Yugoslav Journal of Operations Research
35 (2025), Number 2, 365-382
DOI: https://doi.org/10.2298/YJOR240115032J

Research Article

MULTI-OBJECTIVE MATHEMATICAL PROGRAMS TO MINIMIZE THE MAKESPAN, THE PATIENTS' FLOW TIME, AND DOCTORS' WORKLOADS VARIATION USING DISPATCHING RULES AND GENETIC ALGORITHM

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Received: January 2024 / Accepted: July 2024

Abstract: The World Health Report estimated that 20-40% of health sector resources are wasted globally. Balancing many conflicting objectives such as clinical excellence, cost containment, and patient satisfaction can be challenging. In fact, multiple objective programming is one of the best tools that can be used for logistics optimization in many organizations. The aim of our paper is to propose a multi-objective mixed integer linear program that satisfies the goals of two important actors in the healthcare system: patients and doctors. The problem considers a parallel machine scheduling model that integrates simultaneously the following most known objectives in healthcare systems: minimization of the makespan, the patients' total flow times, and the doctors' workloads variations. The current paper deals with a real case study where the number of doctors exceeds the number of machines. A mathematical model combined with some dispatching rules was developed

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and solved using the CPLEX software, which shows the practical importance of our approach. For small instances, we use a mathematical programming model and a heuristic method based on the "first come, first served" rule to assign patients to machines and doctors. For larger instances, we use a genetic algorithm to approximately solve our multi-objective model.

Keywords: Makespan, flow time, scheduling, doctors' workload variation, multi-objective mixed integer linear programming, genetic algorithm.

MSC: 90C29.

1. INTRODUCTION

The scheduling theory, which is one of the main operations research subjects, aims to find the best sequences that optimize a single or multiple criteria using one or parallel machines. Several models and approaches were proposed to solve the problems of scheduling, namely mathematical programming, heuristics, meta-heuristics and simulation optimization algorithms. Some known dispatching rules such as the First Come First Served (FCFS) method are frequently used and easily applied. However, they do not always guarantee the optimality, except for some special cases. In the healthcare field, the scheduling theory has been used by many authors such as [1, 2, 3, 4, 5, 6]. Appropriate algorithms and heuristics have been applied to solve this problem taking into account its complexity, the number of machines, the scheduling system and the static or dynamic nature of patients' arrivals. Various optimization methods were applied in the healthcare field ranging from simple rules to multi-objective programming. [7] developed a robust possibilistic programming framework for designing an organ transplant supply chain under uncertainty. As to [8],they proposed an automatic computer system for diabetic retinopathy affected patients. In turn, [9] used mathematical modeling and Monte-Carlo simulation to determine screening recommendations for diabetic retinopathy patients. Multi-objective formulations were addressed by many authors in the literature. In this regard, [10] developed efficient multi-objective meta-heuristic algorithms for energy-aware non-permutation flow-shop scheduling problem. In turn, [11] proposed a mixed integer-linear programming model and used a multi-criteria decision analysis for supplier selection purposes in the pharmaceutical industry. As to [12], they used Monte-Carlo simulation coupled with expected value and variance operators to come up with an efficient solution. Multi-objective optimization has been demonstrated to provide efficient solutions [13, 14].

As mentioned above, many researchers proposed various mathematical models and algorithms in order to find efficient schedules where one or parallel machines are used in a flow shop or job shop environment. However, the mathematical models are not suitable to find the optimal solution for large scale problems in reasonable time due to its computational intractability. So, various meta-heuristic algorithms are proposed for solving these problems.

Meta-heuristic methods are largely used for solving NP-hard optimization problems. Its algorithms are faster than exact methods and more generic than heuristic methods. Thus, they give solutions with good quality and in reasonable computation time. In general, meta-heuristic methods end up with an approximate or optimal solution in acceptable time. The hybridization of meta-heuristics can give better results. The well-known and frequently used meta-heuristics in the scheduling problems are the local search methods, [15];

intensification and diversification procedure, [16]; tabu search algorithm, [17]; and genetic algorithms, [18, 19, 20, 21, 22].

Diabetic Retinopathy (DR) is a form of complication of diabetes that usually affects both eyes. This severe anomaly is generally asymptomatic in early stages of the disease, [23]. If someone has DR, he may not notice changes to his vision at first. Over time, DR can get worse and cause vision loss, [24]. Caring of patients attained by DR requires a multidisciplinary team with an active participation of the patient [23]. DR causes retinal disorders which could be managed with Laser photocoagulation treatment (light amplification by stimulated emission of radiation). This technique consists of the application of small burns on the retina to prevent the severe bleeding of the retinal blood vessels. It has been reported that Laser treatment is the best in term of extending the time to develop blindness for patients with signs of retinopathy [25, 26]. Given the critical situation of patients suffering from DR disease and knowing the importance of laser photocoagulation treatment, it is mandatory to provide these patients with good care services. Optimizing the scheduling of patients and improving the management of laser rooms are very important aspects for providing timely services for patients and maximizing the use of available resources. The present study deals with parallel machines scheduling problems. Our purpose here is to introduce three objective functions related to three actors. The doctors are concerned with the minimization of workloads variation and makespan objective functions. The patients are concerned with the minimization of total flow times and makespan objective functions. Then, the third actors, the laser machines, are concerned only with the minimization of the makespan objective function. One main goal of this paper is to show the importance of adding the makespan objective in the formulation of scheduling of patients to machines and doctors, as illustrated in our real case, scheduling diabetic retinopathy patients to parallel machines and doctors on the laser room of Habib Bourguiba Hospital in Sfax. We aim to determine the best schedule of patients to be performed over a day by the available resources, machines and doctors. To guarantee efficient and quality service in the ophthalmology department in Habib Bourguiba hospital the number of the working doctors may exceed the number of machines. This problem can be modeled by scheduling n jobs on m parallel machines where jobs represent patients and machines are replaced once by laser machines and in other times by doctors while minimizing the total flow time, the workload variation and the makes pan, simultaneously.

The paper is organized as follows: section 2 describes the problem. In section 3, we present the mathematical formulation of our problem. Then, the proposed model is illustrated through a real case in section 4 with analysis and discussion of results. Section 5 suggests a genetic type algorithm to deal with larger instances. Section 6 summarizes the computational experiments related to the proposed model. Finally, the last section provides conclusion and perspectives.

2. PROBLEM DESCRIPTION

Although there is a vast amount of literature in scheduling theory adapted in the domain of healthcare, to our knowledge, none of the optimization models have addressed simultaneously the well-known objectives of minimizing the total time to complete the tasks, the total patients’ waiting times for treatment, and doctors' workloads variations. We applied this approach on our real case study of scheduling DR patients in the ophthalmology department in Habib Bourguiba hospital of Sfax, Tunisia, where the number of doctors exceeds the number of laser machines. Both resources work in parallel to serve the patients (jobs) who may have different ready dates. This problem is a generalization of one laser machine and one doctor scheduling problem, which minimizes the three objectives mentioned above.

This paper is an extension to the model of [6], where they presented bi-objective mathematical models to the problem of scheduling DR patients to get laser treatment. In this paper, we consider three objectives in an attempt to improve patients and doctors' satisfactions. Our first objective is to minimize the makes pan. The second one concerns the minimization of the flow time of all the patients treated with laser machines in the ophthalmology department, thereby reducing their overall length of stay. Our third objective is to provide a schedule that levels the workload between doctors in laser photocoagulation room during the day. To the best of our knowledge, our research work is the first to consider the real case where the number of doctors exceeds the number of machines while simultaneously considering the three well known objective functions in healthcare field. Both efficient and approximate solutions are provided for small and large instances respectively.

The problem, of scheduling *n* patients on *m* identical parallel machines to minimize the makes pan is considered to be *NP*-hard [27]. In addition, the goal of minimizing the flow time in scheduling problem of *n* tasks on one machine and with 'ready time' different from zero is an NP Hard problem [28]. The scheduling of patients to machines and doctors is NP Hard [6]. Therefore, the use of exact methods cannot provide efficient solutions in reasonable time**.** For this reason, we opt for heuristics (dispatching rules) and meta-heuristics (genetic algorithm).

3. MULTI-OBJECTIVE MODEL FORMULATION

In the hospital, notably in the Laser room, patients are serviced following the method on a First Come First Served (FCFS) basis, which does not necessarily minimize the total patients' stay time though the social value of the FCFS rule is very high and people struggle to respect it. When the patients' arrival order is ignored, the total cumulative stay time for all patients can increase, which may yield longer waiting time for some patients [25]. Thus, by incorporating a maximum limit on patients' flow time, we can avoid longer waiting times.

3.1. Notation and formulation of the mathematical model

Our multi-objective model of scheduling patients is an extension to the mathematical model of [6], but by adding a third objective function of minimizing the makespan. Our model assumes the following hypothesis:

- Every machine can accommodate only one patient at a time.
- Every patient can be assigned to only one machine.
- Each doctor is to be assigned to at most only one machine at a time.
- Each doctor can treat only one patient at a time.
- The processing time of the patients remains unchangeable for all machines.
- The processing time of the patients depends on the severity of the disease.
- If a patient is assigned to a doctor, he will be with him until the end of the treatment process.
- The planning process is considered as dynamic (all the patients may arrive at any time after the start of the scheduled plan).

3.1.1. Notations

i: the index of patients, i (=1,..., n) $\in P$

j: the index of available machines, j (= 1,..., m) $\in M$

k: the index of available doctors, k (= 1,..., d)$\in D$

3.1.2. Sets and parameters

- D: the set of available doctors.
- P: the set of patients.
- M: the set of available machines.
- S_i: the processing time of patient i.
- R_i: ready time: the date of availability for the treatment of patient i.
- A_k: time of availability of doctor k.
- B_j: time of availability of machine j.
- N: a large positive constant.

3.1.3. Decision variables

$X_{ij} = \begin{matrix} \text{1 if patient i is assigned to machine j} \\ \text{0 otherwise} \end{matrix}$

$Y_{i\kappa} = \begin{matrix} \text{1 if patient i is assigned to doctor k} \\ \text{0 otherwise} \end{matrix}$

F_i: flow time of patient i.
W_i: the waiting time of patient i.
C_i: 'Completion time', which corresponds to the end date of treatment of patient i.
$Time_k = \sum_{i\in P}(Y_{ik} \times \sum_{r\in M}(X_{ir} \times S_i))$: the time worked by each doctor.

3.1.4. Mathematical model

In our models, we consider three independent objectives. The first model minimizes the makespan (Cmax).The second one attempts to minimize the flowtime for all patients(TFT), and the third focuses on equally distributing the total workload among doctors (WL).The analytical form of the model is shown as follows:

$$\text{Cmax} = \{\max\{C_i, i \in P\}\}$$

$$TFT = \sum_{i\in P} F_i$$

$$WL = \sum_{k\in D} \left| \sum_{i\in P} Y_{ik} \times S_i - \left(\frac{\sum_{t\in P} S_t}{card(D)} \right) \right|$$

Subject to the following constraints:

$$\sum_{j\in M} X_{ij} = 1 \qquad \forall\, i \in P \tag{1}$$

$$\sum_{k\in D} Y_{ik} = 1 \qquad \forall\, i \in P \tag{2}$$

$$C_i - S_i \geq 0 \qquad \forall\, i \in P \tag{3}$$

$$C_l - C_i - S_l + N \times (2 - Y_{ik} - Y_{lk}) \geq 0$$

$$\forall\, i \in P, l \in P, order\ of\ i < order\ of\ l, k \in D \quad (4)$$

$$C_l - C_i - S_l + N \times \left(2 - X_{ij} - X_{lj}\right) \geq 0$$

$$\forall\, i \in P, l \in P, order\ of\ i < order\ of\ l, j \in M \quad (5)$$

$$W_i - C_i + \sum_{r \in M} X_{ir} \times S_i + R_i = 0 \quad \forall\, \mathrm{i} \in \mathrm{P} \quad (6)$$

$$F_i = C_i - R_i \ \forall\, i \in P \quad (7)$$

$$C_i - S_i - A_k + N \times (1 - Y_{ik}) \geq 0 \quad \forall\, i \in P, k \in D \quad (8)$$

$$C_i - S_i - R_i + N \times \left(1 - X_{ij}\right) \geq 0 \quad \forall\, i \in P, j \in M \quad (9)$$

$$C_i - S_i - B_j + N \times \left(1 - X_{ij}\right) \geq 0 \quad \forall\, i \in P, j \in M \quad (10)$$

$$X_{ij} \in \{0,1\}, Y_{ik} \in \{0,1\}, N\text{: a large positive constant}, \forall i \in P, j \in M, k \in D \quad (11)$$

The constraints descriptions are as follows:

- Constraints (1) guarantee that each patient is assigned to exactly one machine.
- Constraints (2) guarantee that each patient is assigned to exactly one doctor.
- Constraints (3) guarantee that the completion time of patient i is more than its processing time.
- Constraints (4), (5) guarantee that there is no overlapping for any two patients assigned to the same doctor or the same machine, respectively.
- Constraints (6) define the completion time as the release time of the patient plus his/her service time plus his/her waiting time.
- Constraints (7) guarantee that the flow time of a patient is equal to the completion time minus the release time of the same patient.
- Constraints (8) guarantee that the completion time of a patient i is more than the availability of doctor k plus his/her service time (provided that patient i is assigned to doctor k)
- Constraints (9) guarantee that the completion time of a patient i is more than the availability of patient i plus his/her service time (provided that patient i is assigned to machine j)
- Constraints (10) guarantee that the completion time of a patient i is more than the availability of machine j plus his/her service time (provided that patient i is assigned to machine j)
- Constraints (11) define the decision variables.

3.1.5. Normalization of ideal solutions

In this subsection, we propose three mathematical programs in order to determine the three ideal solutions for each objective of the model. We have an ideal solution for the maximum completion time, other for the total flow time objective, also another one for the doctors' workloads variations objective.

Program 1:

$$min\,\{max\{C_i, iinP\}\} = Cmax *$$

Subject to the following constraints:

Constraints (1) to (11).

After adding the parameter Cmax*, the solution to the above program, we obtain program 2 presented as follows:

$$min \sum_{i \in P} F_i = TFT^*$$

Subject to the following constraints:
Constraints (1) to (11)

Now solving Program 2, we obtain the best solution represented by the parameter TFT*.

Since the Cmax, TFT and WL objectives are of different magnitude (uncommon measurability), the idea of including the normalization of the goal program is highly recommended. What remains at stake is how the decision maker values the different objectives. An initial way is to assume that he/she assigns the same value for all objectives. [21] have provided significant justifications in their paper on the double role of the weight factor in the goal programming.

- $d3$ is the deviation variable from the best objective of Program 1.
- $d4$ is the deviation variable from the best objective of Program 2.

Note that we cannot obtain a value of makespan and flow time less than the ideal solutions obtained in programs 1 and 2, respectively.

The constraint $Cmax$ - $d3$ = $Cmax^*$ has only one variable (without counting the deviation) and therefore the norm of the vector is 1.

The constraint $\sum_{i=1}^{n} F_i - d4 = TFT^*$ has n variables (without counting the deviation) and therefore the norm of the vector is $\sqrt{n}$.

For each k, the constraint $\sum_{k \in D} \left| \sum_{i \in P} Y_{ik} \times S_i - \left(\frac{\sum_{t \in P} S_t}{card(D)} \right) \right| + d1_k - d2_k = 0$

has n variables (without counting the deviation. Therefore, the norm of the vector is $\sqrt{S_1^2 + S_2^2 + \cdots . + S_n^2}$ and the objective function in program 3 becomes:

$Min \ \sum_{k \in D} \frac{1}{\sqrt{S_1^2 + S_2^2 + \cdots . + S_n^2}} (d1_k + d2_k) + 1d3 + \frac{1}{\sqrt{n}} d4$.

Let $w1 = \frac{1}{\sqrt{S_1^2 + S_2^2 + \cdots . + S_n^2}}$, $w2=1$, $w3=\frac{1}{\sqrt{n}}$

Program 3 is a weighted normalized goal program presented as follows:

$$Min \ \sum_{k \in D} w1(d1_k + d2_k) + w2 \times d3 \ + \ w3 \times d4$$

Subject to the following constraints:
Constraints (1) to (11)

And adding the constraints

$$Cmax - d3 = Cmax^* \quad (12)$$

$$\sum_{i=1}^{n} F_i - d4 = TFT^* \quad (13)$$

$$\sum_{k \in D} \left| \sum_{i \in P} Y_{ik} \times S_i - \left(\frac{\sum_{t \in P} S_t}{card(D)} \right) \right| + d1_k - d2_k = 0, \text{ for each } k \text{ in } D \quad (14)$$

F_i designates the flow time of patient i, which is $C_i - R_i$, i in P.
$d1_k$ is the negative deviation from a balanced workload for doctor k, k in D.
$d2_k$ is the positive deviation from a balanced workload for doctor k, k in D.

In the next section, we use a mathematical programming model and a heuristic method based on the FCFS rule to assign the patients to machines and doctors for small instances. Regarding larger instances, we use a genetic algorithm to approximately solve our multiobjective model.

4. NUMERICAL EXAMPLES OF A REAL CASE

We assist the ophthalmology department of Habib Bourguiba hospital by working on a real example of patients' treatment by laser photocoagulation machines in the laser room. The machines are daily used by 4 doctors (3 seniors and 1 resident doctor). Every day, we have n patients who must be scheduled and treated by laser photocoagulation. This number is selected in advance with respect to the daily machines' capacities.

A more general description of the problem in the ophthalmology department is to schedule everyday n patients ($P_1,\ldots,P_n$) on m identical parallel machines ($M_1,\ldots,M_m$) using k identical doctors ($D_1,\ldots, D_k$). In this regard, it is worth considering an example of scheduling fifteen patients in a typical day on three machines using four doctors so as to optimize the total flow time of patients, doctors' workloads variations, and the makespan, simultaneously. The patients' ready times (in minutes) are uniformly randomly selected integers from 0 to 2 hours. Patients coming later than two hours are not accepted. The processing times (in minutes) are integers uniformly and randomly selected between 13 and 16 minutes. We assume also that the presence of doctors could take values uniformly as much as 60 minutes. Due to machine maintenance, setup, and breakdowns, the availability of each machine could be delayed with time distributed uniformly from 0 to 180 minutes.

Table 1: Indicates the patients, their processing times and their ready times

Patient	Processing time	Ready time
P1	109	14
P2	13	16
P3	79	15
P4	83	16
P5	67	14
P6	16	15
P7	74	14
P8	7	16
P9	107	15
P10	84	14
P11	7	14
P12	102	14
P13	104	15
P14	8	15
P15	118	13

Table 2 and Table 3 present machines' and doctors' availabilities in minutes, respectively.

Table 2: Machines' availabilities

Machine	Availability
1	99
2	8
3	108

Table 3: Doctors' availabilities

Doctor	Availability
1	29
2	30
3	52
4	48

Table 4 represents the order (Patient 8 is scheduled first), the ready, the processing and the completion times of each patient.

Table 4: Patients ordered by first come first served rule

Patient	Readytime	Processing time	Completion time
8	7	16	45
11	7	14	59
14	8	15	74
2	13	16	90
6	16	15	105
5	67	14	113
7	74	14	119
3	79	15	123
4	83	16	129
10	84	14	133
12	102	14	137
13	104	15	144
9	107	15	148
1	109	14	151
15	118	13	157

Step 2:
After optimization, we obtain the FCFS sequences.

- Sequence of patients on machine 1: = 5 4 13 15;
- Sequence of patients on machine 2: = 8 11 14 2 6 7 10 9;
- Sequence of patients on machine 3: = 3 12 1;
- Sequence of patients on doctor 1: = 8 2 4;
- Sequence of patients on doctor 2: = 11 7 12 1;
- Sequence of patients on doctor 3: = 14 5 10 9;
- Sequence of patients on doctor 4: = 6 3 13 15;
- Cmax = 157
- sum {i in Patients} F[i] = 749

The values of doctors' workloads are presented in Table 5.

Table 5: The working time of each doctor

Doctor	Working time
1	48
2	56
3	58
4	58

4.1. Scheduling with different dispatching rules

We explore different cases where a rule, a number of patients and a number of doctors are fixed for each case, but the number of machines varies between 1, 2 and 3. The rules are FCFS, SPT, and LPT. We record the values of the makespan, the flow time of each patient and the difference between the maximum and the minimum time worked by each doctor.

Table 6: Results with FCFS rule

Machine	Cmax	Total Flow	Workload variation
1	319	2293	12
2	177	850	11
3	157	749	10

Table 7: Results with SPT rule

Machine	Cmax	Total Flow	Workload variation
1	332	2377	10
2	214	1434	11
3	171	1008	12

Table 8: Results with LPT rule

Machine	Cmax	Total Flow	Workload variation
1	319	2332	11
2	200	1242	10
3	171	1052	11

As mentioned in Tables 6, 7 and 8, we find that for M = 3 the FCFS rule dominates the other two scheduling procedures, although it was interesting to see how the schedules of patients work with reduced resources and how the LPT and SPT could be affected by the ready time. We also ran the program for a varying number of machines and set the number of doctors to 4.

Table 9: Results of the objectives under different rules for m=1, d=4

Rule	Cmax	Total Flow	Workload variation
FCFS	319	2293	10
SPT	332	2377	10
LPT	319	2332	11

Table 10: Results of the objectives under different rules for m=2, d=4

Rule	Cmax	Total Flow	Workload variation
FCFS	177	850	10
SPT	214	1434	11
LPT	200	1242	10

Table 11: Results of the objectives under different rules for m=3, d=4

Rule	Cmax	Total Flow	Workload variation
FCFS	157	749	10
SPT	171	1008	12
LPT	171	1052	11

We run the optimization programs with *m*=3, *d*=1,2, 3, 4.

Table 12: Results of the objectives under different rules for *m*=3, *d*=1,2,3,4

Number of doctors	Cmax	Total Flow	Workload variation
d=1	311	2173	-
d=2	232	1765	12
d=3	170	831	1
d=4	157	749	10

It is possible to work with the four configurations. It is worth noting that working with three doctors could yield the best work balance when compared to working with four and two doctors. Hence, for the time being, we opt for using m=3 and d=4 with different new dispatching rules.

4.2. Results with different variants of the FCFS rule

From the analysis displayed in the previous section, three different more logical rules come to mind: The first one is “First Come First Served” (FCFS). This rule is justified by the fact that a patient coming earlier should not be delayed when being treated. A critic to this method is that this early comer may take longer time in the treatment process. The second rule assigns priority to the patient who has the least possible sum of both arrival time and processing time (R+S). The third rule assigns high priority to the patient with least arrival time plus twice his/her processing time (2R+S).

We run the program with the FCFS, R+S, 2R+S rules and different sets of data. Each set is run for the three rules to obtain Table 13.

Table 13: Three rules’ results

Rule	Cmax	Total Flow	Workload variation
FCFS	133	661	12
R+S	134	661	13
2R+S	134	661	13

Table 13 confirms the good performance of the FCFS rule.

Tables 14 and 15 present the availabilities of doctors and machines while Table 16 gives ready times, processing times and the FCFS completion times of patients.

Table 14: Doctors’ availability

Doctor	Availability
1	48
2	1
3	50
4	43

Table 15: Machines’ availabilities

Machine	Availability
1	32
2	96
3	24

Table 16: The FCFS results

Patient	Ready time	Processing time	Completion time
7	4	15	39
10	5	15	54
6	29	14	57
12	32	14	68
1	36	15	72
3	39	15	83
5	41	16	88
11	44	15	98
14	44	14	102
2	49	15	111
4	67	14	112
13	67	16	118
8	72	15	126
9	93	14	126
15	104	15	133

The FCFS sequences of patients scheduled on machines and doctors are as follows:

- Sequence of patients on machine 1:= 10 12 3 11 4 9;
- Sequence of patients on machine 2 := 2 8;
- Sequence of patients on machine 3 := 7 6 1 5 14 13 15;
- Sequence of patients on doctor 1 := 12 3 11 4;
- Sequence of patients on doctor 2 := 7 10 2 9;
- Sequence of patients on doctor 3 := 5 13 15;
- Sequence of patients on doctor 4= 6 1 14 8;
- Cmax = 133
- sum{i in Patients} F[i] = 661

Table 17 presents the doctors' workloads values given by the FCFS rule.

Table 17: Doctors' workloads

Doctor	Working time in minutes
1	58
2	59
3	47
4	58

4.3. Sensitivity analysis

In this section, we consider different examples of scheduling patients on machines and doctors in order to optimize the total flow time of patients (TFT), doctors' workloads variations (WLV), and the makespan (Cmax) when using the FCFS rule. At each iteration, we vary the number of patients or the number of machines, and then we discuss the impact of each variation on the values of different objective functions (see Table 18). We give the program a time limit equal to 360 seconds. It is also noteworthy that the best values are obtained for six out of seven cases.

Considering the case with the same number of machines (three) and the same number of doctors (four), the increase in the number of patients has a negative impact on the values of average flow time and Cmax. However, the objective of WLV attains its best value (zero) when n is equal to an intermediate value (twenty) and is deteriorated with a higher number of patients (thirty). When we consider the example of scheduling thirty patients and four doctors while increasing the number of machines from three to four, we note the improvement of average flow time and Cmax values, but not those of the WLV. The decrease in the number of machines to one and two have a negative impact on all considered objectives. The addition of a fourth machine to the real case study has a slight positive effect on the average flow time and the Cmax objectives, but it deteriorates the value of WLV.

Table 18: Sensitivity analysis through varying the patients' and machines' numbers

M	N	D	WLV	TFT	TFT/n	Cmax	FCFS rule and normalized objective function
3	15	4	10	749	49.93	157	Best solution
3	20	4	0	1148	57.4	183	Best solution
3	30	4	12	4394	146.46	292	Feasible solution
4	30	4	13	4158	138.6	261	Best solution
1	15	4	12	2293	152.87	319	Best solution
2	15	4	11	850	56.67	177	Best solution
4	15	4	12	739	49.27	150	Best solution

5. LARGE INSTANCES AND GENETIC TYPE ALGORITHM

Since the problem of allocating jobs on parallel machines is NP hard, the more complex problem of scheduling patients by adding the availability of constraints on patients, machines, and doctors is also NP-hard and may not be solved in a reasonable amount of time, notably for large instances. For this reason, we use meta-heuristic algorithms such as GA which is an evolutionary algorithm created by John Holland in 1960. According to [30], GA can be defined as a stochastic process based on a population of individuals. It is known as a problem-solving system based on the principles of evolution and hereditary [31]. Each system starts with an initial set of random solutions and uses a process similar to biological evolution to improve upon them, which encourages the survival of the fittest. Thus, the best overall generated gene becomes the candidate solution. This algorithm operates as follows: First, each individual represents a potential solution. Then, the program generates randomly an initial population of individuals. After that, the individuals are classified in the sampled population according to a fitness function. Next, the selection is applied relatively to the calculated probabilities. Then, the crossover operator produces new individuals called "off spring" from the selected parents in order to obtain better sequences in the scheduling problem. After the crossover, the obtained sequences are subject to mutation. The mutation operator helps the algorithm to break out from a local optimum through the process of diversification. Several mutation operators are possible: the inversion of two jobs between two randomly chosen positions, the insertion of a randomly chosen job at a random position, the swap of position of two randomly chosen jobs, and the shift of the selected randomly job to a random position right or left of its initial position. Finally, the replacement consists of updating the population. Ultimately, to finish the GA process, a stopping condition, such as the maximum number of iterations without improvement, the maximum number of generations, and a fixed time should be setup.

5.1. Application of genetic algorithm

We develop a program that generates random permutations of patients ignoring the FCFS rule for larger instances. A permutation of patients is considered a chromosome. In the selection process, an initial population of 200 chromosomes is generated and two random chromosomes based on their fitness are selected so as to be crossed. The crossover process consists of generating a random chromosome of zeros and ones. According to [32], the chromosome takes a value of 1 when there will be a change in the parent position (filled from the second parent), and 0 if it will not be a change (see Figure 1). The method was described to be successful to solve TSP, VRP, and so on.

The two worst generated genes according to fitness are eliminated and the process is repeated 100 times until we obtain a “good population”.

Crossover:

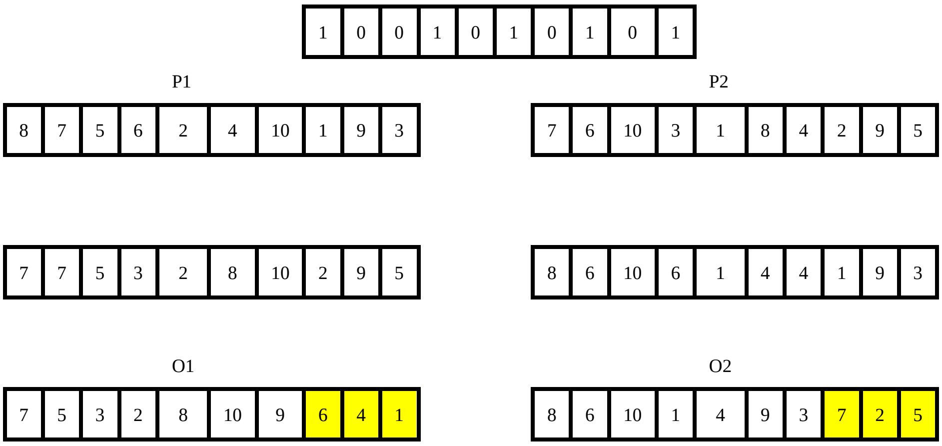

Figure 1: The development of crossover chromosomes

Later the mutation process starts with the creation of an integer random number to indicate from which area the transposition (swapping) of two consecutive patients is made to obtain the new chromosome. In fact, the new chromosome suggests another different schedule on machines and doctors and it gives a different Cmax (maximum time to complete all tasks), total flowtime of all patients and workload variation. We define the fitness of each chromosome as the sum of Cmax, total flowtime and variation of the doctors' workload. The program runs a number of iterations. At each iteration, the location of where the transposition is made is changed, a different chromosome is created and new schedule is evaluated. From all those schedules, we choose the one with the least fitness. This program can give a schedule of up to 100 patients in an acceptable amount of time (see Figure 2).

9

8	11	14	2	6	5	7	3	4	10	12	13	9	1	15

Cmax=
133

TFT=246

8	2	7	10	9
11	6	3	12	1
14	5	4	13	15

Workload of doctors=

1 59

2 58

3 59

4 43

8	5	4	9
11	6	10	1
14	7	12	15
2	3	13	

Mutation

Mutated Chromosome

8	11	14	2	**5**	**6**	7	3	4	10	12	13	9	1	15

Cmax =133

TFT=248

8	2	7	10	9
11	5	3	12	1
14	6	4	13	15

Workload of doctors

1 57

2 60

3 59

4 43

8	5	4	9
11	6	10	1
14	7	12	15
2	3	13	

Figure 2: The development of the mutated schedule

6. RESULTS AND DISCUSSION

In this section, we test small instances for n=5, 6,7,8,9 using mixed integer linear programs and the normalized goal program for randomly generated instances and fixed FCFS sequence. The CPU time required for FCFS permutation of patients to be assigned to machines and doctors is shown in Table 19.

Table 19: Average CPU times in seconds to solve randomly generated problems using programs 1, 2 and 3

n	5	6	7	8	9
Program 1	0.012	0.0574	0.0246	0.0356	0.7186
Program 2	0.015	0.0712	0.07	0.0548	1.655
Program 3	0.0234	0.0416	0.23	0.2506	4.843667
Sum	0.0504	0.1702	0.3246	0.341	7.217267

Table 19 reveals the CPU time required to solve each of the three different optimization problems of the goal program. Notice that it takes 0.0504 seconds to solve a problem of size 5 patients: 0.012 to find the best makespan (Program1), 0.015 seconds to find the best total flowtime (Program2) and 0.0234 seconds to find the compromise solution that tries to be as close as possible to the best makespan, to the best total flowtime and to the best workload variation (Program3). It is also worth noting that beginning from n=9, the solution to the three programs takes a long CPU time. Hence, there is a need for an efficient heuristic or a meta-heuristic to solve these types of problems. For larger instances, we allow time to obtain the results from the genetic algorithm method. The FCFS condition is relaxed and we try to find the best permutation and hence the assignment to machines and doctors. The average CPU time required to solve the problem for randomly generated instances is shown in Table 20.

Table 20: Average CPU time in seconds to obtain solutions for large instances using genetic algorithm

n	10	20	50	100	200	300	400	500
Genetic algorithm	0.45	0.89	3.05	8.36	32.4	64.7	114	184.61

7. CONCLUSION AND PERSPECTIVES

The aim of this paper is to provide a schedule of patients who require laser photocoagulation treatment using special machines and qualified doctors. The number of doctors may exceed the number of machines though they both work in parallel. For this purpose, we formulated mixed integer mathematical models for scheduling patients to machines and doctors in which we incorporated different ready times of patients and availabilities of both machines and doctors. Three programs are presented to give efficient values for the three considered objectives of minimizing the makespan, patients' total flow times and doctors' workloads variations. For a real case of scheduling fifteen patients to at most four doctors and at most three laser machines, we compare the FCFS rule with its variants and with other dispatching rules. The FCFS rule which is fair and commonly used in healthcare systems has shown good performance. This result encourages hospital decision makers to adapt it for small and medium instances. By allowing the variation in the number of machines and the dispatching rules, the good performance of the three-machine model using the FCFS rule is demonstrated. However, when we vary the number of doctors while setting the number of machines to three, we noticed the advantage of using four doctors in the model for the makespan and the total flow time values but not for the workload variation which reached its best value when the number of doctors is equal to the number of machines. A sensitivity analysis has revealed that an addition of one machine (from three to four) slightly improves the average of the flow time and the Cmax, but deteriorates the workload variation value. For larger instances, and due to the complexity of the problem, a genetic type algorithm is constructed to achieve approximate solutions and to be able to deal with the problem in a reasonable CPU time. In a forthcoming work, we can extend our algorithms to solve such parallel machines scheduling problem with additional objectives, particularly minimizing tardiness and earliness while including manager preferences and uncertainty. We propose to design simulation models to find the best configurations of resources that maximize the satisfaction functions related to different healthcare actors. We can also identify other real case applications where machines are used along with other variable resources.

Funding: This research received no external funding.

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