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AN EXPERT SYSTEM FOR NATIONAL ECONOMY MODEL SIMULATIONS

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Abstract: There are some fundamental economic uncertainties. We cannot forecast economic events with a very high scientific precision. It is very clear that there does not exist a unique "general" model, which can yield all answers to a wide range of macroeconomic issues. Therefore, we use several different kinds of models on segments of the macroeconomic problem. Different models can distinguish/solve economy desegregation, time series analysis and other subfactors involved in macroeconomic problem solving. A major issue becomes finding a meaningful method to link these econometric models.

Macroeconomic models were linked through development of an Expert System for National Economy Model Simulations (ESNEMS). ESNEMS consists of five parts: (1) small-scale short-term national econometric model, (2) Methodology of Interactive Nonlinear Goal Programming (MINGP), (3) data-base of historical macro-economic aggregates, (4) software interface for interactive communications between a model and a decision maker, and (5) software for solving problems. ESNEMS was developed to model the optimum macro-economic policy of a developing country (SFRY-formerly Yugoslavia).

Most econometric models are very complex. Optimizing of the economic policy is typically defined as a nonlinear goal programming problem. To solve/optimize these models, a new methodology, MINGP, was developed as a part of ESNEMS. MINGP is methodologically based on linear goal programming and feasible directions method. Using Euler's Homogeneous Function Theorem, MINGP linearizes nonlinear homogeneous functions. The highest priorities in minimizing the objective function are the growth of gross domestic product and the decrease of inflation.

In the core of the optimization model, MINGP, there is a small-scale econometric model. This model was designed through analysis of the causal relations in the SFRY's social reproduction process of the past 20 years. The objective of the econometric model is to simulate potential short term (one-year) national economic policies. Ex-ante simulation and optimization of economic policy for 1986 showed that, in SFRY, non-consistent macro-economic policy was resolute and led to both slower economic development and more rapid growth of inflation.

Keywords: Expert systems, econometric model, national macro-economic policy, multicriterial decision-making, interactive nonlinear goal programming, Pareto optimality, Cobb-Douglas's production function, Euler's homogeneous function theorem.

1. INTRODUCTION

Expert systems are computer programs that use a collection of facts, rules of thumb and other knowledge about the given field, coupled with methods of applying those rules to make inferences. Expert systems can be effectively used to solve problems in such specialized fields as optimal short term economic policy choice.

The interface between a decision-maker and the ESNEMS is through two software subsystems, which communicate by simple questions and answers. A question could be: Which would you choose: a combination of 3% unemployment rate and an annual inflation rate of 5% – or – a combination of 10% unemployment rate and an inflation rate of 1%?

The Expert System will construct the decision-maker's preference function in free form regardless of answers to a complete system of such partial questions. Decision-makers will be able to go back to their PCs where both the Expert System and the entered data regarding the core of the economy reside. They can now add or change any formal preferences in a quantitative form. The Expert System will then solve, utilizing the algorithms built into other econometric models (through use of MINGP) to obtain an optimal development path for the economy under the given external circumstances and stated preferences.

This work is the result of a long term research into the application of goal programming to economic modeling [81], [84], [87], and [91]. This paper's aim is to examine abilities and possible advantages of econometric model-based Expert System in economic policy decision-making. A new small-scale national econometric model has been designed to analyze post-hoc economic policy. Embedded econometric models are used to simulate future national economic behavior. As the economic policy choice has been defined as an optimization problem, a nonlinear goal-programming model and a new interactive goal programming methodology for problem solving have been developed and are presented here. These approaches are more efficient than existing ones (both model and methodology). In order to develop an algorithm for solving nonlinear goal programming models where Cobb-Douglas type nonlinear constraints exist, a new gradient nonlinear programming algorithm then was constructed with feasible direction methods built in.

An interactive methodology is used here as an interface for support in preparing decisions when a decision-maker is uncertain in ranking his/her priorities. It is also used to define weighting coefficients in the objective function of goal programming problems. This methodology examines compromise solutions for a decision-maker and, if necessary, sets up a new objective function structure. This new structure should lead to a more satisfying and executable solution. There are new applied methods and methodologies built into this Expert System. The preliminary results and tests of this methodology in real decision situations have been encouraging.

2. ECONOMETRIC MODEL

What we attempt to measure and the way we measure the economy is strongly influenced by the conceptual framework we have developed for analyzing the economy. The appropriate amount of detail included in a macro-economic model depends on questions being addressed. The method used to construct the econometric data is also reflected in the structure of the model. Investigations have demonstrated that econometric models are able to analyze present behavior and to carry out simulations of the future. They are able to use knowledge about the present to make "baseline projections", i.e., basic or "standard" forecasts that assume a continuation of present trends. At the same time, there is a possibility of influencing the economic environment by macroeconomic policies. Desirable policies can push the country's economy into an orbit of greater and more sustainable growth.

An econometric model consists of equation sets, each of which is mathematical complex containing "stochastic members". These members are the results of complex relationships in a nation's economy. In simpler mathematical models, the relationships between variables are exclusively functional. In econometric models, the relations are stochastic. Econometric techniques help to partially compensate for a lost precision, caused by our model's simplification of economic reality.

There are two types of equations in econometric models: stochastic, or behavioral, and identities. Stochastic equations are estimated from historical data. Identities are equations that hold by definition; they are always true.

The typical econometric problem should consist of:

- statement of the issue to be investigated,
- specification of the model,
- preparation of data base,
- estimation of the model,
- validation of the model, and
- use of the model for policy and other forms of analysis.

Design of an econometric model is complex. Identification of key factors influencing a specific economy is necessary. The model must examine and analyze separate segments (niches) of that mechanism (resources distribution, investments, export-import, prices, etc). Practically, each segment defines one partial model. After all the sub-models are built, the econometric model integrates them in a meaningful fashion. Relevant statistical data from the past provides empirical quantification of established and/or hypothesized relationships. Separate economic policies must be quantified and separated into meaningful aggregates (tax rate, monetary policy, etc.). Empirically evaluated relationships in the model show how, in a particular economy, in a particular period, specified macroeconomic parameters are related to economic policy instruments. The economic policy goals can be approximated through the quantitative expression of the economic parameters targeted in the econometric model.

The basic data source underlying almost every economy-wide model in the SFRY is the SFRY Statistics of Social Accounts. The National Income Product Accounts (NIPAs) are commonly used in the U.S.A. These accounts are a series of statistics generally considered indicative of the economic health and wellbeing of the nation. The aggregate of the national income accounts is the gross national product, the sum of all productive activity in the country.

3. SHORT REVIEW OF SOME EMPIRICAL MODELS OF ECONOMY SIMULATION IN THE WORLD

Empirical models use analytical and methodological tools which enable analyses of key relationships within some economy and by which quick simulations of alternative economic development policies can be evaluated. The mathematician F.R. Ramsey (1928), partly stimulated by J.M. Keynes, raised the fundamental issue of trying to determine an optimal rate of saving or accumulation (see: Johansen L., 1977, p.24). The first attempt at using a macroeconomic model for elucidating the influence on economic development by possible government instruments was done by Jan Tinbergen in Netherlands (1936). The first attempts at constructing national accounts came in Norway (1936) by Ragnar Frish. Economic planning ideas also emerged in the U.S. (W.C. Mitchell's contribution in 1937). Parallel with the development of econometric methods, simulation models of economic development were created using simultaneous equations.

The use of econometric models by government planning bureaus for the purposes of forecasting and policy guidance has become widespread in recent years. Following the type of models developed by Tinbergen and Klein and Goldberger many years ago, model projects have been developed and implemented for the U.S., United Kingdom, Canada, the Netherlands, Sweden, Norway, Peru, Italy, France, India, Japan and many other countries on a continuous basis. For many other countries these models are used on an occasional basis. Econometric models have also been applied to smaller government units (eg. states) and/or industries. The relative success of forecasting suggests that econometric model building might be applied profitably to a broader range of countries. A sample of past usage includes the following:

Econometric Models in the World	Built	Number of equations total	Behavior equations	Definable Equations or identities
Dynamical multisectoral model of India development *Mahalonobis P.C.*	1953	228	-	-
Klein-Goldberg's model of U.S. economy development *Klein L.R. and A.S. Goldberger*	1955	23	15	8
Brooking's quartal econometric model of U.S. *Fromm G. and L.R. Klein*	1965	-	-	-
Alternative model of Peru economy development *Thorbecke E. and Condos A.*	1966	-	-	-
Regional-National Econometric Model of Italy *Chenery H. and Druno M.*	1966	12	7	5
A short-term econometric model of French economy *Evans K Michael, OECD*	1969	55	34	21
U.S. Macroeconometric Model and Multicountry Econometric Model (MC) *Ray C. Fair, Yale University*	1976 1980	131 1440	30 328	101 18+1094
Econometric Model of Great Britain *Terry Barker et. al. University of Cambridge*	1980	24	16	8
Japan's FUGI global model of World economy	1986	12700	-	-
KKRI - Macro-econometric Model of Japan, *Naoki Tanaka, Nariyasu Ito and Mamory Obayashi et. al.*	1988	86	37	49
Arkansas Econometric Model, *Institute for Economic Advancement-University of Arkansas at Little Rock*	1995	160	-	-
Oklahoma State Econometric Model, *Office of Business and Economic Research*	1998	-	-	-
ASPEN-New economic model simulates U.S. Economy, *Sandia National Laboratories, Livermore, California*	1998	-	-	-
Kansas Econometric Model *The Institute for Public Policy and Business Research, University of Kansas*	1998		-	-

Econometric models developed in other countries have been used primarily for the following three uses: 1) forecasting, 2) multiple analysis and policy simulation, and 3) simulation of past periods. The last of these can be used as a useful diagnostic tool in order to understand how previous recessions, inflation, or other undesirable elements of economic activity might have been prevented. This type of analysis has considerable interests in some context and has been used extensively for the U.S. economy.

Fair's models, like instant models, are continually updated based on NIPA data and have been available to Internet[1] users and others (to use), on a client-server basis, as a tool to forecast, do policy analysis, and examine historical episodes.

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[1] *http://fairmodel.econ.yale.edu/info/whatis.html.*

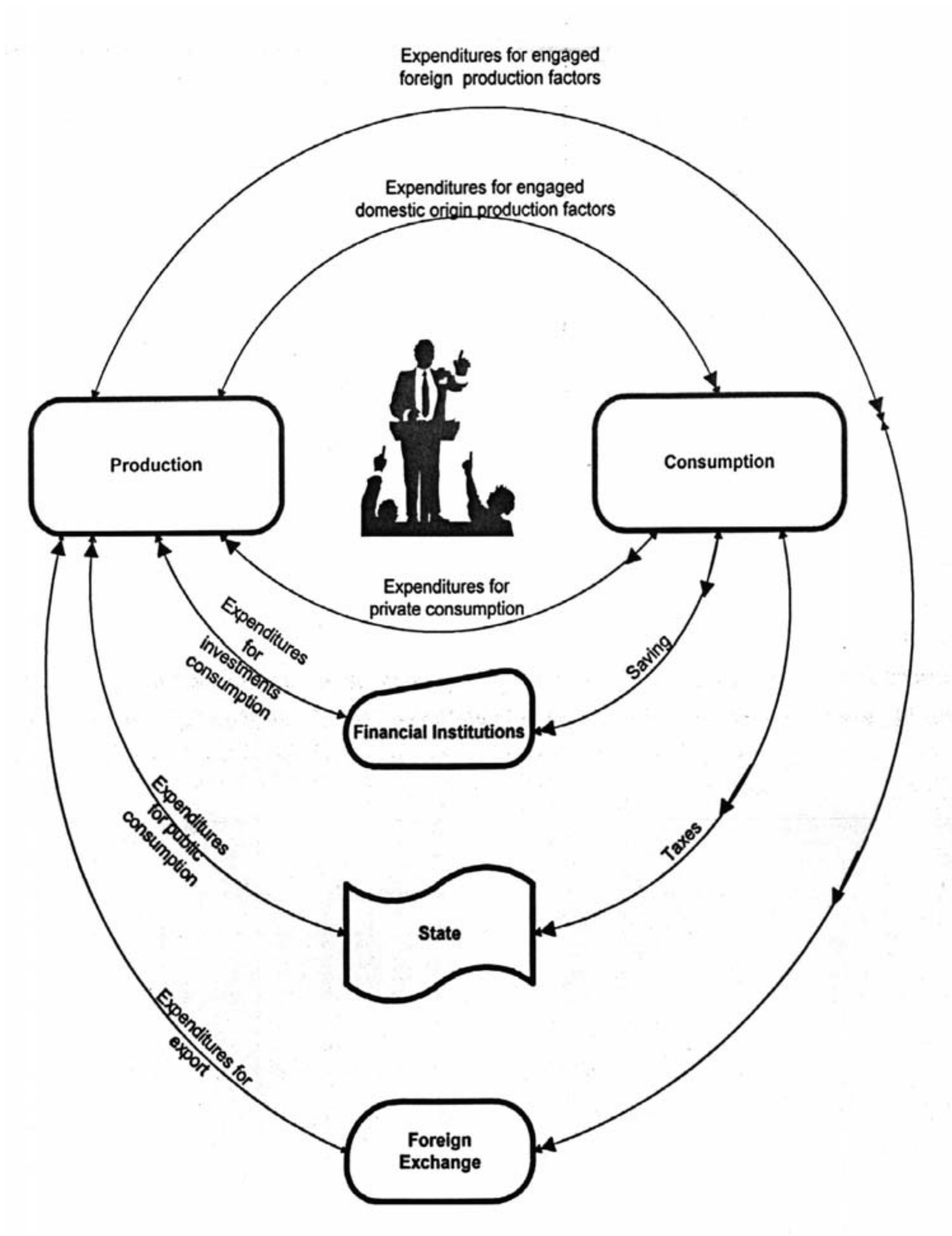

Figure 1: Cyclical flow of open economy's economic activities

4. ECONOMETRIC MODEL AS THE CORE OF THE ESNEMS

The econometric model developed here can be described as a productional, nonlinear, aggregate, macroeconomic, dynamic simultaneous equations small-scale model. The model is not in equilibrium because it does not contain an explicit formulation of equality between supply and demand. The model is dynamic because it contains time as a variable and also lagged endogenous variables. It is nonlinear because Cobb-Douglas's production function is used and also because two other non-linear equations are used to link constant and variable prices. The main equation in this model, as it is mentioned earlier, is the production function which starts and ends the cyclical flow of an open economy's economic activity (Figure 1.). This function specifies the maximum aggregate output, which is divided between consumption and investment.

This is, of course, a very simplified presentation of the cyclical flow of open economy's economic activities. This process is too complex to allow the inclusion of all elements of the economy and thereby form action-consequence conclusions. The number of relations by which this process is interconnected is so high that it is unable to recognize it. But some basic and common values and relations have been identified and crystallized. These are: gross national product, fixed capital, employment, investments volume, prices, personal income, export and import, as well the relationships that exist between them. The starting hypothesis in this research (the statement of the issue to be investigated) is that the essence of economic disturbances in the Yugoslav economy was more due to insufficient supply of goods, while excessive demand was only a secondary occurrence.

The equations, which constitute our econometric model are:

Row #	Equations of econometric model	The name of dependent variable	Notation
1.	GDPTCP = GDPSSCP+GDPPSCP	Total gross domestic product at constant price (CP, 1972=100)	x_1
2.	GDPSSCP = *f*(FCAPCP, <u>EMPSS</u>)	Gross domestic product of social sector at CP	x_2
3	GDPPSCP = *f* (GDPSSCP)	Gross domestic product of private sector at CP	x_3
4	FCAPCP =*f* (GICP, FCAPCP$_{-1}$)	Fixed capital of social sector at CP	x_4
5	EMPPRS = <u>EMPSS</u> + EMPIS	Employment people in productive sector	x_7
6	EMPPS = *f* (GDPPSCP, EMPPS$_{-1}$)	Employment people in private sector	x_8
7	EMPNPS = *f* (GDPTCP, EMPPRS)	Employment people in nonproductive sector	x_9
8	EMPT = EMPPRS + EMPNPS	Total employment people	x_{10}
9	EMPTSS = <u>EMPSS</u> + EMPNPS	Total employment people in social sector	x_{20}
10	IDGDP = *f* (PIIG, PIPR, ILC, GDPTCP)	Implicit deflator of gross domestic product	x_{11}

Row #	Equations of econometric model	The name of dependent variable	Notation
11	PIPR = f (MMFP, INVFP, PIIMP)	Price index of producers	x_{12}
12	PIIG = f (PIPR, EXRIMP)	Price index of investment goods	x_6
13	ILC = f (PIPR, ILC_{-1}, PIIG)	Index of living costs	x_{13}
14	PCFP = f (ILC, NPINFP, GDPTCP)	Personal consumption expenditures at flow prices (FP)	x_{18}
15	GICP = f (GDPSSCP, $GICP_{-1}$, PIIG, $GDPTCP_{-1}$)	Gross investments at CP	x_{19}
16	NPIFP = f (GDPSSCP, ILG, EMPTSS)	Net personal income at FP	x_{15}
17	EXPCP = f (IMPCP, EXREXP PIEXP)	Export at CP	x_{21}
18	IMPCP = f (GDPTCP, EXRIMP, PIIMP)	Import at CP	x_{22}
19	ORFSFP = FSRFP - TXFP - STFP - DUTFP	Other revenues of fiscal system at FP	x_{25}
20	FSRFP = f (GDPTCP, IDGDP)	Fiscal system revenues at FP	x_{24}
21	TXFP = f (NIFP)	Taxes and contributions at FP	x_{26}
22	STFP = f (PCFP)	Sales tax at FP	x_{27}
23	DUTFP = f (EXRIMP, IMPCP)	Duty at FP	x_{28}
24	NIFP = GDPTCP * IDGDP - AMFP	National income at FP	x_{29}
25	AMFP = f (GDPTCP * IDGDP)	Amortization at FP	x_{30}
26	PIIMP = f (TIME)	Prices index by import	x_{31}
27	PIEXP = f (TIME)	Prices index by export	x_{32}

The notation $f(\ldots)$ means that the equation is stochastic, otherwise it is an identity equality. The variables inside the parentheses are explanatory variables. Exogenous variables in equations are underlined, and lagged variables are subscripted by $_{-1}$.

Notation	Exogenous variable	The name of dependent variable
x_5	EMPSS	Employment people in social sector
x_{14}	EXRIMP	Exchange rate by import
x_{16}	MMFP	Monetary mass at FP
x_{17}	INVFP	Inventories at FP
x_{23}	EXREXP	Exchange rate by export
x_{33}	TIME	Time (1966th – 0.l)
	Lagged variable	**The name of lagged variable**
x_{34}	$FCAPCP_{-1}$	Fixed capital at CP $(t-1)$
x_{35}	$EMPPS_{-1}$	Employment people in productive sector $(t-1)$
x_{36}	ILC_{-1}	Index of living costs $(t-1)$
x_{37}	$GICP_{-1}$	Gross investment at CP $(t-1)$
x_{38}	$GDPTCP_{-1}$	Total gross domestic product at CP $(t-1)$

The econometric model presented here provides a better understanding of the stochastic aspects of a developing economy by examining the changes that occur over the time. The kernel of the econometric model is a sort of combination of demand and supply models. The model consists of 27 equations, 21 of which are stochastic and 6 are defining.

In order to have a better insight into the system of relationships and relations, we could present our model roughly by seven blocks:

BI – block of production, which is described by the real gross national product in both social (government) and individual sectors. Econometrically, gross national product is explained by fixed capital in constant prices and the number of employed people.

B2 – block of prices, formalizes an implicit gross national product deflator, as a representative measure of inflation.

B3 – block of investments, although it is given in high aggregated form, presents a very substantial (vital) model segment by which the connection of block of prices and block of production is assured.

B4 – block of personal-consumption, describes the consumer behavior in this particular economy and is theoretically linked to the block of personal incomes.

B5 – block of personal incomes.

B6 – block of foreign exchanges, gives data of dependency degree between production and import of goods, the data about balancing exchange with the World, and how these factors influence fiscal revenues.

B7 – block of fiscal revenues. Reflects some assumptions which are the starting points of this econometric model:

- that the balance between fiscal's revenues and expenses should be assured (what is *conditlo sine qua non* of this model),
- that the expenses of the fiscal system are incurred in part due to interaction with the earlier specified "blocks."
- that the fiscal policy instruments are endogenized by a classical approach. That is to say that the fiscal policy has neglected economical and social effects, while ensuring (providing) enough funds for the functioning and maintenance of the social (government) sector.

The blocks are mutually connected by some variables (see Figure 2).

Most of the macroeconomic aggregates are influenced by monetary policy measures (financial market), fiscal policy measures (country revenues and expenses) and by foreign policy measures (import, export, exchange rate).

As it is known, economic policy measures in econometric models are presented by instrumental variables. Therefore the solution to the problem of economic policy choice in such models has been reduced to determining the instrumental variable values.

The equations of the econometric model are estimated by the ordinary least square method along with the F-test of significance of the estimated coefficients and regression validation. The auto-correlation has been tested by Durbin-Watson statistic defined by 5%. The presence of auto-correlation is rejected by the Cochrane-Orcutt procedure.

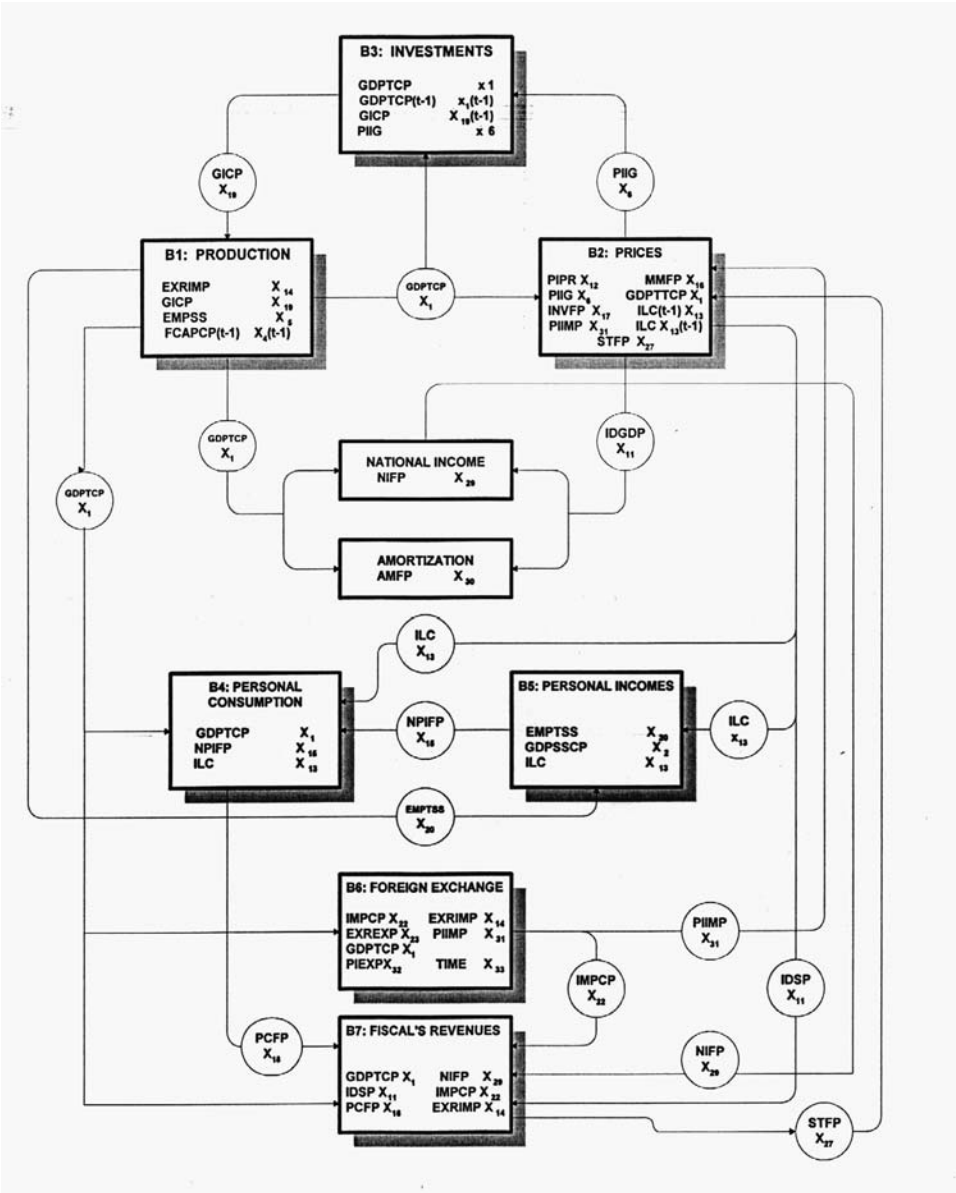

Figure 2: Block-recursive econometric model structure

For econometric validation for forecasting (model calibration) and consistency evaluation, by ex-post simulation[2] we have employed constant term adjustment (see: Klein and Young, 1981).

Since the values of the model variables were measured in different measures, before MINGP application, we ensured the "normalization" of some of the equations by reducing the multidimensional feasible solution area on the dimension scale between 0 and 20.

Once the statistical tests on the data were satisfactorily completed, input parameters estimated, and several ex post tests performed to validate the model and to estimate the minimal relative size of estimation error, the econometric model was run for predicting optimal economic policy during the next one year planning period (horizon).

Dynamically, we first applied MINGP as a test of consistency of the planned goals for the next year. In the planning documents the main quantitative goals were:

- increase in gross domestic product by 3%,
- increase in inflation by 45%, and
- increase in gross investments by 2%,

which in essence are conflicting goals.

The dynamic simulation of econometric problem, solved by MINGP as a single objective nonlinear programming problem, showed that such a goal constellation of the Yugoslav economic policy would not be realized without destroying the structural constraints of the existing econometric model.

Therefore, the input file of the optimization policy choice problem, consists of the following relaxed goals:

- increase in gross domestic product by a rate greater than 2.2%,
- increase in inflation by a rate greater than 43% but less than 88%, and
- increase in gross investments by 2%.

The mathematical model formulation of the choice of optimal economy development policy, constructed via a combination of dynamic simulation and optimization techniques, can then be formulated as a nonlinear goal programming problem as below:

Minimize

$$P_1(d_{39}^+ + d_{40}^- + d_{43}^- + d_{44}^-) + P_2(d_2^+ + d_2^- + d_{24}^+ + d_{25}^+ + d_{25}^-) + P_3(d_{28}^+ + d_{28}^-) +$$
$$P_4(d_{29}^+ + d_{29}^-) + P_5(d_{30}^+ + d_{30}^-) + P_6(d_{31}^+ + d_{31}^-) + P_7 d_{46}^-$$

Subject to the constraints from (1) to (46)

[2] The solution of a model over a historic period, where the actual values of exogenous variable are known, is called ex-post simulation. In this case, we do not have to guess values of the exogenous variables because all of these variables are known. One can thus use ex post simulation to test a model in the sense of examining how well it predicts historical episodes.
The solution of a model for a future period, where "guessed" values of the exogenous variables are used, is called an ex ante simulation.

1)	$100x_1-100x_2-10x_3$		=	0.0
2)	$100x_2-155.27821x_4^{0.538509}x_5^{0.461491}$	$+d_2^- - d_2^+$	=	0.0
3)	$10x_3-8.12x_2$		=	24.351247
4)	$1000x_4-5.92362x_{19}-975.69x_{34}$		=	0.210555
5)	$x_7-x_5-x_8$		=	0.0
6)	$x_8-0.0021x_3-0.9524x_{35}$		=	0.002298
7)	$x_9-0.061x_1-0.15327x_7$		=	0.007853
8)	$x_{10}-x_7-x_9$		=	0.0
9)	$x_{20}-x_5-x_9$		=	0.0
10)	$10x_{11}-2.22463x_6-1.16842x_{12}-8.31871x_{13}-0.0843x_1$		=	0.037344
11)	$10x_{12}-2.25x_{16}-6.69x_{17}-0.27215x_{31}$		=	0.747693
12)	$10x_6-5.41342x_{12}+0.5382x_{14}$		=	0.6783085
13)	$10x_{13}-9.46445x_{12}-0.1047x_{27}-4.80405x_{36}$		=	0.4591351
14)	$-1000x_{18}+12.1346x_1+604.3191x_{15}+1399.81x_{13}$		=	417.229687
15)	$10x_{19}-59.3205x_1-2.45899x_6-9.91684x_{37}$		=	417.229687
16)	$100x_{15}+7.27287x_{20}-13.131x_2-88.6279x_{13}$		=	4.813987
17)	$-100x_{21}+40.792x_{22}+10.609x_{23}+38.01588x_{32}$		=	20.114828
18)	$-100x_{22}+54.817x_1-38.716x_{14}+76.22366x_{31}$		=	84.655504
19)	$1000x_{25}-1000x_{24}+1000x_{26}+100x_{27}+100x_{28}$		=	0.0
20)	$1000x_{24}-1276.2744x_{11}-87.9184x_1$		=	284.451904
21)	$1000x_{26}-186.97x_{29}$		=	3.951925
22)	$100x_{27}-131.354x_{18}$		=	3.951925
23)	$100x_{28}-9.37x_{22}-74.7098x_{14}$		=	12.79104
24)	$1000x_{29}+1000x_{30}-1000.27821x_1x_{11}$	$+d_{24}^- - d_{24}^+$	=	0.00022
25)	$-1000x_{30}+105.43x_1x_{11}$	$+d_{25}^- - d_{25}^+$	=	25.303613
26)	x_5		=	5.40294
27)	$2.86816x_{14}-3.556x_{11}$		=	0.0
28)	x_{16}	$+d_{28}^- - d_{28}^+$	=	2.664805
29)	$2.86816x_{17}-2.3651x_{11}$	$+d_{29}^- - d_{29}^+$	=	0.0
30)	$2.86816x_{23}-3.353x_{11}$		=	0.0
31)	x_{10}	$+d_{31}^- - d_{31}^+$	≤	7.7
32)	$-x_{31}+2.1896x_{33}$		=	0.65596
33)	$-x_{32}+1.8761x_{33}$		=	0.71605
34)	x_{33}		=	2.1
35)	x_{34}		=	1.01952
36)	x_{35}		=	0.138
37)	x_{36}		=	5.89485
38)	x_{37}		=	8.06203
39)	x_{38}		=	3.93648
40)	x_1	$+d_{40}^- - d_{40}^+$	≤	4.0221
41)	x_{11}	$+d_{41}^- - d_{41}^+$	≤	5.4
42)	x_{11}	$+d_{42}^- - d_{42}^+$	≥	4.1
43)	x_{19}		=	8.22327
44)	x_5	$+d_{44}^- - d_{44}^+$	≤	5.5
45)	x_1	$+d_{45}^- - d_{45}^+$	≥	3.98336
46)	x_{14}	$+d_{46}^- - d_{46}^+$	≥	5.334

In the first 25 terms the equations of the econometric model have been given. In terms from 26th to 30th rows the functions of instrumental variable values are given. The term in the 31st row contains the upper bound of available working people (in millions). In the 32nd row, is the variable "prices by import." In the 33rd row is the "prices by export" variable. Although these rows (26-33) belong to the econometric model, they are not treated as goals but as structural constraints, which are functions of time, thus making the model time-dependent. In the 34th row the time variable assumes that first year $(x_{33} = 0.1)$ is 1966, and rows 35-39 give the value for time lagged variables: $x_{4(-1)}, x_{8(-1)}, x_{13(-1)}, x_{19(-1)}$, and $x_{1(-1)}$ respectively. The terms from the 41st to 46th row contain the boundary values for instrumental variables.

The first priority in the initial objective function is given to satisfying the goal of gross national product growth of both sectors of capital ownership (social and private) – of variable x_1, and to keeping the inflation rate (x_{11}) in previously given bounds. The second priority goal is the satisfying of all nonlinear functions in the model.

In the 3rd, 4th, 5th and 6th priority in the objective function are the requirements for minimization of deviations from goal instruments of economic policy and in the 7th priority is the goal requirement for national gross investment growth.

5. RESULTS OF ECONOMY MODEL SIMULATION

This problem is solved after the 3rd interactive trial and 15 iterations of the NGP algorithm to determine the best solution (so called Pareto's optimum[3]):

$x_1 = 4.02210$	Total gross domestic product of both social and individual sectors at constant prices
$x_2 = 3.49481$	Gross domestic product of social sector at constant prices (CP)
$x_3 = 5.27291$	Gross domestic product of individual sector at CP
$x_4 = 1.04443$	Fixed capital of production purchase value of social sector at CP
$x_5 = 5.50000$	Number of employees in social sector of economy
$x_6 = 1.98261$	Index of growth of investment goods at CP
$x_7 = 5.64480$	Number of employees in both social and individual sector
$x_8 = 0.14480$	Number of employees in individual sector
$x_9 = 1.118380$	Number of employees in nonproductive sector
$x_{10} = 6.76318$	Total number of employees in both productive and nonproductive sector
$x_{11} = 5.40000$	Implicit deflator of gross domestic product at CP

[3] The solution of multi-objective programming problem in which satisfying of less important goal could be reached only by doing not (worse) on expense of the goals on higher priority. In other words: "Under what configuration of the economic system it would be able to make one person better off only at the expense of making someone else worse off".

$x_{12} = 4.06739$	Producer's price index at CP
$x_{13} = 5.42617$	Costs of living index at CP
$x_{14} = 5.33400$	Average exchange rate of dinar (domestic currency) in import
$x_{15} = 4.83480$	Net incomes of both productive and nonproductive sector of social sector
$x_{16} = 3.50849$	Monetary mass
$x_{17} = 4.62770$	Stock growth increase
$x_{18} = 10.14894$	Individual consumption expenditures
$x_{19} = 8.35289$	Gross investment in fixed assets
$x_{20} = 6.61838$	Employed people in social sector in both productive and nonproductive sectors
$x_{21} = 2.63153$	Export of goods and services at CP
$x_{22} = 2.29802$	Import of goods and services at CP
$x_{23} = 6.31283$	Average exchange rate of dinar (domestic currency) at export
$x_{24} = 7.52995$	Revenues of fiscal system
$x_{25} = 2.06817$	Other revenues of fiscal system
$x_{26} = 3.69190$	Taxes, fees and contributions, excluding sales taxes
$x_{27} = 13.37056$	Sales tax
$x_{28} = 4.32826$	Customs revenues
$x_{29} = 19.45477$	National income
$x_{30} = 2.26457$	Amortization
$x_{31} = 3.94220$	Index of gross import prices at CP
$x_{32} = 3.22376$	Index of gross export prices at CP

This solution has been obtained in the 3rd interactive trial, after the two previous interactions have not given a satisfactory solution for achievement of the 2nd and 3rd goals, but in such a way that the 3rd goal in the initial objective function was renamed as the first and the 2nd as the 3rd, while the order of other priorities remained unchanged.

This fact points to the importance and potential role of the approach offered by MINGP in relation to ad hoc, often pragmatic approaches. Namely, it must be totally clear that a decision maker could not, by a simple deductive procedure, reach such a conclusion as the improvement of satisfying the 2nd goal it could be accomplished indirectly by renaming the 3rd goal as the first one.

In essence, mathematically speaking, MINGP has found three local minimums among which, in collaboration with the decision maker, it has selected the most satisfactory solution.

The optimum solution of economic policy for the planned year_1 as predicted by the model, has revealed that there were possibilities for a further increase of production activities in the Yugoslav economy but not with such intensity of social product and inflation growth as it had been predicted. In other words, it has been shown that the Yugoslav economy, wishing to stabilize, must look for a trade-off between growth and stabilizing under limited investments (this being the problem of most developing countries) at the expense of social product growth. In addition to the solution of instrumental variables of monetary and fiscal policy the specific solution given in the paper demonstrates that trade-off is an economic policy which would give a social product growth of 2.2% and inflation growth of 54% (not 45%). In other words, inflation of 45% cannot be maintained at a social product growth of 3% but at a level almost double.

6. GOAL PROGRAMMING AND GRADIENT METHOD OF FEASIBLE DIRECTIONS

Nonlinear models are difficult or impossible to solve analytically, but they can usually be solved numerically. Like using the Gauss-Seidel technique. We used here goal programming and feasible directions methods.

The goal programming method has been considered the oldest method of multiple objective programming. It has been very popular in the applicative sense but waited far too long to pass from a theoretical basis (Charnes and Cooper, 1961) into the sphere of practical applications (Lee, 1972, and Ignizio, 1976).

The problem of nonlinear goal programming can be expressed in the following way:

$$\min F(d) = \sum_{l=1}^{k}\sum_{i=1}^{n} w_i P_l (d_i^- + d_i^+) \tag{1}$$

s.t.

$$G_i(x) = \sum_{j=1}^{n} g_{ij} x_j + h_{ij} \prod_{j=1}^{n} x_j^{e_{ij}} + d_i^- - d_i^+ = c_i \tag{2}$$

$$A_i(x) = \sum_{j=1}^{n} a_{ij} x_j \le b_i \tag{3}$$

$$x_j, d_i^-, d_i^+ \ge 0,\ d_i^- \cdot d_i^+ = 0 \text{ , for all } i = 1,\dots,m,\ j = 1,\dots,n \text{ ,} \tag{4}$$

where x_j are decision making variables, d_i^- and d_i^+ represent negative and positive deviation variables from the goals (underachievement and overachievement), respectively, g_{ij} are coefficients of linear portions of goals (constraints (2)), a_{ij} are coefficients of structural constraints (3), h_{ij} are coefficients of nonlinear portions of goals (2), and e_{ij} are exponents, c_i and b_i are constants of right hand sides in (2) and (3) respectively.

P_l, $l = 1,\dots,k$ in objective function (1) are preemptive priority factors, so that the following is valid:

$$P_j >>> P_{j+1} \quad \text{for all} \quad j = 1,2,\dots,k-1 \text{ .}$$

The highest priority is indicated by P_1, the next highest by P_2, and so fort. w_i are weights assigned to some priority factors. The notion of priorities holds that P_1 is preferred to P_2 regardless of any weights w associated with P_2.

In the great majority of up-to-date developed methods used for solving problems of this very type, among the most significant ones is the gradient method combined with the feasible directions method.

The feasible directions[4] methods have been primarily designed for nonlinear programming problems and they are the iterative ones whose solutions in certain iterations have the following recursive form:

$$x_{p+1} = x_p + l_p s_p(x) \quad \text{for all} \quad p = 1,2,...,m$$

where $s_p(x) = s(x_0, x_1, x_2,..., x_p)$ is a direction and $l_p \geq 0$ is a step size, which is chosen so that:

$$F(x_p + l_p s_p) \leq F(x_p), \quad 0 \leq l_p \leq 1, \quad x_p + l_p s_p \in X,$$

and where:

$$X = \{x \in R^n, \text{ and conditions (2), (3) and (4) are satisfied}\}$$

Gradient methods use the direction of a gradient as a solution improvement direction, thus defining the feasible and usable feasible directions and enabling reduction of the nonlinear problem to an approximate linear problem which is close to the initial one.

The method is iterative and each iteration starts with an initial feasible vector. At each iteration of the feasible directions method a feasible direction of improvement (usable feasible direction) is determined and a new "better" point is found along that direction. Optimality is achieved when no further improvement can be made in any feasible direction

To determine the gradient this method requires that functions are continuous and differentiable. In order to guarantee convergence of the algorithm the gradient method requires that the model constraints form a convex set at each goal level while the objective function is concave

Many methods for solving linear or non-linear programming problems will appear to be methods of feasible directions. They only differ in the extra requirements for fixing the starting point x_0, the directions s_k or the step lengths l_k.

We are in this case particularly interested in a nonlinear function of the Cobb-Douglas type, the generally form of which is:

$$g(x_1, x_2,..., x_n) = h \prod_{i=1}^{r} x_i^{b_i} \tag{5}$$

[4] Feasible direction. Given a feasible point x, a direction vector, say d, is feasible if there exists $s > 0$ such that $x + sd$ is feasible.

where: $x_i \geq 0$, for all $i = 1,2,...,n$, are independent variables, $h \in R$ is coefficient, and $b_i \in R$ are exponents of independent variables. This is the most familiar and most frequently applied form of production function by which the production value in a certain economy is expressed as a function of labor and capital investment.

The idea of linearization of nonlinear constraint and solution of nonlinear programming problems is not new, because Griffith and Stewart first suggested in 1961 that nonlinear problem may be linearized in the region about the particular point by expansion in the Taylor's series, ignoring terms of a higher order than linear and adding two more restrictions for each nonlinear constraint. In that way nonlinear programming problems have been transformed into a form which can be solved by the linear programming algorithm (see: Ignizio [1976], and especially Lee, S.M. and Olson D.L. [1985]).

Our idea was to apply the Euler's theorem for the "total" linearization of nonlinear constraints in the region about the particular point (see: Roljić, L. [1987]). Owing to the Euler's homogenous function[5] theorem it is possible to apply this method to solve the problems of NGP regardless of whether Cobb-Douglas's functions are linearly homogenous (that is: $\sum_{i=1}^{n} b_i = 1$), or positively homogenous (that is: $\sum_{i=1}^{n} b_i \geq 1$).

In economic theory, the production function is frequently assumed to be linearly homogenous because such functions have convenient properties.

The advantage of our approach is that it is not necessary to add two more constraints for each linearized nonlinear constraint in every simplex iteration of NGP algorithm, and because the linearization is more accurate.

7. ALGORITHM OF NGP AND MINGP

As already mentioned, an algorithm of nonlinear goal programming is based on hybrid connection of modified simplex method of goal programming (Lee, 1972) and gradient method of feasible directions (Zoutendijk, 1976). As, in the problem at hand, the objective function is linear and only three constraints are nonlinear, the procedure is simplified when compared with general convex programming problems.

The iterative procedure is given in five steps with an initial step (Step 0) being used only at the beginning i.e. in the initial iteration (Figure 3).

The initial step sets all solution vector values to zero and uses this point as initial x_0 in which all nonlinear portions gradient values of the goals are equal to zero. This reduces the problem of NGP (problem of (1) to (4)) to linear approximation solvable by the modified simplex method of linear goal programming.

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[5] A homogeneous function is a function which has a feature that for some real constant λ satisfies $F(\lambda x, \lambda y) = \lambda^n f(x,y)$ for a fixed n. Then we say that function F is homogenous by degree of homogeneity of n.

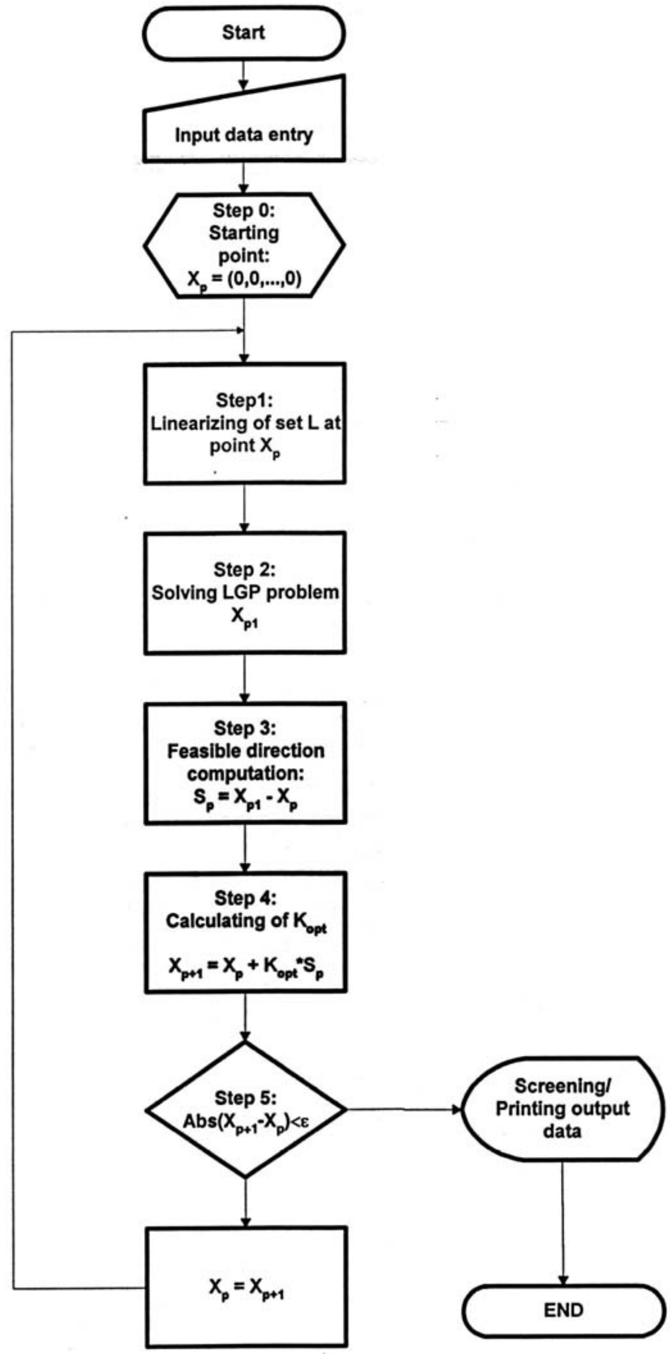

Figure 3: Flow diagram of NGP algorithm

The first step of an NGP algorithm is the computation of gradients of all constraints together with checking the status of nonlinear constraints in order to deflect, if need be, the gradient of active nonlinear constraints and to avoid zigzagging (see: Zoutendijk, 1976). Without this deflecting, the algorithm may converge to a suboptimal point (see: Van de Panne and Popp, 1963).

In the second step the formulated linear goal programming problem (LGP) is solved by a modified simplex method. Namely, nonlinear constraints are transformed to linear on the basis of the Euler's theorem computed gradient value at point x_p (in the initial step it was x_0). The modified simplex method of LGP derives the feasible solution x_p.

In concordance with the Euler's homogeneous function theorem conditions, it is noted that if $f(x_1, x_2, ..., x_n)$ is homogenous of degree r and the first-order derivatives exist, then it can be shown that is:

$$f(x_1, x_2, ..., x_n) = \frac{1}{r} \sum_{r=1}^{n} \frac{\partial f(x_1, x_2, ..., x_n)}{\partial x_i}\bigg|_{[x_p]} \cdot x_i$$

where, with $[x_p]$ is denoted "in particular point x''_p, and where for Cobb-Douglas's functions (5):

$$\frac{\partial f(x_1, x_2, ..., x_n)}{\partial x_i}\bigg|_{[x_p]} = h b_k x_k^{(b_k - 1)} \left(\prod_{\substack{i=1 \\ i \neq k}}^{n} x_i^{b_i} \right) \quad \text{for all} \quad x_i \geq 0$$

are the coefficients of linearized constraints calculated at particular point x_p.

The third algorithm step serves for feasible direction $s_p(x)$ computation:

$$s_p(x) = x'_p - x_p$$

which has to be "searched" according to the constraints and structure of priority in order to improve the simplex solution.

In the fourth step, the optimal solution vector movement size (step size) is determined by linear searching in the feasible direction, identified in the previous step. In this way, first the structural constraints $A_i(x)$ must be satisfied and then the goal ones, starting from the goal constraint $G_i(x)$ in (2) which contains the deviation variable with top priority in the objective function of the NGP problem

To perform the search and to prevent infinite moving in the cases of unbounded problems, it is necessary to determine the lower and upper movement bounds in feasible direction and unit movement size (increment) within these bounds. The convergence of the method for the problem at hand, speed of convergence, and other algorithm features depend on the choice of vectors $s_p(x)$ and bounds for the step size l_p. Initially, the lower limit of l_p in the algorithm was set to 0 and the upper to 1, but it is possible to define them differently if a need be. The unit movement size in the algorithm at first was set on 0.1, but it is possible to increase the search density.

By these procedures we define whether the first goal constraint (per priority and not per order) is satisfied within the initial bounds. If it is satisfied at some smaller interval $[l_{pl}, l_{pd}]$, for all $l_{pl} \geq 0$ and $l_{pd} \leq 1$, the search for further goals constraint goes on exclusively within this interval as its satisfaction, in accordance with so called Pareto optimality, cannot be sought to the detriment of satisfaction of a higher priority goal.

If during the search we come across a constraint which has not been satisfied previously by goal constraint set bounds, then within these bounds the algorithm identifies the value of movement size for which the deviation from the subject constraint is the least l_p, we complete the searching and compute the new solution (successor) as follows:

$$x_{p+1} = x_p + l_p s_p(x_p)$$

If all constraints are satisfied then the new solution is at the same time the optimal solution to the set problem.

In the fifth algorithm step we check the problem convergence by setting previously small value of the desired level of convergence accuracy $-\varepsilon$:

$$|(x_{p+1} - x_p)| \leq \varepsilon, \text{ for all } p = 1, 2, \ldots$$

which means that the algorithm has converged and it stops. Otherwise, the procedure is repeated starting from the first step.

8. INTERACTIVITY IN ESNEM

The interactive work of ESNEM is supported by the algorithm included subsystem for establishing the priority structure-PRIOR and subsystem POST for solution improvement on the basis of post-optimal analysis and priority structure change. A decision maker can but need not be in position to formulate the priority structure in (1) precisely, i.e. to define the objective function or to rank the goals according to their meaning. If the decision maker is not in such a position, the subsystem for establishing the priority structure offers the user a list of possible goals asking the following questions:

- could you rank the goals, at least partially,
- could you group the goals,
- could you select the most important and the least important group of goals,
- do you want to assign different priorities within this group, in an attempt to obtain at least partially a differentiated priority structure.

This subsystem also gives the possibility of assigning different weights to goals of the same priority group whether the weights are determined by the decision maker alone or by a program on the basis of random choice.

If so requested by a decision maker, the POST subsystem can change the priority structure. It starts up and carries out the post-optimal solution analysis and/or keeps searching for a better solution. The work of this subsystem goes on in the next few steps:

1. Analyses of the results obtained by NGP algorithm in order to determine the level of satisfaction of certain priorities in objective function,
2. Diagnosing the conflict within the priority structure and establishing the method for its solution,
3. Memorize the best former solution,
4. Priority structure change in objective function according to the information obtained in steps 1 and 2.
5. Resetting the algorithm NGP, this time from the point of the former best solution with the priority structure defined in step 4,
6. New solution finding and interaction with user to check whether he or she is satisfied with this solution or not,
7. Proclamation of the new solution for the former best solution, if user is satisfied and, otherwise, resetting the memorized former best solution to the operative computer memory,
8. Examination of further possibilities for solution improvement and, if there are some, returning to step 3 or, otherwise, going to step 4,
9. Printing final results.

By the approach "keep finding out better solution" the initial problem solving time is significantly decreased in the case when a decision maker finds, through insight into the former best solution, an often unforeseen dependence between priorities i.e. goals. In that case it is not necessary to change the priority structure and to restart the NGP algorithm from the initial step. The subsystem for solution improvement includes the NGP algorithm from the point of the former best solution and in many ways decreases the necessary number of iterations. Computer programs for realizing ESNEM were coded in Portran7 for PC.

9. CONCLUDING REMARKS

The purpose of this research is to 1) construct a new structure for econometric modeling of the SFRY economy, 2) apply this new approach to analyze SPRY economic activities, and 3) enable the recommendation of economic policy variants which would bring about an optimization of production, consumption and inflation outcomes. This optimal policy variant represents a stabilization policy for the post-war SFRY.

We have shown the possibility of hybridizing many analytical methodologies for a national policy model simulation within ESNEMS - mathematical methods algorithms, econometric methods, particular data base connection, software-driven interactive communication, and an efficient expert system.

One non-typical example of multi-objective decision making was illustrated. For demonstrating the possibilities of ESNEMS, both MINGP and the baseline of a new SFRY econometric model have been established. Periodically updating of the econometric model built in ESNEMS (and linearized by MINGP) becomes a crucial step. The specification of any given econometric model, of course, has certain deficiencies, particularly for developing economies with not established standard econometric approaches. But, as L. R. Klein claimed, "A bad model is better than no model at all."

Information richness within the critical matrix of simplex method has enabled information feedback for the decision model. ESNEMS, as a hybrid methodology solving NGP problems by iteration, seeks the best solution for a specified objective function structure.

Although MINGP is a numeric methodology for solving only certain types of NGP problems, it has been extended to also solve other nonlinear forms. MINGP can be used for NPG problems represented by homogeneous$_1$ continuous, and differentiable functions. The only ESNEMS requirement is separate subprograms for calculation of nonlinear function gradients and for calculation of the solution value(s).

Many decision methods can handle other forms of nonlinear constraints than the Cobb-Douglas type. Using the Euler's homogenous function theorem, it was possible to realize a hybrid connection of the gradient feasible direction method together with linear goal programming method.

10. FUTURE RESEARCH

There is much more work which could be done by multidisciplinary researchers. This work could equip countries and other economic entities to create better formations of realistic systems and more reliable model approximations. This would enable economic policy analysis leading to stable trends instead of ad hoc and sub-optimal economic appraisal. There are no practical limitations to the application of ESNEMS and econometric models to lower organizational forms (regions, enterprises, etc.) if an adequate database is provided. Reliability and availability of appropriate statistically significant data is needed.

There is a great potential in the further hybridization of ESNEMS. Thus, we suggest the connection of MINGP and econometric methods for solving simultaneous equation parameters. Econometric model estimation and solution of some statistical parameters in the objective function of a generalized NGP problem should also bear fruitful results.

Other potentially fruitful research lies in combining ESNEMS (and MINOP) with some methods of multi-objective ranking (Promethee, Electre, Eigenvector, etc.). Particularly if these MOR methods include unbiased approaches to the determination of objective function priorities. Stochastic approaches to the priorities discussed in Section 5 can also be used to create add-on methodologies.

Econometric models should be upgraded regularly. New connections, interrelationships, and regularly updated series of new consecutive ex-post simulation and calibration will enable better (more timely, precise, and updated) economic policy recommendations. The work on features improvement of the ESNEMS should be directed, also, to integrate the database and statistical results of econometric model estimation with MINGP.

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