \tag{23}$$

$$\frac{d}{dt}P_{R,L}(t) = -R\mu P_{R,L}(t) + \lambda_{L-1}P_{R,L-1}(t) + \varrho P_{0,L}(t)\,, \tag{24}$$

The Runge-Kutta technique is utilized to compute the numerical solutions for the above system of equations governing transient-state behavior. In the forthcoming section, with the use of transient state probabilities, we present the expressions for system's queueing metrics and system's reliability function.

3.2. Reliability function and Queueing Indices

In the present section, with the use of transient-state probabilities, we establish the following expressions for critical key metrics including the system reliability function, the average number of failed machines and the average waiting time of failed machines:

- $E(N_s(t)) \equiv$ The average number of failed machines in the system.

  The average number of failed machines in the system is important from the perspective and interest of management as $E(N_s(t))$ is an important metric that represents service quality. The average number of failed machines in the system is the sum of the failed machines in the queue and the failed machine in service. Thus, we have;

$$E(N_s(t)) = \sum_{n=0}^{L} nP_{0,n}(t) + \sum_{n=1}^{N_2-1} nP_{1,n}(t) + \sum_{i=2}^{R-1} \sum_{n=Q_{R-i+1}+1}^{N_{i+1}-1} nP_{i,n}(t) + \sum_{n=Q_1+1}^{L} nP_{R,n}(t) \tag{25}$$

- $\lambda_{eff}(t) \equiv$ The effective failure rate.

Considering the state dependent failure rate λ_n and the probability associated with each state, the effective failure rate is formulated as;

$$\lambda_{eff}(t) = \sum_{n=0}^{L} \lambda_n P_{0,n}(t) + \sum_{n=1}^{N_2-1} \lambda_n P_{1,n}(t) + \sum_{i=2}^{R-1} \sum_{n=Q_{R-i+1}+1}^{N_{i+1}-1} \lambda_n P_{i,n}(t) + \sum_{n=Q_1+1}^{L} \lambda_n P_{R,n}(t) \tag{26}$$

- $E(W_s(t)) \equiv$ The average waiting time of the failed machine in the system.

  Little's formula given by Equation 27 expresses the relationship between the average number of failed machines in the system $E(N_s(t))$ and the average waiting time spent by the failed machine in the system $E(W_s(t))$

$$E(N_s(t)) = \lambda_{eff}(t)\ E(W_s(t)) \tag{27}$$

  Thus, from Equation 27 we compute formulation for $E(W_s(t))$ as;

$$E(W_s(t)) = \frac{E(N_s(t))}{\lambda_{eff}(t)} \tag{28}$$

- $R_Y(t) \equiv$ The system reliability function.

  The servers in the queueing model under investigation are considered to be reliable throughout. But the states, when all the L machines have failed i.e. $P_{0,L}$ and $P_{R,L}$, the system will shut down. Thus, the reliability function for the system is given by;

$$R_Y(t) = 1 - (P_{0,L}(t) + P_{R,L}(t)) \tag{29}$$

4. SPECIAL CASES

In this section we deduce some queueing models on control policies, that have already been studied in published research papers by utilizing the queueing model of machining system with generalized triadic control policy being discussed in the present study. For the purpose we assign specific values to some input parameters, L, M, S, C, R, $Q_i(1 \leq i \leq R-1)$, $N_i(1 \leq i \leq R)$, α, λ, μ, ϑ and ϱ.

- **Case 1** If we take $R = 2$, [$L = M$, $S = 0$, $C = 0$, $\alpha = 0$(i.e. no standby provision)], [$\vartheta = 0$, $\varrho = \infty$(i.e. no multiple working vacation policy)]and also take the notations as $Q_1 = Q$, $N_1 = N$, $N_2 = M$, our model reduces to $M/M/2$ queueing model with finite capacity L operating under the triadic $(0, Q, N, M)$ policy. The numerical results depicting variation of $E(N_s(t))$ and $E(W_s(t))$ with failure rate λ, service rate μ are in agreement with those of *Kuo-Hsiung Wang* and *Ya-Ling Wang*(2002)[[28]].

- **Case 2** Taking $R = 2$, $[L = M,\ S = 0,\ C = 0,\ \alpha = 0$(i.e. no standby provision)],and take the notations as $Q_1 = Q$, $N_1 = N$, $N_2 = M$, our model reduces to $M/M/2$ multiple working vacation model operating under the triadic $(0, Q, N, M)$ policy. The trends of $E(N_s(t))$ and $E(W_s(t))$ with input parameters depicted from our model are numerically parallel to those of *Teketel Ketema*, *Seleshi Demie* and *Melisew Tefera*(2021)[[9]].

In the next section, we present results of numerical simulations performed to analyze the effects of modifying input parameters on the metrics of the system being studied.

5. NUMERICAL SIMULATIONS

We demonstrate some numerical simulations within the current section to examine that impact of changing input parameters on the various metrics of the system under investigation. The system governing transient-state probabilities are computed numerically using the Runge-Kutta technique. We have utilized the ode45 function of the MATLAB software to develop the 4th order Runge-Kutta algorithm for numerical simulation purposes. Kumar and Jain also [29] adopted Runge-Kutta's approach to estimate transient-state probabilities for a redundant multi-component machining system with two unreliable repairmen operating under a bi-level (N, L) control policy. While performing numerical computations for this model, R can be assigned any positive integer value. In the present analysis, all results have been out for $R = 2, R = 3$ and $R = 4$. We also conduct a comparative analysis to look at the manner in which the system's behavior shifts as the number of servers changes.

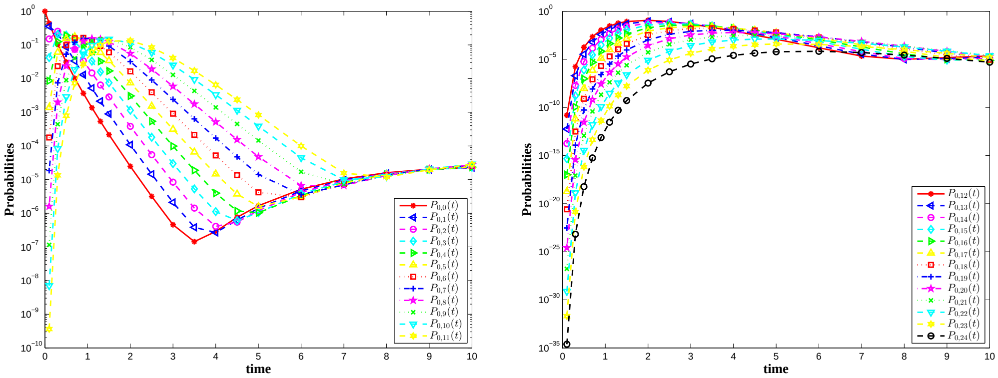

Figure 2: Variation of system probabilities with time. We take $R = 4$; $L = 32$; $M = 28$; $C = 1$; $S = 3$; $Q_1 = 9$; $\tau = \gamma = 3$;$\lambda = 0.4$; $\alpha = 0.2$; $\mu = 2.0$; $\vartheta = 1.0$ and $\varrho = 1.2$.

Figure 2 show the variation of state probabilities of the number of failed machines with time given, $R = 4$. It has been observed that all of the system state probabilities starts from zero except from $P_{0,0}$ which starts from 1. This pattern observed as a consequence of the initial condition we have defined in Equation 2 for our model. Thereon these probabilities rises quickly over a short period of time, and then eventually attain steady-state after some time has elapsed. The patterns of system performance indices have been interpreted in the following manner predicated by the numerical outcomes:

In Figure 3(a-f) the pattern of the average number of failed machines in the system $E(N_s(t))$ and the average waiting time of failed machines in the system $E(W_s(t))$ with failure rate λ is observed. As the failure rate λ rises, a growing number of defective equipments are accumulated in the service facility. Hence, $E(N_s(t))$ and $E(W_s(t))$ will rise. Considering the fixed failure rate, we discover that $E(N_s(t))$ and $E(W_s(t))$ would keep rising with time. Figure 3(a-f) and Figure 4(a-f) displays that as time goes on both $E(N_s(t))$ and $E(W_s(t))$ gradually increase, but then starts to decline. This behavior results from the fact that there is higher count of failed machines over time, and when that number hits certain levels, additional servers must be activated in accordance with the general triadic policy. As the number of active servers in the system increases, both the average number of failing machines and their waiting time will decrease. The aforementioned fact becomes more plainly apparent to the observer with rising R value.

In Figure 4(a-f) the variations in $E(N_s(t))$ and $E(W_s(t))$ with service rate μ is depicted. When the server's service rate spikes, the backlog of failed machines in the service facility is quickly attended. Consequently, the average number of failing machines in the system $E(N_s(t))$ and $E(W_s(t))$ both decline rapidly.

From Figure 5(a-f), we can see that the reliability is inversely related to failure rate λ and linearly related to service rate μ. In the beginning, there is a tendency for the number of malfunctioning machines to steadily rise over time, thereby increasing the likelihood of all L machines experiencing failure. Consequently, the reliability function decreases. The service mechanism of the system under examination is governed by the generalized triadic policy. As the number of failed machines increases, the threshold for server activations will also be reached gradually. This will result in a greater number of active servers in the system and then the failed devices will require less time to be repaired. The repair service for failed machines will be promptly provided, resulting in a reduction in the number of failed machines that have accumulated within the system. This will reduce the likelihood that all L machines have failed in the system and therefore, the reliability functioning improves. While the number of failed machines increases over time, the count of active servers will also increase, resulting in a drop in the number of failed machines due to repair services. Thus, we notice that the reliability function declines at first and subsequently improves overtime. In order to make sense from a logical standpoint, the reliability function is supposed to be convex. Figure 5(a-f) and Figure 6 demonstrate that the reliability function,

$R_Y(t)$ exhibits convex behavior. Because the servers are supposed to be always reliable in our model, the reliability function never attains zero. Figure 6 illustrates that increasing the number of servers in a system improves system reliability.

Table 2–Table 4 displays the variation of $E(N_s(t))$, $E(W_s(t))$ and $R_Y(t)$ with parameters α , ϑ and ϱ. As the failure rate of warm standby machines (α) increases, more failed machines will start to pile up in the service facility. This increases the average number of failed machines $E(N_s(t))$ as well as their average wait time $E(W_s(t))$ in the system. As a result, the system's reliability function ($R_Y(t)$) declines. If the service rate during working vacation (ϑ) increases, the failing machines will not have to wait for a longer period of time to receive repair service, and so the number of accumulated machines will start to decrease. This reduces the average number of failed machines along with their average wait time in the system, improving system reliability. The duration of working vacation decreases as the rate of working vacation (ϱ) increases. This allows for keeping the server available in the servicing facility for a longer period of time. Consequently, the average number of failed machines $E(N_s(t))$ along with their average waiting time $E(W_s(t))$ in the system decreases, thus increasing system's reliability function ($R_Y(t)$).

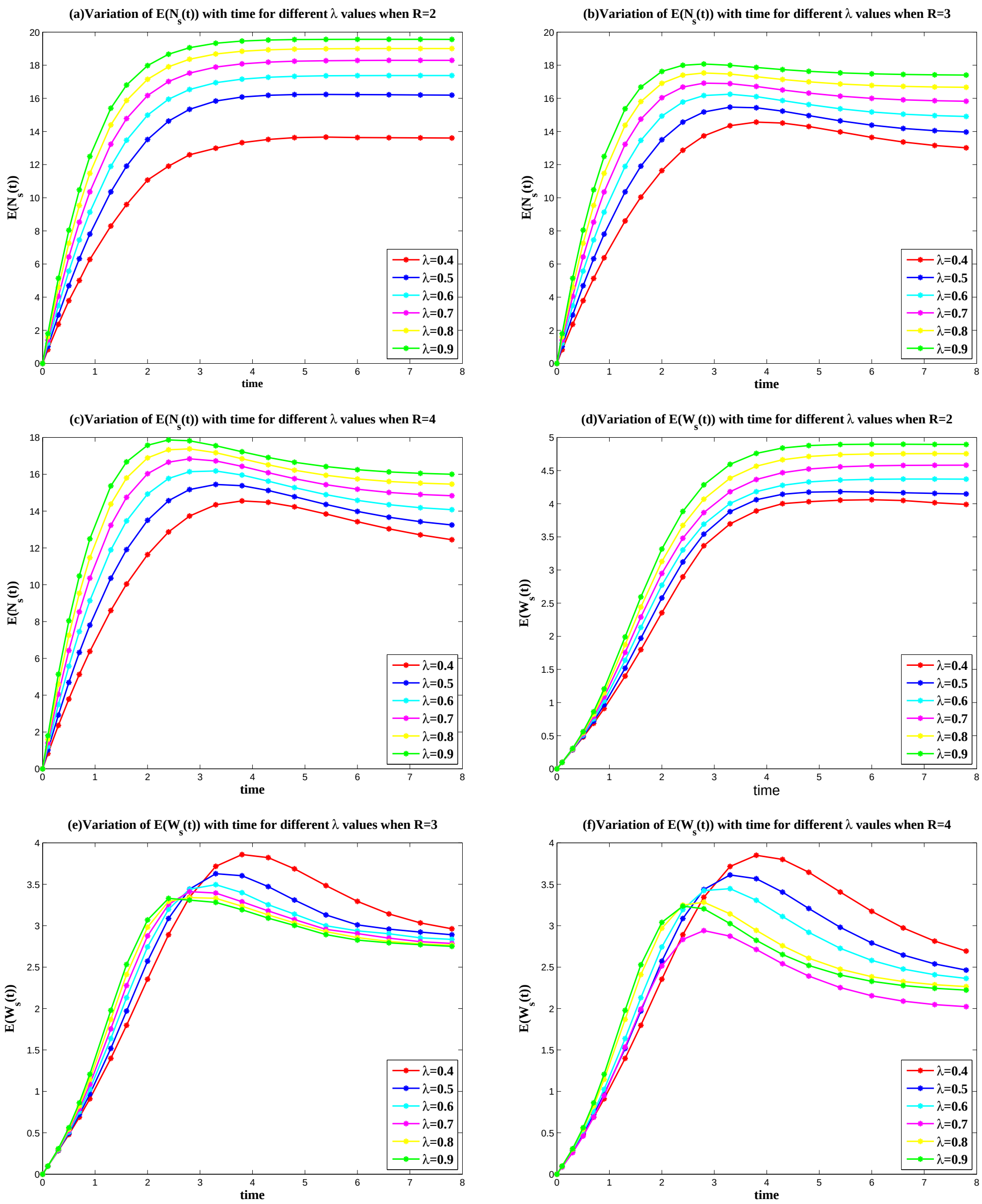

Figure 3: Time-based variation of $E(N_s(t))$ and $E(W_s(t))$ with λ. We take $L = 32;\ M = 28;\ C = 1;\ S = 3;\ Q_1 = 9;\ \tau = \gamma = 3;\ \alpha = 0.2;\ \mu = 2.0;\ \vartheta = 1.0$ and $\varrho = 1.2$.

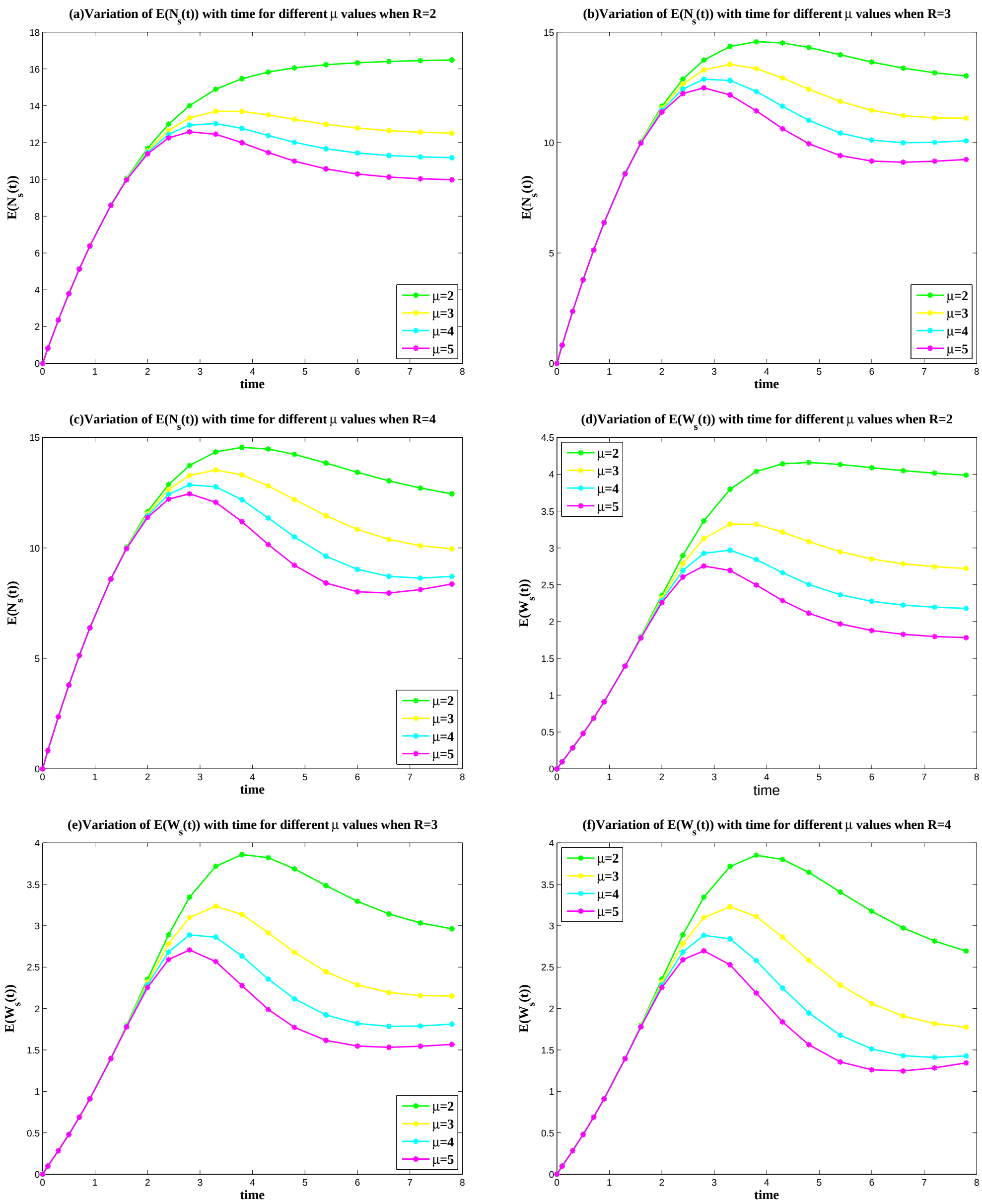

Figure 4: Time-based variation of $E(N_s(t))$ and $E(W_s(t))$ with μ. We take $L = 32$; $M = 28$; $C = 1$; $S = 3$; $Q_1 = 9$; $\tau = \gamma = 3$; $\lambda = 0.4$; $\alpha = 0.2$; $\vartheta = 1.0$ and $\varrho = 1.2$.

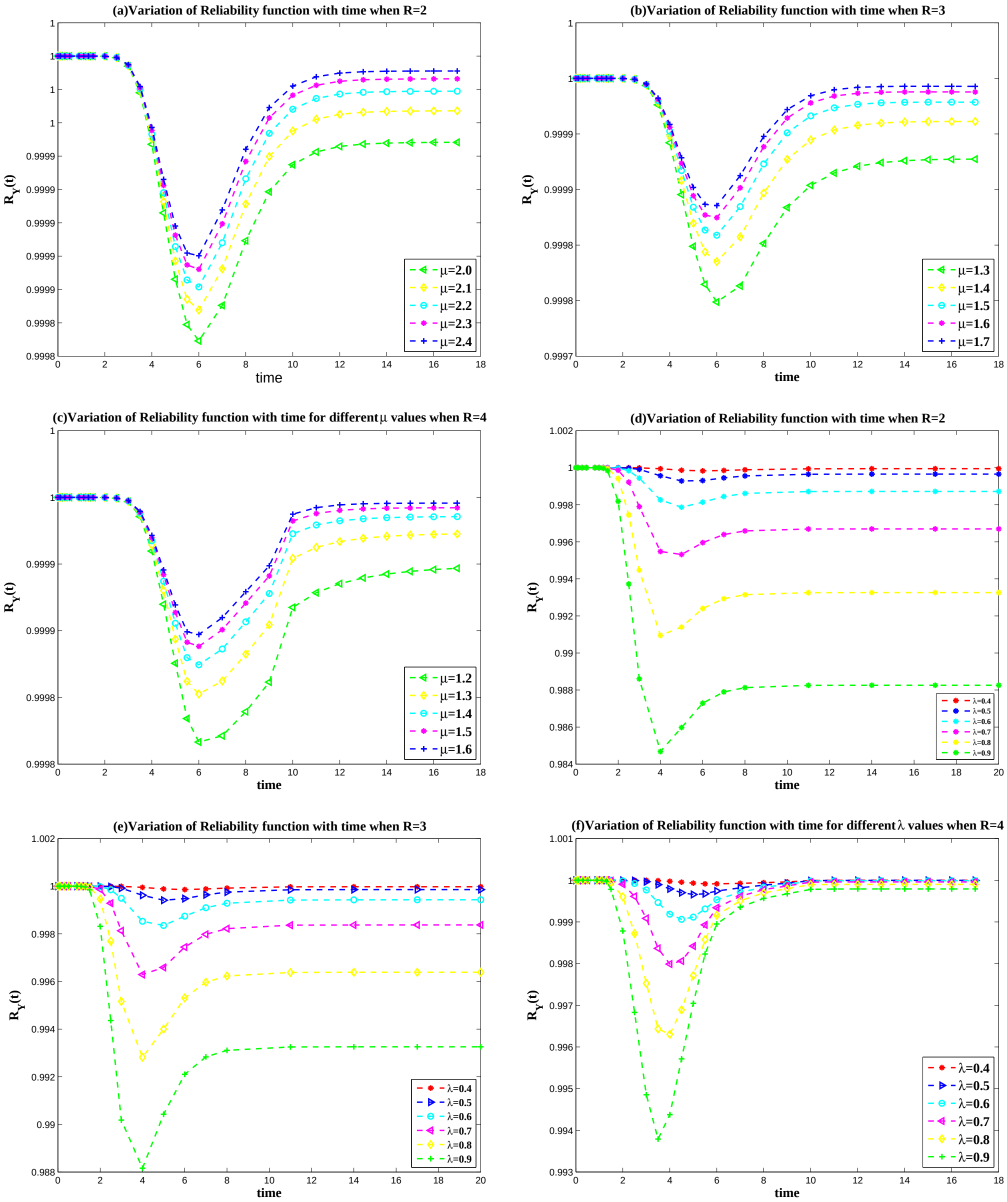

Figure 5: Time-based variation of $R_Y(t)$ with failure rate λ and service rate μ. We take $L = 32$; $M = 28$; $C = 1$; $S = 3$; $Q_1 = 9$; $\tau = \gamma = 3$; $\alpha = 0.2$; $\vartheta = 1.0$ and $\varrho = 1.2$.

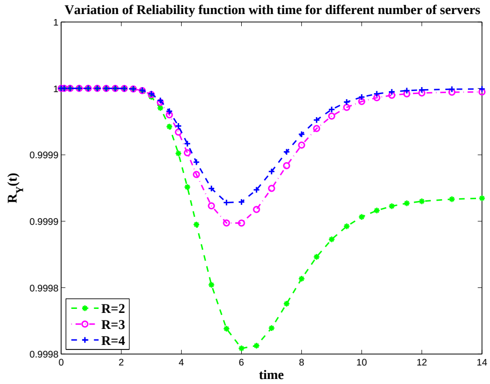

Figure 6: Developments in $R_Y(t)$ over time as a function of the system's server count, R. We take $L = 32$; $M = 28$; $C = 1$; $S = 3$; $Q_1 = 9$; $\tau = \gamma = 3$; $\lambda = 0.4$; $\alpha = 0.2$; $\mu = 2.0$; $\vartheta = 1.0$ and $\varrho = 1.2$.

Table 2: Variation of Performance measures with parameters α for $R = 4$. We take $R = 4$; $L = 32$; $M = 28$; $C = 1$; $S = 3$; $Q_1 = 9$; $x = y = 3$;$\lambda = 0.4$; $\mu = 2.0$; $\vartheta = 1.0$ and $\varrho = 1.2$.

α	$E(N_s(t))$	$E(W_s(t))$	$R_Y(t)$
0.2	13.7900963	2.701346851	0.999995981
0.25	13.81069939	2.710845794	0.999995908
0.3	13.83071598	2.720098856	0.999995836
0.35	13.85017197	2.729155015	0.999995764
0.4	13.86909173	2.737982592	0.999995692

Table 3: Variation of Performance measures with parameters ϑ for $R = 4$. We take $R = 4$; $L = 32$; $M = 28$; $C = 1$; $S = 3$; $Q_1 = 9$; $x = y = 3$;$\lambda = 0.4$; $\mu = 2.0$; $\alpha = 0.2$ and $\varrho = 1.2$.

ϑ	$E(N_s(t))$	$E(W_s(t))$	$R_Y(t)$
1	13.7900963	2.701346851	0.999995981
1.1	13.66800323	2.645800659	0.999996451
1.2	13.54590189	2.591550629	0.999996866
1.3	13.42379652	2.538554177	0.999997232
1.4	13.30169155	2.486770739	0.999997555

To compare the data sets shown in Table 2, Table 3 and Table 4, we utilize the statistical hypothesis with Pearson Correlation Coefficient.

Table 4: Variation of Performance measures with parameters ϱ for $R = 4$. We take $R = 4$; $L = 32$; $M = 28$; $C = 1$; $S = 3$; $Q_1 = 9$; $x = y = 3$;$\lambda = 0.4$; $\mu = 2.0$; $\vartheta = 1.0$ and $\alpha = 0.2$.

ϱ	$E(N_s(t))$	$E(W_s(t))$	$R_Y(t)$
0.05	13.7900963	2.701346851	0.999995981
0.07	13.78284181	2.698008717	0.999996066
0.09	13.77568979	2.694722371	0.999996149
0.11	13.76863856	2.691486838	0.999996231
0.13	13.76168649	2.688301165	0.99999631

Hypothesis Test with the Pearson Correlation Coefficient

A statistical measure referred to as the Pearson's correlation coefficient describes both the strength and the direction of the linear relationship that exists between two sets of data associated with the variables x and y. Here variable x can be one of the parameter among α, ϑ and ϱ and variable y can be one of the performance metric among $E(N_S)$, $E(W_S)$ and $R_Y(t)$. The sample's Pearson correlation value is r which is calculated from the sample data. It is an estimate of the population's Pearson correlation coefficient, ρ which is unknown.

- *Step 1: State Hypotheses and choose the significance level (α)*

  We begin by outlining the null and alternate hypotheses:

  - The null hypothesis states that there is no correlation between the variables, x and y i.e. $H_0 : \rho = 0$.
  - The alternative hypothesis claims that the variables x and y do have a significant correlation i.e. $H_1 : \rho \neq 0$.

  We employ a significance level of 5%.

- *Step 2: Compute the values of sample correlation (r) and the test Statistic (t)*

  We use the formula given in Equation 30 and calculate the sample correlation(r) values, displayed in Table 5

$$r = \frac{1}{n-1} \sum_k \left(\frac{x_k - \bar{x}}{\sigma_x} \right) \left(\frac{y_k - \bar{y}}{\sigma_y} \right) \tag{30}$$

  where, n: Sample size
  $\bar{x}, \bar{y}$: Estimates of the population mean.
  σ_x, σ_y: Estimates of the population standard deviation.
  Table 6 gives the test statistics i.e. $t-$values which are computed using the

formula given in Equation 31.

$$t = \frac{r}{S_r}, \tag{31}$$

where,

S_r is the estimated standard error in r given by, $S_r = \sqrt{\dfrac{1-r^2}{n-2}}$

Table 5: Pearson correlation co-efficient (r) evaluated for variables ($\alpha, \vartheta, \varrho$) and ($E(N_s(t)), E(W_s(t)), R_Y(t)$)

		Performance Metrics		
		$E(N_s(t))$	$E(W_s(t))$	$R_Y(t)$
Parameter	α	0.999859	0.999899	-1
	ϑ	-1	-0.9999	0.997301
	ϱ	-1	-0.99996	0.999904

Table 6: Test statistics $t-$values evaluated corresponding to Pearson correlation co-efficient values given in Table 5 for variables ($\alpha, \vartheta, \varrho$) and ($E(N_s(t)), E(W_s(t)), R_Y(t)$)

		Performance Metrics		
		$E(N_s(t))$	$E(W_s(t))$	$R_Y(t)$
Parameter	α	103.0264	121.8577	-809.045
	ϑ	-844403.5	-125.209	25.52684
	ϱ	-84403.4	-187.88	125.0564

- *Step 3: Calculate the p-value*

The probability value, denoted as p, represents the likelihood of observing the given t-value under the assumption that the null hypothesis is true. We calculate the $p-$value corresponding to the observed test statistic by referring to t-distribution with $n-2$ degrees of freedom. Table 7 shows the p-values calculated in association with test statistics given in Table 6.

Table 7: p-values evaluated using test statistic t-values given in Table 6 for degree of freedom, $df = 3$

		Performance Metrics		
		$E(N_s(t))$	$E(W_s(t))$	$R_Y(t)$
Parameter	α	0.00000202	0.00000122	0
	ϑ	0	0.00000112	0.000132
	ϱ	0	0	0.00000113

- *Step 2: Make the decision and draw the conclusion*

  We utilize the p-value in order to make decisions regarding the hypothesis specified in the beginning.

  - From the Table 7, we observe that the p-value is less than the chosen significance level 0.05 in each case. Thus, the null hypothesis is rejected in favor of the alternative hypothesis. We conclude that sufficient evidence exists to demonstrate that x and y have a significant linear relationship as the correlation coefficient deviates significantly from zero.
  - After confirming the validity of the alternative hypothesis and determining that the correlation is not equal to zero, we proceeded to examine the correlation coefficient values as presented in Table 5. When the observed values of r are close to -1, it signifies a negative correlation. This implies that as the variable x increases, the variable y decreases. When the observed values of the correlation coefficient r are close to 1, it suggests a positive correlation, meaning that as the variable x increases, the variable y also tends to increase.

Consequently, the statistical hypothesis test validated the findings that we had drawn from the data presented in Table 2–Table 4. These findings may be summed up as follows:

- The average number of failed machines, $E(N_s(t))$ tends to increase as the failure rate of standby machines, α increases. However, it demonstrates an inverse relationship with both the working vacation rate ϱ and the service rate during working vacation ϑ.
- The average waiting time of failed machines in the system, $E(W_s(t))$ increases linearly with the failure rate of standby machines, α. However, $E(W_s(t))$ varies inversely with both the working vacation rate ϱ and the service rate during working vacation ϑ.
- As the working vacation rate, ϱ and the service rate during working vacation ϑ, increase, there is an increase in $R_Y(t)$.

6. ADAPTIVE NEURO-FUZZY INFERENCE SYSTEM (ANFIS)

Based on the results obtained from our numerical simulation, it has been observed that the failure rate, λ and the service rate, μ have a substantial influence on the performance metrics of the system. However, both λ and μ are variable parameters that are associated with uncertainty. The utilization of fuzzy sets provides a practical approach for addressing imprecision in a given context. In the present section, the neuro-fuzzy approach has been implemented for performing the numerical assessment for various queueing indices of redundant machining systems, in particular when input parameters are not crisp. In this ANFIS technique, we input the training and testing dataset. The input training dataset is first fuzzy-tuned followed by ANFIS training and then ANFIS testing. After testing is finished, we evaluate the training results. We gather the Runge-kutta input data and then use ANFIS techniques using the MATLAB software to estimate it. The variables λ and μ are fuzzy-tuned utilizing the gaussian membership function. Figure 8 displays the fuzzification of λ and μ by assigning the linguistic values, displayed in Table 8, to membership functions corresponding to the input parameters λ and μ. The training algorithm employs a hybrid approach, combining the least-square and backpropagation gradient descent techniques, to effectively model the training dataset.

In Figure 7– Figure 10, we demonstrate the impact of input parameters λ and μ on the performance metrics of the system. We then proceed to compare the numerical results computed with the Runge-Kutta method, depicted by a continuous line, with the neuro-fuzzy results, represented by tick marks. It is worth noting that all the numerical results obtained using the Runge-Kutta approach are substantially consistent with those obtained using the ANFIS technique.

Table 8: Linguistic values of the membership functions for input parameter λ and μ

	Linguistic values of the membership functions for input parameter	
Input variable	Number of membership functions	Linguistic valuues
Filure rate of operating machine, λ	5	Very low Low Average High Very high
Service rate in normal busy mode, μ	5	Very low Low Average High Very high

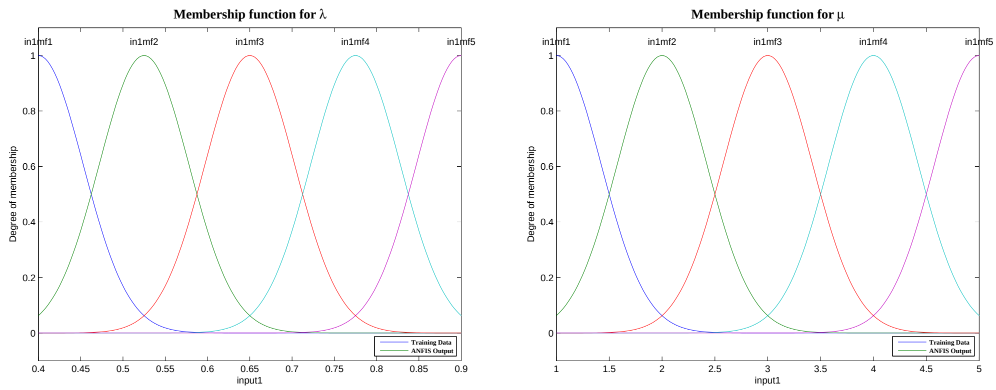

Figure 7: Membership functions for input parameters (a) λ (b) μ.

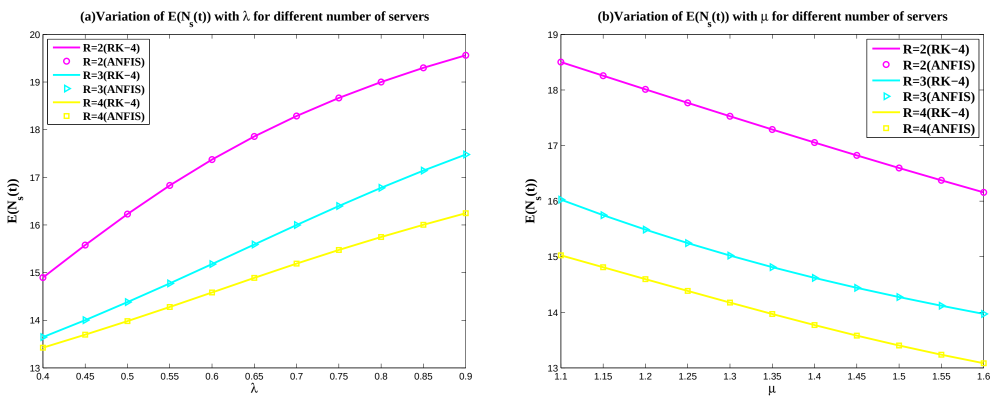

Figure 8: Variation of $E(N_s(t))$ with (a) λ (b) μ for varying R. We take $L = 32$; $M = 28$; $C = 1$; $S = 3$; $Q_1 = 9$; $\tau = \gamma = 3$; $\alpha = 0.2$; $\vartheta = 1.0$ and $\varrho = 1.2$.

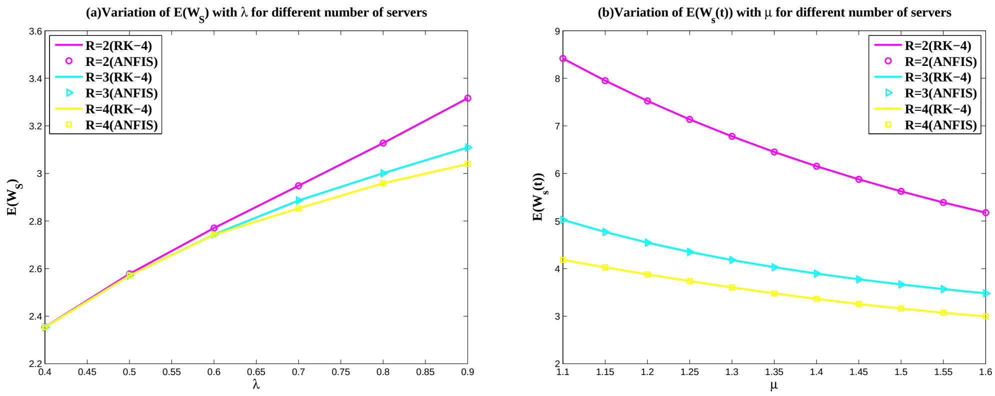

Figure 9: Variation of $E(W_s(t))$ with (a) λ (b) μ for varying R. We take $L = 32$; $M = 28$; $C = 1$; $S = 3$; $Q_1 = 9$; $\tau = \gamma = 3$; $\alpha = 0.2$; $\vartheta = 1.0$ and $\varrho = 1.2$.

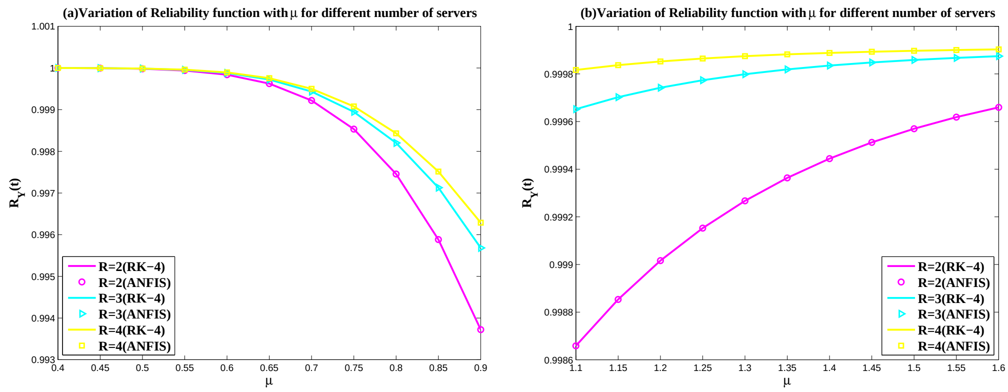

Figure 10: Variation of $R_Y(t)$ with (a) λ (b) μ for varying R. We take $L = 32$; $M = 28$; $C = 1$; $S = 3$; $Q_1 = 9$; $\tau = \gamma = 3$; $\alpha = 0.2$; $\vartheta = 1.0$ and $\varrho = 1.2$.

7. THE OUTCOMES AND MANAGERIAL INFERENCE

This section highlights the findings acquired through numerical simulation and sensitivity analysis. A broad spectrum of managerial consequences and insights are provided in relation to the examined data and statistical findings. The system investigators can capitalize on these insights to their strategic advantage. Our exhibited technique is feasible for the manufacturing, electronic, communication, and hospitality industries, among others.

- As anticipated, the average number of failed units in the system $(E(N_s(t))$ rises in proportion to the failure rate of the operating machines, λ and the failure rate of the warm standby machines, α. However the average number of failed units exhibits an inverse relationship with the increase in the service rate during the regular busy period μ, the service rate during working vacation period ϑ and the working vacation rate ϱ. Understanding the impact of various system parameters on average number of failed machines will contribute to help system engineers and design professionals to regulate queueing characteristics while constructing a generalized triadic policy machine repair model with multiple working vacations.

- The average waiting time of a failed machine in a system $(E(W_s(t)))$ rises as the failure rate of operating units λ and the failure rate of the warm standby machines, α rises, whereas it falls when the service rates μ (service rate during regular busy period) and ϑ (service rate during working vacation period) rises. This finding is consistent with realistic scenarios.

- The reliability function $(R_Y(t))$ initially exhibits convexity with the failure rates of the operating unit (λ), failure rate of the standby unit(α) and the service rate during regular busy period(μ), service rate during working vacation period (ϑ). But eventually the reliability achieves the steady-state with the passage of time. The reliability of the system is proportional to working vacation rate, ϱ. These findings offer insight regarding how to enhance the reliability of the system through assigning adequate values to different system parameters.

- Findings indicate that when the number of servers (R) in a system increases, the average number of failed units in the system will decrease, as will the average waiting time of a failed machine in a system. Likewise, it has been seen that increasing the number of servers in the system improves system reliability. These observations are in line with the real facts.

8. CONCLUSION

This paper has successfully generalized the concept of triadic control policy for multi-server machine repair problem. The adoption of several working vacations and mixed standbys has rendered the system more realistic. This generalization work of triadic policy will be very useful to the researchers in finding various published research related to the triadic policy. The Runge-Kutta's technique is implemented to conduct a numerical analysis for the average number of failed machines in the system, the average waiting time for failed machines, and the system's reliability function. Moreover the performance indicators may be valuable to system organizers and decision-makers. The numerical simulation for various numerous performance indicators demonstrate that Runge-Kutta's computational technique is practical. Statistical hypothesis testing using Pearson correlation coefficient has been carried out to compare the results presented in table. Furthermore, we generalized the notion of triadic policy, constructed various system performance metrics and gave a comparison results research for Runge-Kutta's and ANFIS methodologies in this study. The current study has constraints because it only studies the transient behavior; however, the model could benefit from a steady-state analysis to provide more insightful comprehension long run behavior. The research might potentially take into account a variety of server scenarios. Non-Markovian queueing models can be investigated further by including impatient behavior, server unreliability. Following that we may provide a cost analysis study for machine repair problems under generalized triadic policy. This multi-server redundant machine repair problem can be further enhanced by incorporating additional realistic factors, such as the impatience exhibited by failed machines and the inherent unreliability of servers. The present study could also be conducted by considering heterogeneous servers as an alternative to homogeneous servers.

Acknowledgements. The authors would like to express their gratitude to the editors and anonymous referees for their valuable comments and recommendations. These inputs have greatly contributed to enhancing the overall layout along with the content of the paper.

Funding. This research received no external funding.

REFERENCES

[1] Y. Yadin and P. Naor, "Queueing systems with a removable service station," *Journal of the Operational Research Society*, vol. 14, no. 4, pp. 393–405, 1963. doi: 10.2307/3006802

[2] P. K. Chahal and K. Kumar, "Queueing models of machine repair problems with control policies: a survey and analysis," *International Journal of Operational Research*, vol. In press, 2023. doi: 10.1504/IJOR.2023.10060548

[3] C. E. Bell, "Optimal operation of an M/M/2 queue with removable servers," *Operations Research*, vol. 28, no. 5, pp. 1189–1204, 1980. doi: 10.1287/opre.28.5.1189

[4] H. K. Rhee and B. D. Sivazlian, "Distribution of the busy period in a controllable M / M / 2 queue operating under the triadic (0 , K , N , M) policy," *Journal of Applied Probability*, vol. 27, no. 2, pp. 425–432, 2015. doi: 10.2307/3214662

[5] K. H. Wang and C. L. Chang, "Reliability of a controllable M/M/2 system with warm standbys operating under the triadic (0, Q, N, M) policy," *Computers and Industrial Engineering*, vol. 28, no. 1, pp. 163–178, 1995. doi: 10.1016/0360-8352(94)00035-L

[6] H. I. Huang, P. C. Hsu, and J. C. Ke, "Controlling arrival and service of a two-removable-server system using genetic algorithm," *Expert Systems with Applications*, vol. 38, no. 8, pp. 10 054–10 059, 2011. doi: 10.1016/j.eswa.2011.02.011

[7] C. D. Liou, K. H. Wang, and M. W. Liou, "Genetic algorithm to the machine repair problem with two removable servers operating under the triadic (0, Q, N, M) policy," *Applied Mathematical Modelling*, vol. 37, no. 18, pp. 8419–8430, 2013. doi: 10.1016/j.apm.2013.03.036

[8] V. Goswami and P. V. Laxmi, "Analysis of discrete-time machine repair problem with two removable servers under triadic policy," *International Journal of Mathematical Modelling & Computations*, vol. 5, no. 1, pp. 29–40, 2015. doi: 10.1504/IJOR.2023.10060548

[9] T. Ketema, S. Demie, and M. T. Belachew, "Controllable M/M/2 machine repair problem with multiple working vacations and triadic (0,Q,N,M) policy," *International Journal of Management Science and Engineering Management*, vol. 16, no. 3, pp. 184–196, 2021. doi: 10.1080/17509653.2021.1935358

[10] J. Taylor and R. R. P. Jackson, "An application of the birth and death process to the provision of spare machines," *Journal of the Operational Research Society*, vol. 5, no. 4, pp. 95–108, 1954. doi: 10.2307/3007087

[11] D. Gross, H. D. Kahn, and J. D. Marsh, "Queueing models for spares provisioning," *Naval Research Logistics*, vol. 24, no. 4, pp. 521–536, 1977. doi: 10.1002/nav.3800240402

[12] K. H. Wang, "Cost analysis of the M/M/R machine-repair problem with mixed standby spares," *Microelectronics Reliability*, vol. 33, no. 9, pp. 1293–1301, 1993. doi: 10.1016/0026-2714(93)90131-H

[13] S. Kolledath, K. Kumar, and P. Pippal, "Survey on queueing models with standbys support," *Yugoslav Journal of Operations Research*, vol. 28, no. 1, pp. 3–20, 2018. doi: 10.2298/YJOR170621024K

[14] G. Gao and J. Wang, "Reliability and availability analysis of a retrial system with mixed standbys and an unreliable repair facility," *Reliability Engineering and System Safety*, vol. 205, p. 107240, 2021. doi: 10.1016/j.ress.2020.107240

[15] K. Kumar, M. Jain, and S. Chandra, "Machine repair system with threshold recovery policy , unreliable servers and phase repairs," *Quality Technology & Quantitative Management*, pp. 1–24, 2023. doi: 10.1080/16843703.2023.2232639

[16] Y. Levy and U. Yechiali, "Utilization of idle time in an M / G / 1 queueing system," *Management Science*, vol. 22, no. 2, pp. 202–211, 1975. doi: 10.1287/mnsc.22.2.202

[17] V. M. Chandrasekaran, K. Indhira, M. C. Saravanarajan, and P. Rajadurai, "Queueing models of machine repair problems with control policies: a survey and analysis," *International Journal of Pure and Applied Mathematics*, vol. 106, no. 6, pp. 33–41, 2016. doi: 10.12732/ijpam.v106i6.5

[18] D. Y. Yang and C. L. Tsao, "Reliability and availability analysis of standby systems with working vacations and retrial of failed components," *International Journal of Operational Research*, vol. 182, pp. 46–55, 2019. doi: 10.1016/j.ress.2018.09.020

[19] A. A. Bouchentouf and L. Yahiaoui, "On feedback queueing system with reneging and retention of reneged customers, multiple working vacations and bernoulli schedule vacation interruption," *Operations Research*, vol. 6, no. 1, pp. 1–11, 2017. doi: 10.1007/s40065-016-0161-1

[20] M. Jain, D. Dhibar, and S. S. Sanga, "Markovian working vacation queue with imperfect service, balking and retrial," *Journal of Ambient Intelligence and Humanized Computing*, vol. 13, no. 4, pp. 1907–1923, 2022. doi: 10.1007/s12652-021-02954-y

[21] S. Gupta, P. K. Joshi, and K. N. Rajeshwari, "Optimization of M/M/2 queueing model with working vacations," *Journal of Scientific Research*, vol. 15, no. 1, pp. 31–41, 2023. doi: 10.3329/jsr.v15i1.59348

[22] I. Ziad, P. V. Laxmi, A. A. Bouchentouf, and M. Shakir, "A matrix geometric solution of a multi-server queue with waiting servers and customers' impatience under variant working vacation and vacation interruption," *Yugoslav Journal of Operations Research*, vol. 33, no. 3, pp. 389–407, 2023. doi: 10.2298/YJOR220315001Z

[23] Z. C. Lin and C. Y. Liu, "Analysis and application of the adaptive neuro-fuzzy inference system in prediction of CMP machining parameters," *International Journal of Computer Applications in Technology*, vol. 17, no. 2, pp. 80–89, 2003. doi: 10.1504/IJCAT.2003.000335

[24] K. Kumar and M. Jain, "Threshold n-policy for (M, m) degraded machining system with K-heterogeneous servers, standby switching failure and multiple vacations," *International Journal of Mathematics in Operational Research*, vol. 5, no. 4, pp. 423–445, 2013. doi: 10.1504/IJMOR.2013.054719

[25] M. Jain and R. K. Meena, "Vacation model for markov machine repair problem with two heterogeneous unreliable servers and threshold recovery," *Journal of Industrial Engineering International*, vol. 14, no. 1, pp. 143–152, 2018. doi: 10.1007/s40092-017-0214-x

[26] R. Sethi, M. Jain, R. K. Meena, and D. Garg, "Cost optimization and ANFIS computing of an unreliable M/M/1 queueing system with customers' impatience under N-policy," *International Journal of Applied and Computational Mathematics*, vol. 6, no. 2, pp. 1–14, 2020. doi: 10.1007/s40819-020-0802-0

[27] A. Ahuja, A. Jain, and M. Jain, "Transient analysis and ANFIS computing of unreliable single server queueing model with multiple stage service and functioning vacation," *Mathematics and Computers in Simulation*, vol. 192, pp. 464–490, 2022. doi: 10.1016/j.matcom.2021.09.011

[28] K. H. Wang and Y. L. Wang, "Optimal control of an M/M/2 queueing system with fi-

nite capacity operating under the triadic (0, Q, N, M) policy," *Mathematical Methods of Operations Research*, vol. 55, no. 3, pp. 447–460, 2002. doi: 10.1007/s001860200190

[29] K. Kumar and M. Jain, "Bi-level control of degraded machining system with two unreliable servers, multiple standbys, startup and vacation," *International Journal of Operational Research*, vol. 21, no. 2, pp. 123–142, 2014. doi: 10.1504/IJOR.2014.064540