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CHANGING THE VALUES OF PARAMETERS ON LOT SIZE REORDER POINT MODEL

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Abstract: The Just-In-Time (JIT) philosophy has received a great deal of attention. Several actions such as improving quality, reducing setup cost and shortening lead time have been recognized as effective ways to achieve the underlying goal of JIT. This paper considers the partial backorders, lot size reorder point inventory system with an imperfect production process. The objective is to simultaneously optimize the lot size, reorder point, process quality, setup cost and lead time, constrained on a service level. We assume the explicit distributional form of lead time demand is unknown but the mean and standard deviation are given. The minimax distribution free approach is utilized to solve the problem and a numerical example is provided to illustrate the results.

Keywords: Inventory, service level, minimax distribution free procedure.

1. INTRODUCTION

In the classical production/inventory models, such as the economic order quantity (EOQ) model, the setup/ordering cost and lead time are assumed to be fixed, so does quality of production process (products). In other words, these factors (setup cost, lead time, and quality) are treated as *givens* (Silver [17]) and not subject to control. However, among the modern production/inventory management, the Japanese successful experience of using Just-In-Time (JIT) production has evidenced that the above factors can be controlled through various efforts. Also, accompanying the growth of JIT philosophy, considerable papers discussing the issues related to changing the givens have been presented.

Concerning lead time reduction, Liao and Shyu [8] first presented a probabilistic inventory model in which the order quantity is predetermined and lead time is a unique variable. Ben-Daya and Raouf [1] extended [8] by considering both lead time and order quantity as decision variables. Ouyang et al. [11] generalized [1] by allowing shortages with partial backorders. Moon and Choi [9] and Hariga and Ben-Daya [4] respectively modified [11] to include the reorder point as one of the decision variables. Recently, Ouyang et al. [10] further combined the concepts of setup cost and lead time reductions, and they extended [9] by simultaneously optimizing the lot size, reorder point, setup cost and lead time. Note that the framework of setup cost reduction is initially presented by Porteus [13], and several authors have studied this issue on various production/inventory systems (see, e.g. Keller and Noori [6], Paknejad et al. [12], Sarker and Coates [15]).

In the above mentioned models with controllable lead time [1, 4, 8-11], the quality-related issues are neglected; in other words, quality of production process (products) is implicitly assumed to be fixed at an optimal level and no quality cost is considered. However, in a real production environment, it can often be observed that there are defective items being produced. The results are extra costs incurred, no matter the defective items are rejected, repaired, reworked, or reached to the customer, refunded. Improving quality has been highly emphasized in modern production/ inventory management systems. In addition to the setup cost reduction, Porteus [14] is also the first to explicitly elaborate on a significant relationship between quality imperfection and lot size. Specifically, Porteus extended the classical EOQ model to include an imperfect production process, and based on the modified model, he studied the effects of quality improvement by further introducing the investing options. Besides, there are some authors modified [14] with various settings (see, e.g. Keller and Noori [7], Hong and Hayya [5]).

From the above literature review, it can be found that there is no shortage of inventory models presented for controlling setup cost, lead time or quality, but little work has been done on controlling them simultaneously. In this paper, building upon Ouyang et al.'s [10] modified lot size reorder point (continuous review (Q,r)) inventory model, we extend it to include the possible relationship between quality and lot size and an investing option of quality improvement. Furthermore, instead of having the stock out cost term in the objective function, we employ a service level constraint to control the stock out occasion. Our goal is to minimize the total related cost by simultaneously optimizing the lot size, reorder point, process quality, setup cost and lead time, subject to a service level constraint. We work on a case where the distributional form of lead time demand is unknown but the first and second moments are known and finite. The minimax distribution free approach, originally proposed by Scarf [16] and disseminated by Gallego and Moon [2], is utilized to solve the problem. Also, we develop an algorithm of finding the optimal solution and provide a numerical example to illustrate the results. Finally, the concluding remarks are made.

2. NOTATIONS AND ASSUMPTIONS

First of all, the following notations and assumptions are employed throughout this paper so as to develop the proposed models.

Notations:

Q = lot size

r = reorder point

L = replenishment lead time

D = annual demand rate

h = annual inventory holding cost per unit

s = cost of replacing a defective unit

θ = annual fractional cost of capital investment

β = fraction of the shortage that will be backordered, $0 \le \beta \le 1$

τ = proportion of demands which are not met from stock, $\tau < 1/2$

X = lead time demand which has a distribution function $(d.f.)$ F with finite mean DL and standard deviation $\sigma\sqrt{L}$, where σ denotes the standard deviation of the demand per unit time

A = setup cost per order

A_0 = original level of setup cost

$I_A(A)$ = capital investment required to reduce the setup cost from original level A_0 to target level A, $0 < A \le A_0$

δ = percentage decrease in setup cost A per dollar increase in investment $I_A(A)$

η = probability of the production process that can go 'out-of-control'

η_0 = original probability of the production process that can go 'out-of-control'

$I_\eta(\eta)$ = capital investment required to reduce the 'out-of-control' probability from original level η_0 to target level η , $0 < \eta \le \eta_0$

Δ = percentage decrease in η per dollar increase in investment $I_\eta(\eta)$

Assumptions:

1. The reorder point, $r =$ expected demand during lead time $+$ safety stock (SS), and $SS = k \cdot$ (standard deviation of lead time demand), i.e., $r = DL + k\sigma\sqrt{L}$, where k is the safety factor.
2. The lead time L has n mutually independent components. The i th component has a minimum duration u_i and normal duration v_i , and a crashing cost per unit time c_i . Furthermore, for convenience, we rearrange c_i such that $c_1 \le c_2 \le \ldots \le c_n$. Then, it is clear that the reduction of lead time should first occur on component 1 (because it has the minimum unit crashing cost), and then component 2, etc.

3. If we let $L_0 = \sum_{j=1}^{n} v_j$ and L_i be the length of lead time with components $1,2,...,i$ crashed to their minimum duration, then L_i can be expressed as $L_i = \sum_{j=1}^{n} v_j - \sum_{j=1}^{i} (v_j - u_j)$, $i = 1, 2, \ldots, n$; and the lead time crashing cost $R(L)$ per cycle for a given $L \in [L_i, L_{i-1}]$, is given by

$$R(L) = c_i(L_{i-1} - L) + \sum_{j=1}^{i-1} c_j(v_j - u_j) \text{ and } R(L_0) = 0 .$$

4. The setup cost can be varied through investment. The capital investment, $I_A(A)$, in reducing setup cost is described by a logarithmic function of the setup cost A, and

$$I_A(A) = b\ln(\frac{A_0}{A}) \text{ for } 0 < A \le A_0 \text{, where } b = \frac{1}{\delta} .$$

5. The relationship between quality and lot size is formulated as follows. While producing a lot, the process can go 'out-of-control' with a small probability η each time another unit is produced. The process is assumed to be in control before beginning production of the lot. Once 'out-of-control', the process produces defective units and continues to do so until the entire lot is produced. (This assumption is in line with Porteus [14] and is supported by Hall [3].)
6. The production process can be improved through investment. The capital investment, $I_\eta(\eta)$, in improving process quality by means of reducing the 'out-of-control' probability η (note that the lower the value of η the higher the process quality) is a logarithmic function of η; that is,

$$I_\eta(\eta) = B\ln(\frac{\eta_0}{\eta}) \text{ for } 0 < \eta \le \eta_0 \text{, where } B = \frac{1}{\Delta} .$$

3. MODEL FORMULATION

Recently, Ouyang et al. [10] explored the setup cost and lead time reductions problem on the lot size reorder point inventory system, where shortages are allowed with partial backorders. Symbolically, they formulated an inventory model as follows:

$$\min EAC(Q,r,A,L) = \theta b \ln\left(\frac{A_0}{A}\right) + A\frac{D}{Q} + h\left[\frac{Q}{2} + r - DL + (1-\beta)E(X-r)^+\right] + \frac{D}{Q}[\pi + \pi_0(1-\beta)]E(X-r)^+ + \frac{D}{Q}R(L)$$

subject to

$$0 < A \le A_0 , \tag{1}$$

where π is shortage cost per unit short, π_0 is marginal profit per unit, and $E(X-r)^+$ is the expected shortage per replenishment cycle.

In model (1) the possible relationship between quality and lot size is ignored and no quality improvement planning is considered. These two issues are taken into account here. Firstly, by assumption 5, we note that the expected number of defective items in a run of size Q is approximated to $Q^2\eta/2$ (for more details, see [14]). Suppose the cost of replacing a defective unit is s. Thus, the expected annual defective cost would be $sDQ\eta/2$. Besides, when process quality is no longer considered to be a fixed parameter but a decision variable, the control of quality level is accomplished by varying the capital investment allocated to improve quality level (assumption 6). On the other hand, the stock out cost term is included in model (1). However, the stock out cost often includes intangible components such as loss of goodwill and potential delay to the other parts of the inventory system, so it is difficult to explicitly express the stock out cost. Therefore, we would like to replace the stock out cost term in the objective function by a service level constraint.

With the above modifications, our concerning problem can be formulated as:

$$\min EAC(Q,r,\eta,A,L) = \theta b \ln\left(\frac{A_0}{A}\right) + \theta B \ln\left(\frac{\eta_0}{\eta}\right) + \frac{AD}{Q}$$
$$+h\left[\frac{Q}{2} + r - DL + (1-\beta)E(X-r)^+\right] + \frac{D}{Q}R(L) + \frac{sDQ\eta}{2}$$

subject to

$$\frac{E(X-r)^+}{Q} \le \tau \,,$$

$$0 < \eta \le \eta_0 \,,$$

$$0 < A \le A_0 \,, \tag{2}$$

where τ $(<1/2)$ is the proportion of demands which are not met from stock, and hence $1-\tau$ is the service level.

4. SOLUTION BY MINIMAX DISTRIBUTION FREE APPROACH

Information about the distributional form of lead time demand might be limited in practice. Therefore, in contrast to the traditional approach that the lead time demand X follows a special form of distribution, we assume the *d.f.* F of X belonging to the class $\mathbf{F}$ of *d.f.*'s with finite mean DL and standard deviation $\sigma\sqrt{L}$. In this case, the exact value of the expected shortage per replenishment cycle $E(X-r)^+$ can not be obtained. We then utilize the minimax distribution free approach to solve the problem. The minimax principle for this problem is to find the least favorable *d.f.* F in $\mathbf{F}$ for each (Q,r,η,A,L) and then to minimize the total expected annual cost over Q, r, η, A and L. That is, our problem is to solve

$$\min \max_{F\in\mathbf{F}} EAC(Q,r,\eta,A,L)$$

subject to

$$\frac{E(X-r)^+}{Q} \le \tau ,$$

$$0 < \eta \le \eta_0 ,$$

$$0 < A \le A_0 . \tag{3}$$

We note that to find the least favorable *d.f.* in $\mathbf{F}$ for (3) is equivalent to finding the worst case for $E(X-r)^+$ in model (2). This task can be achieved by utilizing the relationship $r = DL + k\sigma\sqrt{L}$ (assumption 1) and Lemma 1 in Gallego and Moon [2]. That is, we have

$$E(X-r)^+ \le \frac{1}{2}\sigma\sqrt{L}\left(\sqrt{1+k^2}-k\right) \text{, for any } F \in \mathbf{F} . \tag{4}$$

Then by substituting $\sigma\sqrt{L}(\sqrt{1+k^2}-k)/2$ for $E(X-r)^+$ in model (2) and considering the safety factor k as a decision variable instead of the reorder point r (because $r = DL + k\sigma\sqrt{L}$), the problem (3) is reduced to

$$\min EAC^w(Q,k,\eta,A,L) = \theta b \ln\left(\frac{A_0}{A}\right) + \theta B \ln\left(\frac{\eta_0}{\eta}\right) + \frac{AD}{Q} + h\left(\frac{Q}{2} + k\sigma\sqrt{L}\right)$$
$$+\frac{1}{2}h(1-\beta)\sigma\sqrt{L}\left(\sqrt{1+k^2}-k\right) + \frac{D}{Q}R(L) + \frac{sDQ\eta}{2}$$

Subject to

$$\sigma\sqrt{L}\left(\sqrt{1+k^2}-k\right) \le 2\tau Q ,$$

$$0 < \eta \le \eta_0 ,$$

$$0 < A \le A_0 . \tag{5}$$

where $EAC^w(\cdot)$ is the least upper bound of $EAC(\cdot)$.

In order to solve this nonlinear programming problem, we temporarily ignore the restrictions $0 < \eta \le \eta_0$ and $0 < A \le A_0$, and solve the nonlinear programming problem with a single constraint. By adding a nonnegative slack variable, M^2 , to the left-hand side of service level constraint $\sigma\sqrt{L}\left(\sqrt{1+k^2}-k\right) \le 2\tau Q$, we transfer this inequality into equality and formulate the Lagrangean function as follows:

$$\begin{aligned} EAC^{w}(Q,k,\eta,A,L,\lambda,M) &= EAC^{w}(Q,k,\eta,A,L) + \lambda\left[\sigma\sqrt{L}\left(\sqrt{1+k^2}-k\right)+M^2-2\tau Q\right] \\ &= \theta b\ln(\frac{A_0}{A}) + \theta B\ln(\frac{\eta_0}{\eta}) + \frac{AD}{Q} + h\left(\frac{Q}{2}+k\sigma\sqrt{L}\right) \\ &\quad + \frac{1}{2}h(1-\beta)\sigma\sqrt{L}\left(\sqrt{1+k^2}-k\right) + \frac{D}{Q}R(L) + \frac{sDQ\eta}{2} \\ &\quad + \lambda\left[\sigma\sqrt{L}\left(\sqrt{1+k^2}-k\right)+M^2-2\tau Q\right], \end{aligned} \tag{6}$$

where λ is a Lagrange multiplier.

It can be verified that $EAC^{w}(Q,k,\eta,A,L,\lambda,M)$ is not a convex function of (Q,k,η,A,L,λ,M). However, for fixed (Q,k,η,A,λ,M), $EAC^{w}(Q,k,\eta,A,L,\lambda,M)$ is concave in $L\in[L_i,L_{i-1}]$ because

$$\begin{aligned} \frac{\partial^2 EAC^{w}(Q,k,\eta,A,L,\lambda,M)}{\partial L^2} &= -\frac{1}{4}hk\sigma L^{-3/2} \\ &\quad -\frac{1}{8}\left(\sqrt{1+k^2}-k\right)\sigma L^{-3/2}[h(1-\beta)+2\lambda] < 0. \end{aligned} \tag{7}$$

Therefore, for fixed (Q,k,η,A,λ,M), the minimum $EAC^{w}(Q,k,\eta,A,L,\lambda,M)$ will occur at the end points of the interval $[L_i,L_{i-1}]$.

On the other hand, we take the first partial derivatives of $EAC^{w}(Q,k,\eta,A,L,\lambda,M)$ with respect to Q, k, η, A, λ and M, and then set the results equal to zero, respectively. We obtain:

$$\frac{\partial EAC^{w}(Q,k,\eta,A,L,\lambda,M)}{\partial Q} = -\frac{AD}{Q^2}+\frac{h}{2}-\frac{DR(L)}{Q^2}+\frac{sD\eta}{2}-2\lambda\tau = 0\,, \tag{8}$$

$$\frac{\partial EAC^{w}(Q,k,\eta,A,L,\lambda,M)}{\partial k} = h\sigma\sqrt{L}\left[1-\frac{1}{2}\left(1-\frac{k}{\sqrt{1+k^2}}\right)\left(1-\beta+\frac{2\lambda}{h}\right)\right] = 0\,, \tag{9}$$

$$\frac{\partial EAC^{w}(Q,k,\eta,A,L,\lambda,M)}{\partial \eta} = -\frac{\theta B}{\eta}+\frac{sDQ}{2} = 0\,, \tag{10}$$

$$\frac{\partial EAC^{w}(Q,k,\eta,A,L,\lambda,M)}{\partial A} = -\frac{\theta b}{A}+\frac{D}{Q} = 0\,, \tag{11}$$

$$\frac{\partial EAC^{w}(Q,k,\eta,A,L,\lambda,M)}{\partial \lambda} = \sigma\sqrt{L}\left(\sqrt{1+k^2}-k\right)+M^2-2\tau Q = 0\,, \tag{12}$$

$$\frac{\partial EAC^{w}(Q,k,\eta,A,L,\lambda,M)}{\partial M} = 2\lambda M = 0\,. \tag{13}$$

From Eq. (13), we find that $\lambda = 0$ or $M = 0$. However, if $\lambda = 0$, then Eq. (9) will result in $\frac{k}{\sqrt{1+k^2}} = -\frac{1+\beta}{1-\beta} < 0$, which is infeasible since k is a safety factor and the value of k should be nonnegative. Thus, it is clear that the slack variable $M = 0$. Therefore, for given $L \in [L_i, L_{i-1}]$, the optimal solution of (Q, k, η, A) that minimizes the total expected annual cost $EAC^w(Q, k, \eta, A, L)$ and satisfies the constraint $\sigma\sqrt{L}\left(\sqrt{1+k^2} - k\right) \le 2\tau Q$ will occur at the point when this inequality is held at equality.

Simplifying Eqs. (8), (9), (10), (11) and (12), respectively, we get

$$Q = \sqrt{\frac{2D[A + R(L)]}{h + sD\eta - 4\lambda\tau}}, \tag{14}$$

$$\lambda = h\left[\frac{1}{2}(1+\beta) + k\left(k + \sqrt{1+k^2}\right)\right], \tag{15}$$

$$\eta = \frac{2\theta B}{sDQ}, \tag{16}$$

$$A = \frac{\theta b Q}{D}, \tag{17}$$

$$\sqrt{1+k^2} - k = \frac{2\tau Q}{\sigma\sqrt{L}}. \tag{18}$$

Furthermore, solving Eqs. (14)-(18) simultaneously for the relative decision variables (denoted by Q^*, k^*, η^*, A^* and λ^*), we obtain

$$Q^* = \frac{\theta(b-B) + \sqrt{[\theta(b-B)]^2 + h(1-2\tau\beta)[(h\sigma^2 L/2\tau) + 2DR(L)]}}{h(1-2\tau\beta)}, \tag{19}$$

$$k^* = \frac{\sigma\sqrt{L}}{4\tau Q^*} - \frac{\tau Q^*}{\sigma\sqrt{L}}, \tag{20}$$

$$\eta^* = \frac{2\theta B}{sDQ^*}, \tag{21}$$

$$A^* = \frac{\theta b Q^*}{D}, \tag{22}$$

$$\lambda^* = \frac{h}{2}\left\{\beta + \left(\frac{\sigma\sqrt{L}}{2\tau Q^*}\right)^2\right\}. \tag{23}$$

The following proposition shows that, for fixed $L \in [L_i, L_{i-1}]$, when the restrictions $0 < \eta \le \eta_0$ and $0 < A \le A_0$ are ignored, the point (Q^*, k^*, η^*, A^*) is the local optimal solution, which satisfies the constraint $\sigma\sqrt{L}\left(\sqrt{1+k^2}-k\right)\le 2\tau Q$ and minimizes the total expected annual cost $EAC^w(Q,k,\eta,A,L)$.

Proposition 1. When the restrictions $0<\eta\le\eta_0$ and $0<A\le A_0$ are ignored, for given $L\in[L_i,L_{i-1}]$, the point (Q^*,k^*,η^*,A^*) satisfies the second order sufficient condition (SOSC) for the minimizing problem with a single constraint.

Proof: See Appendix.

We now consider the restrictions $0<\eta\le\eta_0$ and $0<A\le A_0$. Firstly, from Eqs. (21) and (22), we note that η^* and A^* are positive as the problem parameters θ, *b, B, s,* and D are positive. Then, we discuss the following four cases where η^* and A^* may occur.

(i) If $\eta^*<\eta_0$ and $A^*<A_0$, then (Q^*,k^*,η^*,A^*) is an interior optimal solution for a given $L\in[L_i,L_{i-1}]$.

(ii) If $\eta^*\ge\eta_0$ and $A^*<A_0$, then it is unrealistic to invest in improving process quality. In this case, the optimal quality level is the original quality level, i.e., $\eta^*=\eta_0$, and the corresponding optimal (Q^*,k^*,A^*) can be determined by solving Eqs. (14), (15), (17) and (18), which results in

$$Q^*=\frac{\theta b+\sqrt{(\theta b)^2+[h(1-2\tau\beta)+sD\eta_0][(h\sigma^2L/2\tau)+2DR(L)]}}{h(1-2\tau\beta)+sD\eta_0}, \tag{24}$$

and k^* and A^* are as those given in Eqs. (20) and (22), respectively.

(iii) If $\eta^*<\eta_0$ and $A^*\ge A_0$, then it is unrealistic to invest in setup cost reduction. In this case, the optimal setup cost is the original setup cost level, i.e., $A^*=A_0$, and the corresponding optimal (Q^*,k^*,η^*) can be determined by solving Eqs. (14), (15), (16) and (18), which results in

$$Q^*=\frac{\sqrt{(\theta B)^2+h(1-2\tau\beta)\left\{2D[A_0+R(L)]+(h\sigma^2L/2\tau)\right\}}-\theta B}{h(1-2\tau\beta)}, \tag{25}$$

and k^* and η^* are as those given in Eqs. (20) and (21), respectively.

(iv) If $\eta^*\ge\eta_0$ and $A^*\ge A_0$, then we should not make any investment to change the current setup cost and process quality. In this case, the optimal $A^*=A_0$ and $\eta^*=\eta_0$, and the optimal (Q^*,k^*) can be determined by solving Eqs. (14), (15) and (18), which results in

$$Q^* = \sqrt{\frac{2D[A_0 + R(L)] + (h\sigma^2 L / 2\tau)}{h(1-2\tau\beta) + sD\eta_0}} \tag{26}$$

and k^* is the same as that given in Eq. (20).

By the above discussions, we now develop an algorithm to find the optimal values for lot size, reorder point, process quality, setup cost and lead time.

Algorithm.

Step 1. For each L_i, $i = 0,1,2, \ldots, n$, utilize (19) to determine Q_i^*, and then substitute Q_i^* into (20), (21) and (22), respectively, to evaluate k_i^*, η_i^* and A_i^*.

Step 2. Compare η_i^* and η_0, and A_i^* and A_0, respectively.

(i) If $\eta_i^* < \eta_0$ and $A_i^* < A_0$, then the solution found in Step 1 is optimal for the given L_i. Go to Step 4.

(ii) If $\eta_i^* \geq \eta_0$ and $A_i^* < A_0$, then for this given L_i, let $\eta_i^* = \eta_0$ and utilize (24) to determine the new Q_i^*, then substitute it into (20) and (22) respectively, to update k_i^* and A_i^*. If the new $A_i^* < A_0$, then the optimal solution is obtained, go to Step 4; otherwise, go to Step 3.

(iii) If $\eta_i^* < \eta_0$ and $A_i^* \geq A_0$, then for this given L_i, let $A_i^* = A_0$ and utilize (25) to determine the new Q_i^*, then substitute it into (20) and (21), respectively, to update k_i^* and η_i^*. If the new $\eta_i^* < \eta_0$, then the optimal solution is obtained, go to Step 4; otherwise, go to Step 3.

(iv) If $\eta_i^* \geq \eta_0$ and $A_i^* \geq A_0$, go to Step 3.

Step 3. For this given L_i, let $\eta_i^* = \eta_0$ and $A_i^* = A_0$, and utilize (26) to determine the new Q_i^*, then substitute it into (20) to evaluate the corresponding optimal k_i^*.

Step 4. Utilize the objective function of model (5) to calculate the corresponding total expected annual cost $EAC^w(Q_i^*, k_i^*, \eta_i^*, A_i^*, L_i)$.

Step 5. Find $\min_{i=0,1,2,\ldots,n} EAC^w(Q_i^*, k_i^*, \eta_i^*, A_i^*, L_i)$.

If $EAC^w(Q_w, k_w, \eta_w, A_w, L_w) = \min_{i=0,1,2,\ldots,n} EAC^w(Q_i^*, k_i^*, \eta_i^*, A_i^*, L_i)$,

then $(Q_w, k_w, \eta_w, A_w, L_w)$ is the optimal solution.

Once k_w and L_w are obtained, the optimal reorder point $r_w = DL_w + k_w\sigma\sqrt{L_w}$ follows.

5. NUMERICAL EXAMPLE

In order to illustrate the above solution procedure, let us consider an inventory system with the data used in Ouyang et al. [10] (the stock out costs are excluded): $D = 600$ units per year, $A_0 = \$200$ per order, $h = \$20$ per unit per year, $\theta = 0.1$ per dollar per year, $b = 5800$, $\sigma = 7$ units per week, and the lead time has three components with data shown in Table 1. Besides, we take $\eta_0 = 0.0002$, $s = \$75$ per defective unit and $B = 400$. Also, suppose the distributional form of the lead time demand is unknown.

Table 1: Lead time data

Lead time component i	Normal Duration v_i (days)	Minimum duration u_i (days)	Unit crashing cost c_i (\$/day)
1	20	6	0.4
2	20	6	1.2
3	16	9	5.0

We solve the cases when the backorder proportion $\beta = 1$ (i.e., complete backorders), and the allowable proportion of demands which are not met from stock $\tau = 1.5\%$, 1% and 0.5% (i.e., the desired service level $1-\tau = 98.5\%$, 99% and 99.5%). Applying the algorithm developed earlier, the computed results are tabulated in Table 2.

Table 2: The computed results of algorithm (L_i in weeks)

Service level $1-\tau$	i	L_i	Q_i^*	r_i^*	k_i^*	η_i^*	A_i^*	$EAC^w(Q_i^*, k_i^*, \eta_i^*, A_i^*, L_i)$
98.5 %	0	8	147	134	2.130	0.0000121	142	3245.25
	1	6	134	104	2.019	0.0000133	129	3036.68
	2	4	122	71	1.779	0.0000146	118	2860.21
	3	3	125	52	1.467	0.0000143	121	2898.02
99 %	0	8	172	148	2.797	0.0000104	166	3670.78
	1	6	154	115	2.685	0.0000115	149	3390.89
	2	4	138	80	2.444	0.0000129	133	3124.51
	3	3	136	60	2.116	0.0000131	132	3098.94
99.5 %	0	8	225	178	4.335	0.0000079	200	4672.30
	1	6	203	141	4.170	0.0000088	196	4230.69
	2	4	175	101	3.932	0.0000101	169	3765.61
	3	3	165	78	3.598	0.0000108	160	3601.21

From Table 2, the optimal operating policy for each desired service level can be found easily by comparing $EAC^{w}(Q_i^*, k_i^*, \eta_i^*, A_i^*, L_i)$, $i = 0,1,2,3$. We summarize the result in Table 3.

Table 3: The optimal operating policy for various service level (L_w in weeks)

Service level $1-\tau$	$(Q_w, r_w, \eta_w, A_w, L_w)$	$EAC^{w}(Q_w, r_w, \eta_w, A_w, L_w)$
98.5 %	(122, 71, 0.0000146, 118, 4)	2860.21
99.0 %	(136, 60, 0.0000131, 132, 3)	3098.94
99.5 %	(165, 78, 0.0000108, 160, 3)	3601.21

Moreover, in order to illustrate the effects of investing in quality improvement and setup cost reduction, in addition to the result of the presented model, we list the optimal operating policies for the cases where process quality or/and setup cost are treated as fixed constant in Table 4. ($\beta = 1$ and $\tau = 1.5\%$)

Table 4: The optimal operating policies for various situations (L_i in weeks)

Decision variables	Q_w	r_w	η_w	A_w	L_w	$EAC^{w}(\cdot)$	*Savings%*
(Q, r, η, A, L)	122	71	0.0000146	118	4	2860.21	14.88
(Q, r, η, L)	136	68	0.0000131	$\underline{200}$	4	2929.75	12.81
(Q, r, A, L)	98	78	$\underline{0.0002}$	94	4	3208.80	4.50
(Q, r, L)	118	72	$\underline{0.0002}$	$\underline{200}$	4	3360.11	-

Notes: (i) *Savings %* is based on the total expected annual cost of the (Q, r, L) model.
(ii) The parameter that has value underlined is fixed and given in the corresponding model.

The results of Table 4 show that no matter quality improvement and setup cost reduction are performed, alone or jointly, the savings of total expected annual cost are realized. Also, the largest % (14.88 %) savings occurs when quality improvement and setup cost reduction are performed simultaneously.

6. CONCLUDING REMARKS

In this paper, we first extend Ouyang et al.'s [10] model to include the possible relationship between quality and lot size. Then we investigate the joint effects of quality improvement and setup cost reduction on the model, where a service level constraint is added to replace the stock out cost term in the objective function. The model, for which the distributional form of lead time demand is unknown but the mean and standard deviation are given, is formulated and solved by the minimax distribution free approach. We develop an algorithm to find the optimal values for the lot size, reorder point, process quality, setup cost and lead time. A numerical example is provided to illustrate the results derived.

The issues of quality improvement, setup cost and lead time reductions studied here belong to the '*changing the givens*' approach. This approach may further invoke some possible research topics and can be applied to other production/inventory models.

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APPENDIX

Proof of Proposition 1: We note that the solution point (Q^*, k^*, η^*, A^*) is obtained at $\sigma\sqrt{L}\left(\sqrt{1+k^2}-k\right)=2\tau Q$ (since the slack variable $M=0$). Therefore, in what follows, we show that (Q^*, k^*, η^*, A^*) satisfies the SOSC for the minimizing problem with a single equality constraint.

For a given value of L, we first obtain the bordered Hessian matrix $\mathbf{H}$ as follows:

$$\mathbf{H}=\begin{bmatrix} 0 & \frac{\partial^2 EAC^w(\cdot)}{\partial\lambda\partial Q} & \frac{\partial^2 EAC^w(\cdot)}{\partial\lambda\partial k} & \frac{\partial^2 EAC^w(\cdot)}{\partial\lambda\partial\eta} & \frac{\partial^2 EAC^w(\cdot)}{\partial\lambda\partial A} \\ \frac{\partial^2 EAC^w(\cdot)}{\partial Q\partial\lambda} & \frac{\partial^2 EAC^w(\cdot)}{\partial Q^2} & \frac{\partial^2 EAC^w(\cdot)}{\partial Q\partial k} & \frac{\partial^2 EAC^w(\cdot)}{\partial Q\partial\eta} & \frac{\partial^2 EAC^w(\cdot)}{\partial Q\partial A} \\ \frac{\partial^2 EAC^w(\cdot)}{\partial k\partial\lambda} & \frac{\partial^2 EAC^w(\cdot)}{\partial k\partial Q} & \frac{\partial^2 EAC^w(\cdot)}{\partial k^2} & \frac{\partial^2 EAC^w(\cdot)}{\partial k\partial\eta} & \frac{\partial^2 EAC^w(\cdot)}{\partial k\partial A} \\ \frac{\partial^2 EAC^w(\cdot)}{\partial\eta\partial\lambda} & \frac{\partial^2 EAC^w(\cdot)}{\partial\eta\partial Q} & \frac{\partial^2 EAC^w(\cdot)}{\partial\eta\partial k} & \frac{\partial^2 EAC^w(\cdot)}{\partial\eta^2} & \frac{\partial^2 EAC^w(\cdot)}{\partial\eta\partial A} \\ \frac{\partial^2 EAC^w(\cdot)}{\partial A\partial\lambda} & \frac{\partial^2 EAC^w(\cdot)}{\partial A\partial Q} & \frac{\partial^2 EAC^w(\cdot)}{\partial A\partial k} & \frac{\partial^2 EAC^w(\cdot)}{\partial A\partial\eta} & \frac{\partial^2 EAC^w(\cdot)}{\partial A^2} \end{bmatrix}, \tag{A.1}$$

where

$$EAC^w(\cdot) \equiv EAC^w(Q,k,\eta,A,L,\lambda),$$

$$\frac{\partial^2 EAC^w(\cdot)}{\partial Q^2}=\frac{2AD}{Q^3}+\frac{2D}{Q^3}R(L),$$

$$\frac{\partial^2 EAC^w(\cdot)}{\partial Q\partial k}=\frac{\partial^2 EAC^w(\cdot)}{\partial k\partial Q}=0,$$

$$\frac{\partial^2 EAC^w(\cdot)}{\partial Q\partial\eta}=\frac{\partial^2 EAC^w(\cdot)}{\partial\eta\partial Q}=\frac{sD}{2},$$

$$\frac{\partial^2 EAC^w(\cdot)}{\partial Q\partial A}=\frac{\partial^2 EAC^w(\cdot)}{\partial A\partial Q}=-\frac{D}{Q^2},$$

$$\frac{\partial^2 EAC^w(\cdot)}{\partial Q\partial\lambda}=\frac{\partial^2 EAC^w(\cdot)}{\partial\lambda\partial Q}=-2\tau,$$

$$\frac{\partial^2 EAC^w(\cdot)}{\partial k^2}=\frac{[h(1-\beta)+2\lambda]}{2(1+k^2)^{3/2}}\sigma\sqrt{L},$$

$$\frac{\partial^2 EAC^w(\cdot)}{\partial k \partial \eta} = \frac{\partial^2 EAC^w(\cdot)}{\partial \eta \partial k} = 0 ,$$

$$\frac{\partial^2 EAC^w(\cdot)}{\partial k \partial A} = \frac{\partial^2 EAC^w(\cdot)}{\partial A \partial k} = 0 ,$$

$$\frac{\partial^2 EAC^w(\cdot)}{\partial k \partial \lambda} = \frac{\partial^2 EAC^w(\cdot)}{\partial \lambda \partial k} = -\left(1 - \frac{k}{\sqrt{1+k^2}}\right)\sigma\sqrt{L} ,$$

$$\frac{\partial^2 EAC^w(\cdot)}{\partial \eta^2} = \frac{\theta B}{\eta^2} ,$$

$$\frac{\partial^2 EAC^w(\cdot)}{\partial \eta \partial A} = \frac{\partial^2 EAC^w(\cdot)}{\partial A \partial \eta} = 0 ,$$

$$\frac{\partial^2 EAC^w(\cdot)}{\partial \eta \partial \lambda} = \frac{\partial^2 EAC^w(\cdot)}{\partial \lambda \partial \eta} = 0 ,$$

$$\frac{\partial^2 EAC^w(\cdot)}{\partial A^2} = \frac{\theta b}{A^2} ,$$

$$\frac{\partial^2 EAC^w(\cdot)}{\partial A \partial \lambda} = \frac{\partial^2 EAC^w(\cdot)}{\partial \lambda \partial A} = 0 ,$$

$$\frac{\partial^2 EAC^w(\cdot)}{\partial \lambda^2} = 0 .$$

For a given value of L, since there are four variables (Q, r, η, A) and one constraint, therefore, we need to check the sign of the last three principal minor determinants of $\mathbf{H}$ at point (Q^*, k^*, η^*, A^*). If the sign of them are all negative, then this solution point satisfies the SOSC for the minimizing problem (see, for example, Taha [18], p.767).

Now we proceed by checking the sign of the last three principal minor determinants of $\mathbf{H}$ at point (Q^*, k^*, η^*, A^*).

$$|H_{33}| = -(\sigma\sqrt{L})^2 \left(1 - \frac{k^*}{\sqrt{1+k^{*2}}}\right)^2 \frac{2D[A^* + R(L)]}{Q^{*3}} - 2\tau^2 \frac{[h(1-\beta)+2\lambda]}{(1+k^{*2})^{3/2}}\sigma\sqrt{L} < 0. \tag{A.2}$$

$$|H_{44}| = \frac{\theta B}{\eta^{*2}} |H_{33}| + \left[\frac{1}{2} sD\sigma\sqrt{L} \left(1 - \frac{k^*}{\sqrt{1+k^{*2}}} \right) \right]^2$$

$$= -\left[\frac{1}{2} sD\sigma\sqrt{L} \left(1 - \frac{k^*}{\sqrt{1+k^{*2}}} \right) \right]^2 \left\{ \frac{2D[A^* + R(L)]}{\theta B Q^*} - 1 \right\}$$

$$- \frac{(\tau s D Q^*)^2 [h(1-\beta) + 2\lambda] \sigma\sqrt{L}}{2\theta B (1+k^{*2})^{3/2}}$$

$$= -\left[\frac{1}{2} sD\sigma\sqrt{L} \left(1 - \frac{k^*}{\sqrt{1+k^{*2}}} \right) \right]^2 \frac{\sqrt{(\theta B)^2 + 2D[A^* + R(L)](h - 4\lambda\tau)}}{\theta B}$$

$$- \frac{(\tau s D Q^*)^2 [h(1-\beta) + 2\lambda] \sigma\sqrt{L}}{2\theta B (1+k^{*2})^{3/2}}$$

$$< 0 . \tag{A.3}$$

$$|H_{55}| = \frac{\theta b}{A^{*2}} |H_{44}| + \left(\frac{D}{Q^{*2}} \right)^2 \frac{\theta B}{\eta^{*2}} \left[\sigma\sqrt{L} \left(1 - \frac{k^*}{\sqrt{1+k^{*2}}} \right) \right]^2$$

$$= -\frac{1}{\theta b} \left(\frac{D}{Q^*} \right)^2 \left[\frac{1}{2} sD\sigma\sqrt{L} \left(1 - \frac{k^*}{\sqrt{1+k^{*2}}} \right) \right]^2 \frac{\left[(h - 4\lambda\tau) Q^* + \theta B \right]}{\theta B}$$

$$+ \frac{1}{\theta B} \left(\frac{D}{Q^*} \right)^2 \left[\frac{1}{2} sD\sigma\sqrt{L} \left(1 - \frac{k^*}{\sqrt{1+k^{*2}}} \right) \right]^2 - \frac{(\tau s)^2 D^4 [h(1-\beta) + 2\lambda] \sigma\sqrt{L}}{2\theta^2 b B (1+k^{*2})^{3/2}}$$

$$= -\left(\frac{D}{Q^*} \right)^2 \left[\frac{1}{2} sD\sigma\sqrt{L} \left(1 - \frac{k^*}{\sqrt{1+k^{*2}}} \right) \right]^2 \frac{\sqrt{[\theta(B-b)]^2 + 2DR(L)(h - 4\lambda\tau)}}{\theta^2 b B}$$

$$- \frac{(\tau s)^2 D^4 [h(1-\beta) + 2\lambda] \sigma\sqrt{L}}{2\theta^2 b B (1+k^{*2})^{3/2}}$$

$$< 0 . \tag{A.4}$$

From (A.2), (A.3) and (A.4), since the sign of $|H_{33}|$, $|H_{44}|$ and $|H_{55}|$ are all negative, hence, it can be concluded that (Q^*, k^*, η^*, A^*) satisfies the SOSC for the minimizing problem with a constraint.