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APPLYING COEFFICIENTS OF PREFERENCE IN RANKING (CPR)*

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Abstract: In marketing or medical research, especially in psychiatrics, it is very often necessary to define preference of examinees against defined object (persons, products, or phenomena). A question that is related to the object of preference is defined as like degree of like (positive preference) or as like degree of dislike (negative preference), where estimation is done as in scholar system (nominal or ordinal characteristics), with marks 1 through 5. Rank of objects achieved is very often expressed as average, which is not a good measure for realistic object ranking.
In this paper, a coefficient of preference is presented as an effort to rank object more efficiently than average or other methods for ranking, especially in the meaning of preference. Preference is essential for humankind for decision making. One of the measures is Coefficients of Preference in Ranking (CPR) as shown.

Keywords: Research, ranking, coefficient, preference.

1. INTRODUCTION

Ranking of specific marks is very often done in a way that its results can cause serious consequences like entering exams, competitions, UN participation, medicine selection and many others.

Ordinary ranking problem is based on:

- $E = \{e_1, e_2, ..., e_n\}$ - a group that has to be ranked,
- $X = (x_1, x_2, ..., x_p)$ - criteria for a group E ranking.
- Group E has to be selected, as a smaller sub-group, based on realization of vector X on the group E.

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There are two linked problems:
1. Ranking,
2. Selection.

If $p = 1$, which means there is just one characteristic (one variable), ranking problem can be solved in a few different ways, but because of the type of research it is necessary to do ranking that will represent the observed problem in the most realistic way. [6, 10]

2. STATISTICAL METHODS IN RANKING

Let's suppose that with one criteria we can group elements of group E. Each element of group A_i formed like that is a sub-group of E.

Suppose that D is a given group of sub-groups of E. If the union of all parts is equal to the group E, D group can be said to be representing one cover of group E. If not all sub-groups of D are full, without any interrelation, and their union is equal to the group E, D is said that to be one division of group E. For parts of one division, it is said they are one class of group E.

Let's assume that the variable has ordered class and ordered elements in each class. To each element two indexes can be given so that the first one presents the rank of the class of elements in the class it belongs to. As an example, e_{ij} would present the j element in i class.

Division with class and elements ordered and marked like that is called classification of group E and is marked with K;

$$K = \langle A_1, A_2, \ldots, A_{k1} \rangle;\ K = \langle \langle e_{11}, e_{12}, \ldots \rangle, \langle e_{21}, e_{22}, \ldots \rangle, \ldots, \langle e_{k1}, e_{k2}, \ldots \rangle \rangle .$$

where k presents the total number of class, and $\langle\ \rangle$ is a mark of some ordered group, e_{ij} is not a divided element of group E, and for A_i as agreed it is taken that it is not a divided element of some particular group of group K.

If $k = n$, each class will contain one element, and classification of K will present one rank-list of elements of group E. [6]

If for the elements of group E the mark X is measured and if they are ordered according to the size of that mark, each element will have its own rank in the order formed like that. If the X value of an element is marked as i in the order according to the size in the group E, that element can be marked as e_i and its value from X to x_i. It is:

$$\forall_i [i \in \{1, \ldots, n-1\} \Rightarrow x_i \geq x_{i+1}] .$$

In this way the classification list can be formed (rank-list, schedule list) for elements of group E related to the value of X. The named process presents ranking. The procedure that is used for the ranking process can be organized in many different ways, depending on the mathematical model that forms the rank list [6].

2.1. Average Values Method

This is one of the most often used methods in ranking in which average value has been taken as a rank. This method is used in the scholar system where the average mark of students is an index for ranking to be done. Average k of that element is reached by the following

$$\overline{X_k} = \frac{1}{n_k} \sum_{i=1}^{n_k} x_i; \quad k = \overline{1, j},$$

and rank based on

$$\overline{X}_j > \ldots > \overline{X}_{k+1} > \overline{X_k} > \overline{X_{k-1}} > \ldots > \overline{X_1}.$$

In some cases, this method is not a good solution as there are large variations, which mean variance not equal for each $\overline{X_k}$, or if the values are discreet. [10]

2.2. Method sum of ranking

A method of sum ranking is based on previous determination of schedule classification of group P elements for each mark of group X.

If rank of P_p element related to X_s characteristic is marked as i_{sp} then the rank matrix can be given as

	X_1	X_2	...	X_k
P_1	i_{11}	i_{21}	...	i_{k1}
P_2	i_{12}	i_{22}	...	i_{k2}
...	...	...	...	...
P_n	i_{1n}	i_{2n}	...	i_{kn}

So that

$$\forall\, s, p, q\ [s \in \{1, \ldots, k\} \text{ and } \{p, q\} \subseteq \{1, \ldots, n\} \Rightarrow i_{s,p} \neq i_{sh} \text{ and } \{i_{s1}, \ldots, i_{s,n}\} \subseteq \{1, \ldots, n\}].$$

By summarizing all ranks of P_j elements, a number is obtained which is considered by economists as the mark of F factor, that means,

$$F_j = \sum_{s=1}^{k} i_{sj}, \quad j \in \{1, \ldots, n\}.$$

This method is easy and simple to understand; still it gives no unique solution, which means that the relation between two elements depends on the third one.

The method that gives no unique solutions and allows it to be misused has no authority and objectivity. Its usage is very dangerous if some important decisions have to be made based on it. [4]

2.3. Bennett's method

Bennett's method is used in the UN for comparison of achieved level of standard of examined group of countries based on measuring of some non-monetary values. By giving an index 100 to the country forehead in one characteristic and related to the values of the index in some other countries, global index of standard in a country is defined by sum of indexes of all other chosen characteristics.

If $X = \{X_1, X_2, ..., X_k\}$ is a group of chosen indexes that gives information about standard of group of countries $P = \{P_1, P_2, ..., P_n\}$, and if x_{ij} is a mark value X_j for country P_i and if

$$x_j^+ = \max_{1 \le i \le n} \{x_{ij}\}, \quad j \in \{1, ..., k\}$$

life standard of the country P_i is defined by Bennett with

$$F_i = \sum_{j=1}^{k} \frac{100}{x_j^+} x_{ij}, \quad j \in \{1, ..., n\}$$

This method gives no unique solution. If one or more basic maximal values exposed to some changes are different, contradictory results can come up even if the changeable values are maximal. [2]

2.4. Cseh-Szombathy's method

This method is an advanced Bennett's method. The group of chosen characteristics can be divided into homogeneous groups and then absolute maximum is defined. [4]

2.5. Niewiarski's Method

Niewiarski has given an advanced Bennett's method. He combined different characteristics with the aim to achieve maximal correlation between different effectively used characteristics and global synthetic characteristics. [4]

2.6. Beckerman's methods

The idea of W. Beckerman was to establish linear regression between the most important criteria and group of the other chosen criteria, so

$$X_0 = a_1 X_1 + a_2 X_2 + \cdots + a_k X_k$$

where X_0 is the most important criteria, $X = \{X_1, ..., X_k\}$ is the group of the other criteria, and $a = \{a_1, ..., a_k\}$ is the adequate group of regression coefficients.

If $S = \{P_1, ..., P_n\}$ is a viewed group of elements and if $n > k$, then regression coefficients can be determined $a_1, ..., a_k$. As none of criteria, not even the most important ones, gives no complete information about elements, Beckerman suggests it to be marked through the other criteria.

Main deficiency of this method is that it is not sensitive to the most important criteria. If two elements have all criteria equal except the most important ones, then they will have the same rank no matter of the advantages of the one that has higher value of the most important criteria. [4]

2.7. Method of the fixed start

This method is based on theoretical minimum of each criterion and the distant of results from it:

$$F_i = \sum_k a_k (x_{ik} - x_k^-)^2 / \sigma_k^2$$

where a_k is a ponder (coefficient of importance) of X_k criteria, x_{ik} is realization of X_k at e_i, x_k - minimal possible value of X_k criteria and σ_k is standard deviation of X_k criteria.

Fixed start method gives good results in ranking if all the criteria are considered adequate and chosen ponder of a_k criteria. [4]

2.8. Methods of factor analysis

Factor analysis method is very important for the problems of ranking of the same group of viewed elements (companies, countries, and regions) for the most different synthetic indicators (factors) based on the same group of chosen characteristics. Using of this method asks for some conditions to be considered. The main one is total so that it does not depend on all elements. [7]

2.9. One-dimensional cluster analysis

One-dimensional cluster analysis is the one used for ranking with methods of cluster analysis at one variable. A problem occurs related to the process of executing cluster analysis, hierarchical and non-hierarchical classification. [1]

2.10. I-distance method

I-distance (Ivanovic distance) enables making of the rank-list of observed units. It is necessary to fix a unit (entity) that will be a referent point on the scale. One fictive unit is often taken as a referent point with minimal values of variables for the observed group. It is the value defined with:

$$x_i^- = \min_{1 \le r \le n} \{x_{ir}\}, \quad i \in \{1,2,\ldots,k\}$$

Formula for I-distance is given:

$$D_r^- = \sum_{i=1}^{k} \frac{x_{ir} - x_i^-}{\sigma_i} \prod_{j=1}^{i-1} (1 - r_{ji.12\ldots j-1})$$

where $r_{ji.12\ldots j-1}$ are partial correlation coefficients.

In this way, the distance for each element of the group can be defined. If all elements (entities) are ordered according to the size of their so estimated I-distances, the rank list will be given. [6]

3. PREFERENCE COEFFICIENT

It is very often difficult to choose the adequate methods for ranking in marketing research as an average mark (or value) is used for ranking of answers to the questions asked. Most often a direct question is asked about the degree of preference of the examinee towards the given object, and the scholar system is used by the marks from 1 to 5 (of 1-min., 5-max.), as it is a system which is most understandable to the one asked. This system is often modified by introducing zero (0) in cases when the examinee does not know or cannot do the ranking, as he has not enough information. These marks, because of great oscillations and subjectivity of those being examined do not give quite a clear view of preferences towards the problem [8]. Preference coefficients defined during the panel research of Laboratory for Statistics, STATLAB, try to give adequate solutions to the problem. [5]

Calculation of preference coefficient is done in two steps:

1. For each viewed object, relative frequencies (%) of defined mark presence (V_i) are calculated, arranged from the smallest to the biggest mark.
2. Calculation is done by the given formulas so the preference coefficient is reached.

Six (6) preference coefficients are defined as it is shown in table 3.1.

Table 3.1: Definition of preference coefficient

Coefficient of maximal preferences CP1	$CP1=\frac{V_n}{V_1}$,
Coefficient of minimal preferences CP1	$CP1'=\frac{V_1}{V_n}=\frac{1}{CP1}$
Coefficient of maximal average ability of preferences CP2	$CP2=\dfrac{\sum_{i=1}^{dev\left\rangle\frac{n-1}{2}\right\langle}\frac{1}{i}V_{n+1-i}}{\sum_{i=1}^{dev\left\rangle\frac{n-1}{2}\right\langle}\frac{1}{i}V_i}$
Coefficient of minimal average ability of preferences CP2	$CP2'=\frac{1}{CP2}$
Adjusted coefficient of maximal/minimal average ability of preferences CP3	$CP3=CP2*\left(1+\dfrac{V_{n-dev\left\rangle\frac{n}{2}\right\langle}+V_{dev\left\rangle\frac{n}{2}\right\langle+1}}{2*n*100}\right)$
Exclusively coefficient of preferences CPI	$CPI=CP3*\left(1-\frac{V_0}{r*100}\right)$

Procedures for calculating are:

- Marks are $i = 1,...,n,$ where $1 < \cdots < n$.
- The relative frequencies of marks are $V_1,...,V_n$ and V_0 is the participation of not given answers.
- *Dev* is a full-number division result, and *n* is a number of possible marks (without 0), while r as the exclusive coefficient of preferences is the negative degree of loading of answers that are not given of viewed object from the one being asked for.

In summary CPR presents relations of maximal versus minimal preferences of examinees compared to the observed phenomena. Preference, according to some phenomena very often presents the critical factor for customers to make decisions. [3]

An example for calculating CP and coefficients are given in the table 3.2.

Table 3.2: Example for calculating CP

Marks	Frequency	Valid Percent
0	26	8.75
1	4	1.35
2	8	2.69
3	32	10.77
4	80	26.94
5	147	49.49
Total	297	100.00
Average	3.94	

n (no. of marks)=5

CP1	CP1'	CP3
36.75	**0.03**	**23.50**
CP2	CP2'	CPI
23.38	**0.04**	**21.44**

n=5; *r=1

CP1=49.49/1.35

CP2=(1*49.49+0.5*26.94)/(1*1.35+0.5*2.69)

CP3=CP2*(1+((10.77+10.77)/(2*5*100)))

CPI=CP3*(1-(8.75/(*1*100)))

4. CASE STUDY

4.1. Marketing study

For choosing a person who would do advertising for Procter&Gamble products, research is made in SM&MR Institute. Those who have been asked had to answer the question which person would make them buy mentioned products.

The question was: "**Please, give mark as in the scholar system to the persons that would make you buy products if you were to do the advertising for** (*1-Negative, 5-Positive, 0-Can't give any mark*)". Based on achieved results and the procedure, the preference coefficient values of coefficients are given in a table just as ranks in the table 4.1.

Table 4.1: Preference coefficient values and rank achieved

Coefficients of Preference						*r=1		Rank				
CP1	CP1'	CP2	CP2'	CP3	*CPI**	AVERAGE	Person	CP1R	CP2R	CP3R	CPIR	AVGR
110.00	0.01	7.50	0.13	7.55	**5.87**	3.30	Dragan Jovanovic	1	1	1	**1**	4
36.75	0.03	6.18	0.16	6.21	**5.66**	3.94	Branka Katic	3	2	2	**2**	1
42.00	0.02	6.04	0.17	6.08	**5.40**	3.93	Anica Dobra	2	3	3	**3**	2
9.54	0.10	2.07	0.48	2.10	**1.85**	3.59	Predrag Mijatovic	7	4	4	**4**	3
10.00	0.10	1.67	0.60	1.69	**1.33**	3.09	Ana Sofrenovic	6	6	6	**5**	7
11.80	0.08	2.06	0.48	2.08	**1.30**	2.37	Igor Milanovic	4	5	5	**6**	12
10.67	0.09	1.53	0.65	1.56	**1.22**	3.04	Dubravka Mijatovic	5	7	7	**7**	8
7.00	0.14	1.34	0.74	1.37	**1.05**	2.77	Jasna Sekaric	9	8	8	**8**	10
4.47	0.22	1.10	0.91	1.12	**0.98**	3.22	Stefan Milenkovic	10	10	10	**9**	5
2.91	0.34	0.68	1.48	0.69	**0.61**	3.13	Ivana Bojic	12	12	12	**10**	6
2.06	0.48	0.65	1.54	0.66	**0.55**	2.83	Dejan Tomasevic	13	13	13	**11**	9
7.33	0.14	1.16	0.87	1.16	**0.38**	1.16	Snezana Dakic	8	9	9	**12**	16
1.35	0.74	0.43	2.35	0.44	**0.24**	1.76	Bojana Maljevic	14	14	14	**13**	14
0.94	1.07	0.25	4.01	0.26	**0.22**	2.43	Sestre K2	16	16	16	**14**	11
3.33	0.30	1.00	1.00	1.00	**0.19**	0.64	Tamara Paunovic	11	11	11	**15**	18
0.57	1.77	0.21	4.71	0.22	**0.17**	2.16	Leontina V.	17	17	17	**16**	13
1.00	1.00	0.39	2.56	0.40	**0.14**	1.08	Marija Macic	15	15	15	**17**	17
0.21	4.79	0.07	14.88	0.07	**0.04**	1.43	Dejan Pantelic	18	18	18	**18**	15

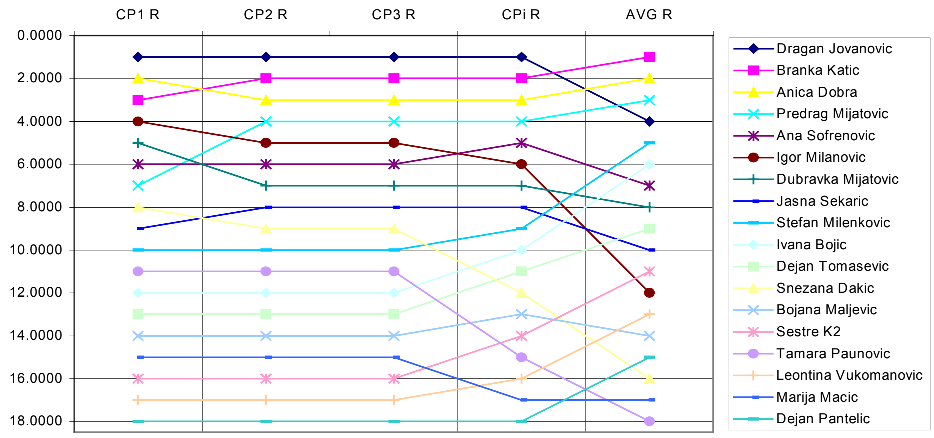

Figure 4.1: Persons' rank based on the preference coefficient (CPx R) and average (AVG R)

At the Figure 4.1 the values of persons are given based on the rank of Coefficient of preference and fluctuation from average. If the value of coefficient of preference is bigger than one (1) CP>1, it means the phenomenon is much preferred (much like than unlike). Vice versa, if CP<1, it means the phenomenon is not preferred (much unlike than like).

4.2. CPR of medications efficacy - medical research

Considering the pain intensity in different time points through the four categories (0-none, without pain; 1-mild; 2-moderate; 3-severe; 4-very severe), we can obtain coefficient of preference in ranking, which gives us the range of quality of analyzed analgesic agents.

Table 4.3 shows the results of coefficient of preference in ranking (Adjusted coefficient of maximal average ability of preference - CP3, rank of CP3 - RCP) for different analgesic agents in time points: 30, 60, 120, 180 and 240 minutes. (Diagram 4.2.) Stability of action of COM150 was demonstrated through this analysis, while other agents showed changeable results in different time points, which impaired its overall efficacy. That is represented most efficiently through SRCPO. (Table 4.2.)

Results obtained in this manner, enable insight in the degree of range in time points with a follow-up presented on Figure 4.2. To achieve the overall ranging, the procedure of summarizing of range values has to be conducted:

$$SRCP = \sum_t RCP$$

where the lowest value of sum of ranges (SRCP) represents the best medication in terms of efficacy regarding the pain evaluation, followed by other medications respectively. The result of overall sum of ranges is given in Table 4.2.

Table 4.2: Overall ranging

Time point Rank	Pain After 30'	Pain After 60'	Pain After 120'	Pain After 180'	Pain After 240'	SRCP	RANK
COM150	2	2	3	2	2	11	1
COM250	4	1	2	1	6	14	2
PROPAR	1	3	4	4	3	15	3
ASA	7	6	1	3	1	18	4
IBU	3	4	5	5	5	22	5
PAR	5	5	6	6	4	26	6
PLA	7	7	7	7	7	35	7

According to this table, it can be concluded that COM150 demonstrates 27.27% better range comparing to COM250, while the COM250 achieves 7.14% better range comparing to PROPAR, etc.

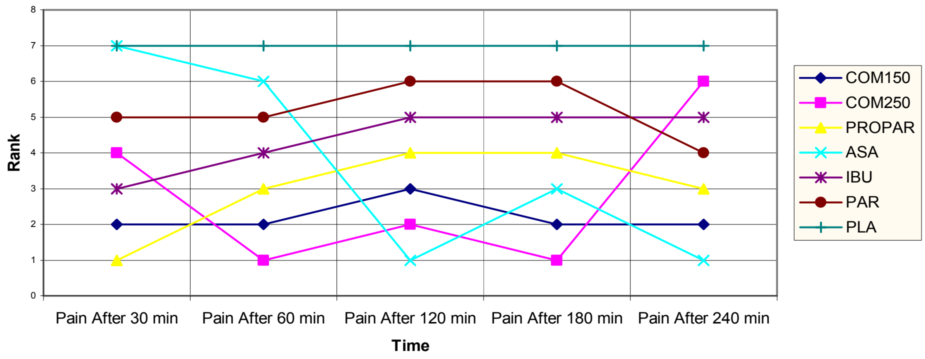

Figure 4.2: Rank of medication in time (pain after minutes) based on CP3

Table 4.3: Coefficient of preference in ranging regarding to time points

Drugs	PA	CP3	RANK
PROPAR	Pain After 30 min	2.86	1
COM150	Pain After 30 min	1.04	2
IBU	Pain After 30 min	0.68	3
COM250	Pain After 30 min	0.44	4
PAR	Pain After 30 min	0.34	5
PLA	Pain After 30 min	0.24	6
ASA	Pain After 30 min	0.00	7
COM250	Pain After 60 min	7.63	1
COM150	Pain After 60 min	3.69	2
PROPAR	Pain After 60 min	3.42	3
IBU	Pain After 60 min	1.76	4
PAR	Pain After 60 min	0.75	5
ASA	Pain After 60 min	0.61	6
PLA	Pain After 60 min	0.25	7
ASA	Pain After 120 min	7.42	1
COM250	Pain After 120 min	4.88	2
COM150	Pain After 120 min	3.77	3
PROPAR	Pain After 120 min	2.34	4
IBU	Pain After 120 min	2.10	5
PAR	Pain After 120 min	1.03	6
PLA	Pain After 120 min	0.20	7

Table 4.3 (Cont.): Coefficient of preference in ranging regarding to time points

Drugs	PA	CP3	RANK
COM250	Pain After 180 min	5.01	1
COM150	Pain After 180 min	2.49	2
ASA	Pain After 180 min	2.19	3
PROPAR	Pain After 180 min	1.35	4
IBU	Pain After 180 min	0.78	5
PAR	Pain After 180 min	0.66	6
PLA	Pain After 180 min	0.10	7
ASA	Pain After 240 min	3.60	1
COM150	Pain After 240 min	1.18	2
PROPAR	Pain After 240 min	0.63	3
PAR	Pain After 240 min	0.60	4
IBU	Pain After 240 min	0.36	5
COM250	Pain After 240 min	0.31	6
PLA	Pain After 240 min	0.10	7

5. CONCLUSION

In marketing research or in social sciences, giving marks to examinees as a establish system to appraise is an obstacle in some occasions. Preference coefficients (CP) are not the final and the best solution for ranking in these cases but offer high level of reality in problem consideration. By now, they have been used in many marketing, political and medical research and have given good results in situation examination.

Coefficient of preference in ranking (CPR) represents the relationship between the categories of answers with categorical or ordinal character. In this study, all relations between appearance of certain answer categories were taken into account with the purpose of achieving unique value i.e. coefficient, which shows preference regarding the observed phenomenon.

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